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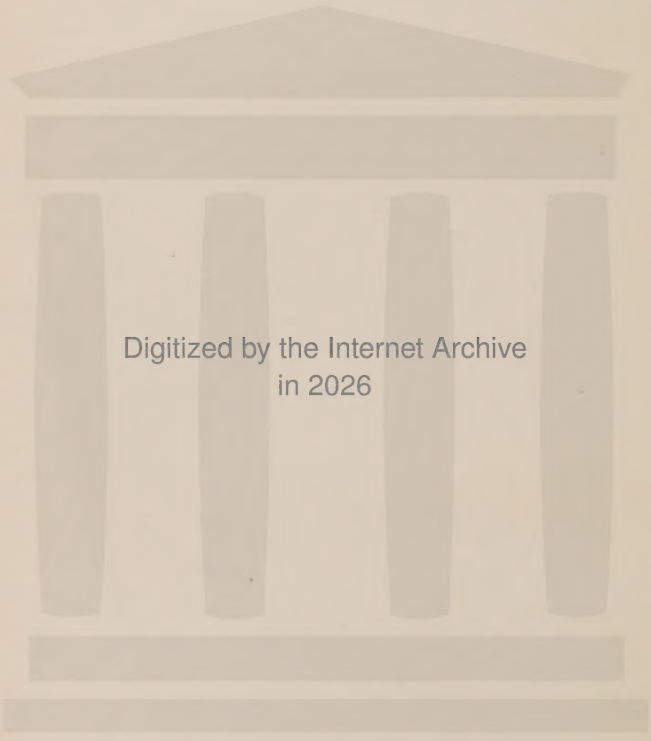

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AN INTRODUCTION

TO

LOGIC

BY

H. W. B. JOSEPH

FELLOW AND TUTOR OF NEW COLLEGE

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PREFACE TO THE SECOND EDITION

The present edition, the preparation of which has been somewhat delayed by reason of the war, has been carefully revised throughout, and is enlarged to the extent of some 40 pages. Though it does not differ in its main teaching from the first edition, there are very few pages without some slight alteration, if only for the sake of greater clearness, or of more consistency in phraseology. I hope the alterations are improvements; I must confess to some dissatisfaction at finding how many I desired to make. The following list of some of the principal alterations and additions may be of use to any reader who is interested in a comparison.

c. i. On p. 5, n. 1 (upon the distinction between the form and matter of thought), I have endeavoured to show that the matter of thought, in this antithesis, is not the same as its 'subject-matter'; and I have altered the language of many passages accordingly. The latter part of p. 10, n. 1, and the last page and a half of the chapter, are new.

c. ii. Much of this chapter has been rewritten. In particular, the general discussion of the nature of a term on pp. 14–28, and that of the distinction between abstract and concrete terms, on pp. 32–35, are largely new. The former includes some discussion of concepts; and both have involved small consequential alterations at many subsequent points.

c. iii. The note on p. 52, on the position of 'first substances' in Aristotle's doctrine of categories, dissents from p. 39, n. 1, of the first edition.

c. v. In this chapter I have laid more stress on the makeshift character of most classification, and have done more justice to the use of negative differentiae.

c. vi. This chapter has been largely rearranged and rewritten. I now prefer, in order to express the truth in the contention

of those who have denied connotation to proper names, to distinguish between intension and connotation, allowing them the former, but not the latter.

c. viii. This chapter also has been largely rewritten, particularly in the discussion of the modality of judgements, and of the distinction between analytic and synthetic judgements. The close print on pp. 192–195, 201–205, 214–215, is new matter. In this and subsequent chapters I think that much which in the first edition was said about judgements should properly have been said about propositions, and correction has been made accordingly. I have made use in certain places of expressions borrowed from Prof. Cook Wilson.

c. ix. On p. 228, and also on p. 120, n. 1, I have inserted a few remarks on Symbolic Logic, which will in some measure explain why the book does not deal more fully with that subject.

c. x. The discussion of the inferential character of so-called Immediate Inference (pp. 240–247) has been enlarged and recast ; pp. 241–242 are new matter.

c. xii. The discussion of the Fourth Figure of Syllogism, pp. 280–284, has been largely rewritten.

c. xiii. In the note beginning on p. 296, the discussion of the passage *Cat.* iii. 1^b 10–15 (pp. 298–299) has been emended.

c. xiv. In pp. 310–311 I have emphasized the subsumptive and therefore inferior character of syllogistic thinking. The close-print discussion on pp. 331–334 is new.

c. xviii. The attempt in the last three pages, 397–399, to characterize the difference between inductive and deductive reasoning is new.

c. xix. The new matter in this chapter, which has also been considerably rewritten, is chiefly in pp. 403–404, 410–413, 418–419, 421, n. 1.

c. xx. I have corrected language which spoke of an event causally connected with another as its cause, both here and subsequently ; the point is discussed on pp. 426–428.

c. xxi. In pp. 475–476 I have dwelt on one or two further matters belonging to the pursuit of the inductive sciences which are no part of their reasoning.

c. xxii. A rather obscure passage in the previous edition (pp. 459–460) is replaced by pp. 495–496.

c. xxiii. I have tried to improve the statement of what explanation is on pp. 502, 521–522; and p. 523, n. 1, is new. The close print on pp. 524–527 is an answer to a criticism which Dr. Bosanquet has made upon the view of induction taken in this book.

c. xxv. I have on pp. 546–549 rewritten and to some extent altered the tenour of what was said before (on pp. 506–509 of the first edition) concerning the foundation of our power to generalize in mathematics.

c. xxvii. Some additional matter is contained in the first notes on pp. 572, 580, 582, 589, 595.

I have taken account of such criticisms as I have seen in print, though I have not thought all equally well founded; for these, and also for various criticisms privately communicated, I desire to express my thanks. I should like here to name again Prof. J. A. Smith, Mr. H. H. Joachim, and Mr. H. A. Prichard, who were all good enough to send me comments on divers points. Besides these, Prof. W. G. de Burgh, of University College, Reading, very kindly helped me with a list of criticisms and suggestions based on his use of the book with students; and Miss Augusta Klein sent me a series of most careful notes upon the first eight chapters. These were particularly helpful upon points of science referred to by way of illustration, and upon the theory of classification, with the logical doctrines on which it rests; and the principal changes which I have introduced on those topics are due to her criticism, though not involving a full acceptance of it.

But chiefly here I desire to put on record the debt which I owe, in common with so many other of his older or younger pupils, to Prof. J. Cook Wilson, whose death occurred while these sheets were passing through the press. Various footnotes will show

the use that I have made of his unpublished teaching; but his illness prevented me from submitting to him what I have written, and his authority must be made responsible for no errors that I may have made. His few and scattered publications can do little to convey to strangers the power and stimulus of his personal teaching. And there are subjects on which, by his combination of scholarly and mathematical with philosophic insight, he was qualified as few have been to produce new work of real value. The hope has vanished that he might put in permanent form the full results of his thinking. But those who knew him well will not misjudge this failure. For they will remember him as not more patient and eager in philosophic reflection than in his devotion during many years to a suffering wife and in his endurance afterwards of his own wasting and fatal illness.

H. W. B. J.

Advantage has been taken of reprinting to correct a number of small errors, mostly typographical, to which my attention has been called by the kindness of readers.

H. W. B. J.

January, 1925.

PREFACE TO THE FIRST EDITION

If an apology that precedes it could mitigate an offence, I should be inclined to convert my preface into an apology for publishing this book. Progress, and the hope of progress, in logical investigations, have lain perhaps during the last three generations chiefly in two directions, either of analysing more closely the processes of thought exhibited in the sciences, or of determining what knowledge is, and the relation of the knowing mind to what it knows. Though I have been compelled to deal in some degree with the first of these questions, I am well aware that it demands a scientific knowledge which I do not possess ; the second I have not attempted systematically to discuss. The aim of the following book is more modest. There is a body of what might be called traditional doctrine in Logic, which is not only in fact used by itself as an instrument of intellectual discipline, but ought also to be in some degree mastered by those who would proceed to the higher and abstruser problems. It is of this traditional doctrine that Benjamin Jowett is recorded to have said, that Logic is neither a science, nor an art, but a dodge. I could perhaps best describe the motive with which this work was begun, as the desire to expound the traditional Logic in a way that did not deserve this accusation. The accusation was doubtless provoked by the attempt to force into a limited number of forms processes of thought, many of which can only with pretence and violence be made to fit them : an attempt, it may be added, at least as characteristic of ' Inductive Logic ' as of any other.

In the course of centuries, the tradition has become divergent, and often corrupt. In this difficulty, I have ventured, like one or two other modern writers, to go back largely to its source in Aristotle. Problems of thought cannot in any case be studied without careful regard to their terminology, and their terminology

cannot be understood without reference to its history. The terminology of Logic owes more to Aristotle than to any one else; but there is this further reason for attention to what he said, that much prevalent falsehood or confusion in the tradition is a corruption of truths expressed by him. At the same time, I have not pretended to believe in the verbal inspiration of his writings.

I have in particular been anxious to teach nothing to beginners which they should afterwards have merely to unlearn. They may of course come to dissent from the positions here taken up; but only, I hope, because they think I have the worst of the argument on a proper issue, and not because, as meat for babes, I have been dogmatically expounding acknowledged fictions.

While dealing largely with the more technical parts of logical tradition and terminology, I have done my best to avoid a superfluity of technical terms; and the subjects discussed have been for the most part discussed in detail, and the principles involved in them debated. The dryness with which the more formal branches of Logic are often charged springs, I think, in part from their being presented in too cut-and-dried a manner; those who go beyond the jejune outline, and get into an argument, often find the subject then first begin to grow interesting. At any rate I have tried to secure this result by greater fullness, and attention to controversial issues. In every study there must be something to learn by heart; but Logic should appeal as far as possible to the reason, and not to the memory. Thus such a question as the 'reduction' of syllogisms has been dealt with at length, not from any wish to overrate the importance of syllogistic reasoning, or burden the student with needless antiquarianism, but because the only thing of any real value in the subject of reduction is just that investigation of the nature of our processes of thinking which is involved in asking whether there is any justification for reducing all syllogisms to the first figure.

Topics whose main interest is obviously historical or antiquarian have been either relegated to footnotes or placed in closer type and between brackets; and as I have followed the advice to translate what Greek I quote, I do not think that there is anything in these

discussions which a reader need be altogether precluded from following by ignorance of that language. I have also put between brackets in closer type other passages which, for one reason or another, might be omitted without spoiling the argument ; among the matters so treated is the fourth figure of syllogism ; for I have reverted to the Aristotelian doctrine of three figures, with the moods of the fourth as indirect moods of the first.

I hope that I have sufficiently acknowledged all detailed obligations to previous writers in the places where they occur. But I owe here a more comprehensive acknowledgement both to the published work of Sigwart, Lotze, Mr. F. H. Bradley, and Professor Bosanquet, and to the instruction received in private discussion with various friends. Among these I should like to mention in particular Mr. J. Cook Wilson, Fellow of New College, Wykeham Professor of Logic in the University of Oxford, whose reluctance to write is a source to many of serious disappointment and concern ; Mr. J. A. Smith, Fellow of Balliol College ; Mr. C. C. J. Webb, Fellow of Magdalen College ; Mr. H. H. Joachim, Fellow of Merton College ; and Mr. H. A. Prichard, Fellow of Trinity College, Oxford. To the last three of these, and also to Mr. C. Cannan, Secretary to the Delegates of the University Press, I am further indebted for the great kindness with which they read large portions of the work in MS. or in proof ; without their suggestions and corrections it would be even more imperfect than it is. Lastly, I have to thank my sister, Miss J. M. Joseph, for the help she gave me in reading the whole of the proof-sheets and in undertaking the laborious and ungrateful task of checking the index.

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CHAPTER I

OF THE GENERAL CHARACTER OF THE ENQUIRY

It is a common practice to begin a treatise on any science with a discussion of its definition. By this means the reader's attention is directed to the proper objects, and to those features of them, with which the science is concerned; a real advantage, when, as with Logic, those objects are not apprehended through the senses, and for this reason ordinarily attract little notice. But the same reason which makes a definition of Logic at the outset useful, makes controversy about its definition comparatively useless at such an early stage. The reader is too unfamiliar with the subject-matter of his science to be able to judge what definition best indicates its nature; he cannot expect thoroughly to understand the definition that is given, until he has become familiar with that which is defined. The definition will at first guide more than enlighten him; but if, as he proceeds, he finds that it helps to shew unity in the different enquiries upon which he successively enters, it will so far be justified.

Logic is a science, in the sense that it seeks to know the principles of some subject which it studies. The different sciences differ in the subjects which they so study; astronomy studies the nature, movements, and history of the heavenly bodies, botany the structure, growth, history, and habits of plants, geometry the properties and relations of lines, surfaces, and figures in space; but each attempts to discover the *principles* underlying the subjects with which it has to deal, and to explain their great variety by the help of one set of principles. These principles are often spoken of as laws; and in the physical sciences that deal with change, as 'laws of nature'. The phrase may suggest that 'nature' is not the sum of things and of events in the physical universe, but a sort of power prescribing to these the rules which they are to follow in their behaviour; as the King in Parliament prescribes rules of conduct to his people. That, however, is not what we have to understand in science by a 'law'; a law in science is not, like human laws, a rule enjoined but sometimes disregarded; it is a principle illustrated—and existing

only in the necessity of its being illustrated—in the department of fact to which it belongs. There are therefore no breaches of scientific law, or of a law of nature [1]; if events are observed which do not conform to what we have hitherto called a law, we conclude not that the law is broken, but that we were ignorant of the true law; if water, for example, were observed to boil on the top of Mont Blanc at a lower temperature than 212° Fahr., we should infer not that the law that water boils at 212° Fahr. was broken but that it is not a law of nature that water boils at 212° Fahr.,—that there are other conditions which have to be fulfilled, if water is to boil at that temperature; and the 'law' is that it should boil only when those conditions are fulfilled. Such laws, the general principles to which things in their properties and their behaviour do actually conform, are what the physical sciences seek to discover, each in its own department, and if Logic is a science, it must have a subject of its own, in which it seeks for principles and laws.

That subject is thought, but thought is always thought about something; and thinking cannot be studied in abstraction from anything thought about. But yet in the same way that we may study the laws of motion, as they are exemplified in the movement of all bodies, without studying all the bodies that ever move, so we may study the laws of thought, as they are exemplified in thinking about all subjects, without studying all the subjects that are ever thought of. This comparison may be pushed further. Just as we must have experience of moving bodies, before we can investigate the laws of their motion, so we must have experience of thinking about things, before we can investigate the principles of thinking; only this means, in the case of thinking, that we must *ourselves* think about things first, for no one can have experience of thinking except in his own mind. Again, although, in studying the laws of motion, we do not study every body that moves, yet we must always have before our minds some body, which we take as representing all possible bodies like it; and in the same way, when we investigate the principles that regulate our thinking, though we do not need to study all things ever thought of, we must have before our minds something thought of, in order to realize in it how we think about it and all possible things like it. For example, it is a general principle of our thought, that we do not conceive of qualities

[1] The question of the possibility of a 'miraculous' breach of natural law need not be considered here; something is said of it in c. xix, *infra*, pp. 417–421.

except as existing in some substance; and that nevertheless the same quality is thought to exist in many substances; green is a quality, which exists not by itself, but in grass and leaves of trees and so forth; at the same time, green may exist in many different leaves or blades of grass. The general principle which is thus illustrated in the quality green is readily understood to be true of all possible qualities; but unless we were able to think of some particular quality to illustrate it, we could not understand the general principle at all.

What has been now said will serve to remove an objection which Locke brought against the study of Logic. 'God', says Locke[1], 'has not been so sparing to men, to make them barely two-legged creatures, and left it to Aristotle to make them rational.' He is urging that men thought rationally, or logically, i.e. in accordance with the principles that Logic discovers to regulate all sound thought, long before those principles were recognized; and that this is still so with each of us; we do not therefore need Logic to teach us how to think. That is quite true, and would be a pertinent criticism against any one who pretended that no one could think rationally without studying Logic; but it is not the business of Logic to make men rational, but rather to teach them in what their being rational consists. And this they could never learn, if they were not rational first; just as a man could never study (say) the principles of voluntary motion, if he was not first accustomed to move his limbs as he willed. Had God made men barely two-legged creatures, Aristotle would in vain have taught them to be rational, for they would not have understood his teaching.

Logic, then, is the science which studies the general principles in accordance with which we think about things, whatever things they may be; and so it presupposes that we have thought about things. Now our thought about them is expressed partly in the daily conversation of life or musings of our minds; partly and most systematically in the various sciences. Those sciences are the best examples of human thinking about things, the most careful, clear, and coherent, that exist. In them, therefore, the logician can best study the laws of men's thinking; and it is in this sense that we may accept the old definition of Logic, *scientia scientiarum*.[2] What 'the courses of the stars' are to astronomy, what figures, lines, and

[1] *Essay*, Bk. IV. c. xvii. § 4.
[2] Joannes Philoponus cites it *ad* Ar. *Anal. Post. a.* ix. 76[a] 15.

surfaces are to geometry, what plants are to botany, or the calendar of Newgate to the criminologist, that the other sciences are to the logician: they are the material which he has to investigate, the particular facts which are given him, in order that he may discover the principles displayed in them. He has to ask what knowledge is as knowledge, apart—so far as possible—from the question, what it is about; and he must therefore examine divers 'knowledges', and see in what they are alike; and among the best pieces of knowledge that exist, the best 'knowledges', are the various sciences. But he is not concerned with the detail of any particular science; only with those *kinds* (or *forms*) of thinking which are exemplified in all our thinkings—though not necessarily the same in all—but best exemplified in the sciences.

It is important to understand what is meant by saying that Logic is concerned with *forms* of thinking; for many logicians who have laid stress on this, and pointed out that Logic is a formal science, have understood by that expression more than seems to be true. There is a sense in which Logic is undoubtedly formal. By *form* we mean what is the same in many individuals called *materially* different—the device, for example, on different coins struck from the same die, or the anatomical structure of different mammalian vertebrates, or the identical mode in which the law requires the different Colleges of the University of Oxford to publish their accounts. And all science is formal, in the sense that it deals with what is common to different instances. A scientific man has no interest in a specimen exactly similar to one which he has already examined; he wants new types, or fresh details, but the mere multiplication of specimens all alike does not affect him.[1] So the logician studies the forms of thinking, such as that involved in referring a quality to a substance possessing it; but when he has once grasped the nature of this act of thought, he is quite uninterested in the thousand different such acts which he performs during the day: they differ only materially, as to what quality is referred to what substance; formally, so far as the notion of a quality as existing in a substance is concerned, they are the same; and the forms that run through all our thinking about different matters are what he studies.

But those who have insisted most that Logic is a formal science, or the science of the formal laws of thought, have not merely

[1] Unless indeed he is collecting statistics as to the comparative frequency of different types.

meant that Logic is in this like other sciences, which all deal with what is formal or universal in their subject-matter.[1] They have meant to exclude from Logic any consideration of forms or modes of thinking which are not alike exemplified in thinking about absolutely every subject. It is as if the botanist were to regard only those laws which are exemplified in every plant, or the geometer were to consider no properties of figures, except what are common to all figures. They have thought that one might abstract entirely from and disregard all question as to what he thinks about, and still find that there are certain principles in accordance with which, if he is to think about anything, he will think. But the truth is, that we think in different ways about subjects of different kinds, and therefore we must, if we wish to study the principles that pervade our thinking, consider to some extent the differences in our thought arising from differences in that about which we think. The distinction between form and matter may as it were be taken at different levels. This is plain in a science that deals with some order of sensible things, like zoology. We may say of all men and all horses that they have severally a common form, that as compared to a man a horse is formally different, but as compared to one another all horses are formally the same, though each horse in his body is materially different from every other. Or we may consider not the form of horse common to Black Bess

[1] It is important to realize that the subject-matter of Logic is our thinking about divers things, not the things which that thinking is about; they are the subject-matter of that thinking. Just as the form and matter of a coin are both in the coin, so the form and matter of thought (if we are to keep the meaning of the antithesis) must be both in the thought; we must not suppose that the formal identity is in the thought, the material differences in the things thought about. An analogy may help to make this point clearer, on which I confess that there was some confusion in the first edition of this book. Hunger and thirst are formally the same as being both appetites, materially different as being the one for food, the other for drink; but the material difference is not the difference between food and drink, nor the matter of the appetites food and drink respectively; the matter is rather the special character which the appetites have through being *for* these objects. So the matter of a thought is the special character which it has through being *about* a certain subject, not the subject which it is about. It may be added, as a caution to the unwary, that the antithesis of form and matter is used in various ways by no means all of them analogous; and its application to thought is not really the same as its application to coins or animals. What is different in particular thoughts is not related to their common form as the gold or silver of two coins to their common device, or the flesh and bones of two animals to their common structure, but rather as the specialty of their structures to the generic identity, or as particular instances to the common nature of which they are instances. Cf. *infra*, pp. 75–77.

and Bucephalus and Rosinante, but the form of vertebrate common to man, horse, eagle, crocodile, &c.; and now man and horse (as compared with oysters for example) are formally alike. Or we may take the four orders in Cuvier's now obsolete division of the animal kingdom, vertebrata, coelenterata, radiata, and annulosa, and regard them as only different examples of the common form of animal; and from this point of view a horse and an oyster differ materially, but not formally. When however we have reached this stage, and achieved the conception of animal, as something exemplified equally in kinds of animal so different, it is clear that we can only understand what animal structure is by seeing it as it exists in all the different orders of animals; whereas we can understand fairly the structure of a mammalian vertebrate without seeing it as it exists in every genus of mammals; still more can we understand the structure of a horse without familiarity with all horses. The higher the level therefore at which in Zoology the distinction between form and matter is taken, the less can we study the form in isolation from variety of matter; no example taken from *one* order of animals, say the starfish, will enable us to realize what animality is. It is the same in studying the forms of thought. The most general forms of thought exist diversely modified in thinking about different subjects; and they can no more be fully known without attending to the different matters in which they appear differently, than animal nature can be fully known without attending to the different orders of animal in which it appears differently. Thus we may take the Proposition, and point out that in every affirmative categorical proposition there is a subject about which something is said, and a predicate, or something which is said about it. This is true equally of the propositions, 'A horse is an animal,' 'First-class railway tickets are white,' and 'Londres is London'. We may if we like, because in all propositions there is formally the same distinction of subject and predicate, take symbols which shall stand for subject and predicate, whatever they are, and say that all affirmative categorical propositions are of the form '*S* is *P*'. But when we ask for the meaning of this form, and in what sense *S* is *P*, it is clear that the meaning varies in different propositions.[1] Londres is just the same as London; but a horse is not just the same as an animal; it may be said that 'animal' is an attribute of horse, and

[1] Professor Cook Wilson has called attention in his lectures to the dangerous ambiguity of this symbolization. Cf. *infra*, pp. 22–24.

'white' of first-class railway tickets, but animal is an attribute belonging to horses in quite a different way from that in which white belongs to first-class railway tickets; these might as well be of any other colour, and still entitle the holder to travel first-class by the railway; a horse could not cease to be an animal and still continue to be a horse.[1] The meaning of the formula '*S* is *P*' cannot possibly be fully known merely by understanding that *S* and *P* are some subject and predicate; it is necessary to understand what kind of subject and predicate they are, what the relation is between them, and in what sense one is the other; and if this sense is different in different cases, just as animal is something different in a dog and a starfish, then the thorough study of the *form* of thought involves the consideration of *material* differences in the thoughts also. But logicians who emphasize the purely formal character of Logic maintain that it can exhaust the form of thought in treating that as one and the same in every possible instance of thinking; an impracticable task, because the form itself (as in the above example of a form of thought which we call a proposition) is modified according to the instance in which it appears. On the other hand, and even although the forms of our thought cannot be studied apart from the differences connected with the particular sort of subject about which we may think, yet Logic is not interested in these differences for their own sake, but only for the sake of the divers forms of thinking involved in them; and so far as the same form is exemplified over and over again in different particular 'bits' of thinking, the study of the common form alone belongs to Logic.

[The truth that form cannot be studied apart from matter might be otherwise expressed by saying, that the general form can only be studied in one or other of the special forms in which it is manifested; and these special forms can only be illustrated in examples that are materially different from one another. The proposition 'Londres is London' is a special form of proposition equally well exemplified in 'Köln is Cologne'; as Bucephalus is an animal of a special form equally well exemplified in Black Bess. What is important to realize is the need of following the common form out into the differences which it displays in different matter.]

The foregoing discussion will probably become plainer if it be read again at a later stage, when the reader is more practised in reflecting on his thoughts. A distinction which is readily seen in

[1] In strictness, the generic nature of a subject should not be called an attribute of it. Cf. *infra*, pp. 82–83.

regard to material subjects, like animals or plants, is not so easily seen in immaterial subjects, like our thoughts. The natural man thinks much about things, and asks and answers questions about them; but it is by an effort that he comes to see how these things are only known to him in his perceptions of them and his thoughts about them, and so comes to turn his attention inward upon the nature of the acts of perceiving or of thinking. Nor can these new objects of his study be preserved and dissected like a material thing; a man cannot catch a thought and bottle it; he must create it by thinking it, if he wishes to think about it; and the task will be found difficult while it is strange, and not altogether easy when familiar.

[Mediaeval logicians sometimes say that Logic deals with *second intentions*; by this is meant what has been pointed out in the last paragraph. The mind intends or directs itself at first upon material things and their qualities or relations; and these are its first intentions; it may afterwards intend or direct itself upon its own modes of thinking as exhibited in its first intentions; and what it then discovers are its second intentions. Thus we observe animals, and give them names according to their kind, calling them stag and ox, worm and lobster; and again we observe how these kinds agree and differ, and call some vertebrate, and some invertebrate, but all animals; and all these names, which are names we give to objects, are names of the first intention. But we may also observe how we have been thinking about these animals, as having some properties common to all, and some peculiar to the members of each kind; and we may call the members of each kind, or their common nature, a species, and the members of the several kinds together, or the wider common nature, a genus; and genus and species are names of the second intention. The unity on the strength of which we call them of one species or of one genus will indeed be something in the animals themselves; and so our names of second intention will in this instance signify something real in things. The distinction therefore presents difficulties which call our attention to the fact, that we cannot altogether keep reflection upon thought apart from reflection upon the nature of things thought about.]

If now we ask for a definition of Logic, to keep before our minds in the following chapters, perhaps it is simplest and least objectionable to call it the Science, or the Study, of Thought; for to say of the Formal Principles of Thought might imply both that there were sciences which did not seek for principles, and that the form of thought can be studied without reference to differences in the matter of it; neither of which things is true.

It is sometimes held that Logic is rather an art than a science, or at any rate that it is an art as well. In considering this question, we must remember that there are two senses of the word art. We may say that a man understands the art of navigation when he is skilful in handling a ship, though he may be unable to explain the principles which he follows; or we may say that he understands it, when he is familiar with the principles of navigation, as a piece of book-work, though he may never have navigated a ship. Thus an art may either mean practical skill in doing a thing, or theoretical knowledge of the way in which it is best done. In the latter sense, art presupposes science; the rules of navigation are based upon a knowledge of astronomical, mechanical, meteorological and physical laws, and presuppose much knowledge of mathematical and other sciences. It is in this sense that Logic is called an art; and hence it is clear that if there is an art of Logic, there must first be a science, for the study of the nature of sound thinking must precede the giving of instructions for thinking soundly. And even granting the existence of such an art, it remains distinct from the science; so that the name Logic would be used of the two in different senses, and we ought rather to say that Logic means the science or the art of thought, than that it is the science and the art thereof. That there is an art of Logic, based on the science of Logic, might be urged on the ground that Logic reveals to us what knowledge about any subject really is, and certain canons of reasoning which no argument can violate and be sound. But more than this would be required in order to constitute an art. There should be rules prescribing measures by which to bring our thought into the forms indicated. An artist, as Aristotle says [1], initiates change in something other than himself: a sculptor e.g. in the clay which he models, a physician in the body of his patient; and if in his own body, he treats himself as he would another. The execution of such changes is indeed different from the rules to be observed in executing them. But the logician's business is not to give rules by following which others or he himself may alter their thought about things, their geometry or chemistry or biology; he offers no prescription for coming to know about all subjects; it is against such pretensions that a protest like Locke's, quoted above, may well be made. His business is to become conscious of the nature of

[1] *Met.* Λ. iii. 1070ᵃ 7 ἡ μὲν οὖν τέχνη ἀρχὴ ἐν ἄλλῳ, ἡ δὲ φύσις ἀρχὴ ἐν αὐτῷ ('Art initiates—*sc.* change—in another thing, the nature ⟨of a thing⟩ in itself').

the thinking carried on in those sciences. Logic, as we have said, studies the way in which we already think about things.

Nevertheless, it is not without effect upon our ordinary thinking. A good deal of our so-called thinking is incoherent, and breaks down when we criticize it. That we can indeed discover for ourselves without learning Logic; an economist can correct his own or his predecessors' errors in political economy, a mathematician in mathematics; they could no more wait for the logician to correct than to construct these sciences.[1] Yet the study of the thinking, good and bad, which has gone to their construction may give us a more lively consciousness of the difference between what its character should be and what it sometimes is, or as the Greeks would have said, between knowledge and opinion. Herein Logic may be compared with Ethics. Ethics investigates human conduct; it discusses the judgements of right and wrong, of good and evil, that we pass upon men's acts and them; it tries to determine what we really mean in calling an act wrong, and what we really require of a man in saying he should do what is right. All this would be impossible unless men already acted wrongly and rightly, and made moral judgements; Ethics does not teach men to do that. But it does bring into clearer consciousness the nature of the ideals which we already have, the grounds of the judgements which we already make, the frequent discrepancy between what is done and what we recognize should be done. To this extent Ethics tells us what to do, though it does not enable us to do it. Similarly Logic helps us to realize what knowledge of a subject is: but it does not enable us to bring our opinions on every subject into the form that knowledge requires. Both Logic and Ethics are thus in some degree practical; but we do not call Ethics an art, and it is not desirable any the more to call Logic so.[2]

[1] The word *logic* is sometimes used not for the study of thought which has been described in this chapter, but for the thinking which it studies: as when we say that some one is a man of powerful logic, or of great logical acumen. It is important to recognize that this is a different sense of the word.

[2] It must not however be supposed either that Ethics can determine what ought to be done in every difficult case of conscience, or that Logic determines exhaustively the forms of reasoning which the sciences must employ. Cf. F. H. Bradley, *Principles of Logic*, pp. 247–249. The phrase *normative science*, which some writers have of late applied to Logic, Ethics and Aesthetics, has perhaps been suggested by the character in them to which this paragraph refers. But it is liable to create misunderstanding, as if it were the business of these enquiries to prescribe rather than to ascertain the principles which our rational thinking, or action, or appreciation of beauty exhibits. The

It is perhaps from a desire to show the practical value of the study of Logic that men have insisted on viewing it as an art. But it would be a mistake to suppose that it can have no practical value unless it can furnish rules for 'the conduct of the understanding'. The direct help that it can give in this way is not very great. Its practical value in general education is firstly this: that it demands very careful and exact thinking about its own subject-matter, and thus tends to produce a habit of similar carefulness in the study of any other subject. In this it only does for the mind what a thorough training in any exact science might do. Secondly, it makes us realize better what the general forms of speech that we habitually use really mean, and familiarizes us with the task of examining our reasonings and looking to see whether they are conclusive. In this it has an effect which the study of some special science like botany is not equally calculated to produce. Thirdly, it brings more clearly into consciousness, as aforesaid, what knowing is, and so far furnishes us with a sort of standard by which to judge what we commonly call our knowledge of things; it makes us more alive to shortcomings in our ordinary opinions. But it does not need for its justification that we should point to effects which it produces upon our thoughts about other subjects; the nature of thought and knowledge is itself a subject worthy of investigation.[1] And, if we are to look also beyond this, its chief value lies in its bearing upon those ultimate problems, concerning the nature of reality, and man's place and destiny in the world, from which at first sight it might seem far

peculiar character of Logic, Ethics, or Aesthetics seems to be this, that we who, in them, reflect upon thought, conduct, or art, ourselves also in other moments of our activity create these objects of our reflection; and because in our reflection we recognize the failure of many of our attempts to think soundly, act rightly, or work beautifully, it is supposed to be the business of reflection, logical, ethical, or aesthetical, to rectify these failures. Such a supposition is in the main erroneous. It is by becoming better men of science that we shall correct our scientific blunders, by becoming better men that we shall correct our moral judgements and choices, by becoming better artists that we shall correct our aesthetical; nor does the recognition of a *should-be* surpassing what is require that we pursue those reflective disciplines. But the exercise of intelligence which they require presupposes the capacity and provokes the activity of that displayed in science, morality, or art themselves; and so there is a connection between them and the improvement of our scientific or moral or aesthetic thinking, such as does not exist between biology and the improvement of species or between dynamics and the improvement of locomotives.

[1] Cf. Bosanquet, *Logic*[2], i. p. 1: 'I am wholly of Hegel's mind when he says that the species of syllogism are at least as well worth discovering as those of parrots or veronicas'.

remote. 'Logic,' says J. S. Mill, in the Introduction to his famous work [1], 'is common ground on which the partisans of Hartley and of Reid, of Locke and of Kant may meet and join hands.' *Conserere manus*—it is only in this sense that rival schools join hands on the field of Logic. The dream of a Logic that shall be 'neutralized' like the physical sciences will not be fulfilled. These may move securely within the limits of certain well-defined assumptions, which all workers, though they may fight over minor points, agree to respect. Logic, which studies the principles of our thought about all things, cannot be content to leave unquestioned the assumptions within the limits of which thought proceeds: for it is those very assumptions that it investigates. The history of Mill's own work disproves his saying, for it is on its metaphysical side that it has been most vehemently attacked. Into metaphysical controversies, however, it is not the aim of this book to enter more than is absolutely necessary. But he would essay a vain task, who should attempt to expound the rudiments of Logic with no presuppositions about the nature of things. We may distinguish thought from the things thought about, but we cannot study it without any reference to what they are. All thought is thinking 'this about that'; and the general nature of the 'this' and the 'that' must be considered, if we are to consider what thought is; otherwise, our subject becomes a blank. The operations of the mind are unintelligible, if we disregard altogether the nature of their objects. To know what desire is, we must know what can be desired; there are some who hold that desire, by its very nature, is for pleasure; if so, could we understand it without considering what pleasure is? So we cannot understand thought without considering in general what thought is of. And consequently Logic, just because it studies our thought about things, is concerned with questions about the general nature of things.[2] Some would dissent from what in the following chapters is said on such questions. The controversies involved are not there pursued as they deserve, for this is not primarily a work on Metaphysics; but they have at any rate been indicated where they arise.

[1] *System of Logic*, Introd. § 7.

[2] Thus recent Symbolic Logic is full of discussions about classes and the relations between classes, because it holds thinking to be fundamentally thinking about the relations of classes. It seems to me that classing and class-relations are a very secondary subject of thought, and that for this reason Symbolic Logic gives a very distorted theory of thinking.

[The connection between questions about our thinking, and what we must think things to be, is excellently shown in the so-called Laws of Thought. These are certain very general principles exemplified in all thinking, and some have supposed the task of Logic to consist merely in developing their implications. They are known as the *Law of Identity*, the *Law of Contradiction*, and the *Law of Excluded Middle*. The Law of Identity may be formulated by saying that 'whatever is, is': or symbolically, that 'A is A'; the Law of Contradiction, that 'a thing cannot both be and not be so and so', that 'contradictory propositions cannot both be true', or that 'A cannot be B and not be B'; the Law of Excluded Middle, that 'a thing either is or is not so and so', that 'contradictory propositions cannot both be false', or that 'A either is or is not B'. In other words, if we think about anything, then (1) we must think that it is what it is; (2) we cannot think that it at once has a character and has it not; (3) we must think that it either has it or has it not. Now though these are called laws of thought, and in fact we cannot think except in accordance with them, yet they are really statements which we cannot but hold true about things. *We cannot think* contradictory propositions, because we see that *a thing cannot have* at once and not have the same character; and the so-called necessity of thought is really the apprehension of a necessity in the being of things. This we may see if we ask what would follow, were it a necessity of thought only; for then, while e.g. I could not think at once that this page is and is not white, the page itself might at once be white and not be white. But to admit this is to admit that I can think the page to have and not have the same character, in the very act of saying that I cannot think it; and this is self-contradictory. The Law of Contradiction then is metaphysical or ontological. So also is the Law of Identity. It is because what is must *be determinately* what it is, that I must so think. That is why we find a difficulty in admitting the reality of absolute change, change when nothing remains the same; for then we cannot say what it is which changes; 'only the permanent', said Kant, 'can change'. The Law of Excluded Middle[1] is so far different as a disjunctive proposition expresses doubt, and doubt belongs to the mind, not to things But to deny that this page need either be or not be white is to deny that it need be anything definite; determinateness involves the mutual exclusiveness of determinate characters, which is the ground of negation; and that is a statement about things. In other words, unless the primary Laws of Thought were Laws of Things, our thought would be doomed by its very nature to misapprehend the nature of things.]

[1] On this cf. further *infra*, p. 41, n. 1.

CHAPTER II

OF TERMS, AND THEIR PRINCIPAL DISTINCTIONS

Logic, we have seen, studies our thought about things ; and that cannot be studied without some consideration of the nature of things ; but further, it cannot be carried on, nor yet studied, without the use of signs—generally written or spoken words, which make what we call language. The relations of thought to things on the one hand and words on the other are difficult and intricate ; but we cannot without some regard to them profitably attack the subject of this chapter.

The true unit of thought, the simplest complete act of thought, or piece of thinking, is the *Judgement*, or *Proposition* : between which where a distinction is intended, it is that the proposition is the expression in words of a judgement. The close connection of language with thought appears already here ; for the utterance of the words, unless we were at the same time meaning with them, or judging, would not really be making a proposition : else the man who repeated the words of an unknown tongue would be ' propounding '. We may indeed understand a proposition without judging it, but only by imaginatively putting ourselves in the situation of a man who is actually expressing his judgement by it.

We may perceive without judging, though our present perception may be possible only through past judgements ; and here as elsewhere the history of how the individual mind has come to be able to do what it now does is elusive ; but that belongs rather to Psychology. I may pass a man in the street, and only afterwards say to myself ' That must have been so-and-so ' ; I may be walking along a railway line in the dark, and hear a sound, and then hear it again, and for the first time think ' That is the noise of a train approaching '. I perceived the man, or heard the sound, the first time ; I judged about them after ; and when I judged (we shall return to this) I distinguished in the ' subject ' I judged about a character which I ' predicated ' of it.

In judging then I always distinguish a particular element, the

predicate, in the being of a subject which I could not think of unless I recognized in it some other than the predicated character.[1] I must *think*, severally yet together, of both ; and if I want to call attention to them separately, I must indicate them by different signs ; but in order to make the judgement, though I need a sign, I do not need to indicate them by different signs. The child that learns to say 'Pussy' when it sees the cat means by the single word what we should express by the proposition 'There is the cat' or 'I love the cat', or whatever it may be ; and Mr. Alfred Jingle expressed his judgements with less than the full complement of words.

Whether any thinking can be carried on without some sort of sensible signs[2] is disputed ; certainly it cannot be carried far. The signs need not be written or spoken words ; they may be gestures, or sensations of touch, by means of which Helen Keller was taught to think. In algebra, though they can be written, they are not words ; in geometry the figure serves to a great extent, and one may think out a demonstration by help of drawing the lines of the construction with less use mentally of words than would be necessary to communicate it. Perhaps, with the figure before one, attending successively to its parts, one may dispense for a time with other signs altogether—*other* signs, because the figure itself is, as Plato noticed, a sort of sign : our demonstration is not true of it, since it is imperfectly drawn, but it helps us to think of the figure whereof it is true.[3] And perhaps when we are perceiving a thing we can make judgements to ourselves about it, without help of any sign, because it is itself sensible ; and when we are not perceiving it, some 'mental image' may serve instead of language. For the imagery which accompanies thinking is not the object of thinking ; I may as a psychologist make it the object of my thinking, and say that it is

[1] Hence a definition is not properly a judgement, as Aristotle saw (v. *Met.* Θ. x. 1051^{b} 17 sq.). For when I define anything—e.g. a triangle, and say that it is a three-sided rectilinear figure—I have not before me a subject already distinguished by some other character than what I predicate. Even here however I distinguish elements in an unity which they constitute ; and hence the definition can be expressed in a proposition. For I give a name to this unity as an unity, and also to the elements distinguished in it. There are some objects of thought which have names, and by the help of instances we come to know them, but because they are simple, or because they are unique in nature, what they are cannot be expressed in a proposition—e.g. *difference*—though judgements may be so expressed which tell us various things about them : e.g. 'difference is a relation' or 'attracts attention'.

[2] I do not imply that some signs are not sensible, but merely wish to call attention to the fact that all are so.

[3] *Rep.* vi. 510 D, E.

vivid, or evanescent, or what not; but that is not the thinking in connection with which it first arises. Its service to thought seems to be comparable with that of words, so that it has been called the 'inner speech-form'; though it is not articulated as language is. These considerations seem to point to the conclusion that language is necessary to thought because so much that we think of in things is not itself sensible, and we cannot fix our attention on what is not sensible, without the help of something that is; but there need be no correspondence in detail between the sensible sign, and the structure of our thought and of its object. This has not always been realized; and because a child first learns separate words, and then learns to combine them in sentences, and then to combine sentences in continuous discourse, it has sometimes been supposed that thought begins with isolated apprehensions of what it afterwards makes subjects and predicates in judgement, and then builds up judgements into reasoning. Such a view is an illusion produced by language, particularly through the consciousness of the separateness of words which modern writing and reading produces. It is indeed supposed by many that in early language words had not a separate existence, but only existed as it were confluently with one another in sentences.[1] Anyhow, there are no 'ideas'[2] which we put together in thinking as we do words in speech and writing.

Though the signs by help of which we think are thus various, words are incomparably the most important: and they are almost always[3] the only ones by help of which we express logical doctrine. Words are signs sometimes of things thought of, sometimes of operations of thinking, sometimes of both together. The subject-word in a proposition is a sign of something thought of, for which it is said to stand, and the proposition is not about it but about what it

[1] I have seen a letter written by an Alpine guide in admirable French, but wildly at fault in its division of words.

[2] No word in philosophy has been responsible for more confusion than the word *idea*. In Plato it meant what is called in Logic an *universal*, the common nature which thought recognizes in different particular things. Nowadays, it sometimes means an opinion (as when I say that my ideas on a subject have changed), sometimes 'mental images', sometimes it is merely an element in a periphrasis: to 'have an idea of' is simply to conceive or think of; then we are apt to suppose that we think of things by means of ideas of them, which is no more an explanation of thinking than if I say that I think of things by means of thinking of them.

[3] Most writers make some use of symbols which are not words to represent objects of thought (e.g. Arabic numerals); and in Symbolic Logic they are extensively used to represent both objects and operations of thought.

stands for: except when we say something about the word itself; an instance of the former is 'Barkis is willin'', of the latter 'Barkis is a proper name'.[1] Words like *if*, *because*, *therefore* are signs of the acts of supposition or inference, and *is* is the sign of the act of judgement, though also implying that something exists.[2] Other verbs, and also adjectives, are signs at once of some object of thought predicated, and of the act of predication; and the same verb may be a sign of the subject of predication as well. Thus in the proposition 'Dogs bark', 'dogs' stands for the things about which the statement is made, 'bark' both is the sign of (or expresses) what is predicated about them, and also of its being predicated; if I wish to disentangle, as it were, the sign of what is predicated from the sign of predication, I must say 'Dogs are barking animals', or something of that sort. The word *Perii* expresses both the subject about which the statement is made, viz. the speaker (though it does not stand for it), what is predicated of it, and the act of predication; and if subject and predicate are to be disentangled, one must say 'I am undone'. Even here the disentanglement is not complete, because 'undone' does not so stand for what is predicated of me that I could make it the subject in another proposition about that; for this purpose I should have to say 'I am a man undone'; I could then go on and say 'A man undone has no energy', or whatever it may be.[3] Words are often made signs of these divers things at once by means of *inflection*.[4] To substitute for a proposition expressing subject or predicate or both by the same word or words that express also the act of predication another in which distinct words express each of the three is called putting it into logical form. Where (as often in Logic) we wish to make subject and predicate separately subjects of logical discussion, this transformation is necessary, though it often does violence to the idiom of language.[5]

Now the subject and predicate (Gk. *ὑποκείμενον* and *κατηγορούμενον*), but not the act of predication, are called the terms in a judgement;

[1] Cf. *infra*, p. 19.

[2] Cf. *infra*, pp. 163–166.

[3] Neither the words 'a man undone' nor (in the previous example) 'barking animals' stand for the *character attributed*; that is 'being undone' or 'the habit of barking'; and if we use words that stand for it, and not for the things characterized by it, it cannot be attributed by the verb *to be*, but by some verb like *have*—e.g. 'Dogs have the habit of barking'. Cf. *infra*, pp. 37–38, 157.

[4] Even in a comparatively uninflectional language like English, in a suitable context, a single word may be a proposition: for example, in a telegram, the word 'coming'.

[5] There is no reason why Logic should 'put into logical form' the examples in which it studies thinking, where this is not wished.

and thus every judgement contains two terms, and they may be called elements in the judgement or the proposition, and it may be said to be resolved into them.[1] This again illustrates how language and thought are bound up together. A proposition is a sentence, but not merely a sentence: it is a sentence expressing or meaning a judgement. Otherwise we could not speak of resolving it into its terms; for the subject and predicate words, at which we thus arrive, need not have been in the unresolved proposition; and a mere sentence could not be resolved into words that were not in it.

It is easy then to see that a term is not the same as a word. In a judgement there are always two terms, but a single word may express both; Caesar's famous message of three words 'Veni, vidi, vici' contains as many distinct propositions, each of which may be resolved into the same subject-term 'I' and a predicate-term which is different. Contrariwise many words may make one term; and this is the commonest case. Subject and predicate may each be expressed by a single word, e.g. 'Tastes differ', 'Regret is foolish'; but in 'Dead men tell no tales', 'The kingdom of heaven is within you', each term consists of several words. Again some words cannot normally be the terms of a proposition at all. They do not indicate by themselves any object of thought, but are either used, like an article, in conjunction with some descriptive word, to designate an object, or, like an adverb, to qualify what another word expresses, or, like a preposition or conjunction, to mark some relation between different parts of a complex object of thought, or

[1] "Ορον καλῶ εἰς ὃν διαλύεται ἡ πρότασις ('I call that a term into which the proposition is resolved'), Ar. *Anal. Pri.* a. i. 24^{b} 16. 'Term' is *terminus*, a translation of the Greek ὅρος. It is not quite easy to see why the parts into which the judgement can be broken up were called ὅροι. The statement that 'a term is so called because it forms one end of a proposition' (*Jevons*) is clearly wrong; for that is an accident of language; even in English 'hungry I was, and ye fed me' would not be impossible, instead of 'I was hungry'. It may be that Aristotle, like the manuscripts of the *Organon*, symbolized the proposition in the form 'A—B' (where we should write 'B is A'), and that the use of the word comes from the position of the symbols. Bonitz (*Index Arist., s.v.* ὅρος, 530^{a} 21) thinks it a metaphor from mathematics, where if the ratio of two quantities was considered, these were called ὅροι, being represented by lines, which are the boundaries of a plane; in the judgement, there is a relation of subject and predicate, which might therefore be called ὅροι too. The word is, however, also used like ὁρισμός, to mean *definition*; and it may be that subject and predicate were called ὅροι as the determinate objects of our thought in a particular judgement, or as together comprising what is propounded, and limiting the judgement in which they occur to its own field.

(as we have seen) to express an operation of thought.[1] Such words are called *syncategorematic* (συγκατηγορηματικά) because only capable of being used along with others in predication; while words which signify what can by itself be a subject or predicate in thought are called *categorematic*. These, indeed, while capable of being used by themselves as terms, may also enter into a term among the words of which it is composed; thus *man* is a term in the proposition 'Man hath found out many inventions', but not in the proposition 'The heart of man is deceitful': *the sea* in the proposition 'The sea shall give up his dead', but not in the line 'She left lonely *for ever the* kings *of the* sea'. In this line the words italicized are syncategorematic; but *sea* is not syncategorematic, because it can stand for a term, though here it does not do so. Terms composed of words of both kinds have been called 'mixed terms'. It is true that syncategorematic words, though standing for nothing whereof anything can be asserted, or which can be asserted of anything, can yet as words be made the subject of linguistic or grammatical discussion, as when we say '*Of* is a preposition', or 'is the sign of the genitive case in English'. When words which stand for no complete object of thought are made objects of our thought themselves as words, it is said to be by a *suppositio materialis*.[2]

[1] With the articles may be coupled words like *some* and *any*; *not*, and *no* in 'no man', are also syncategorematic; so is the copula *is*, as the sign of predication, though not when it means 'exists' and is itself the predicate.

[2] The doctrine of *suppositio*, as of divers other 'properties of terms', has happily fallen into oblivion; but for the benefit of any one who wishes to understand the phrase *suppositio materialis* it may be worth while to add a note on it. All parts of speech were said to have *signification*; then, as sounds having signification, they acquired properties which did not belong to them as mere sounds. These properties were not the same for every part of speech. *Suppositio* belonged to substantives denoting substances, *copulatio* to verbs and adjectives. Substantiality and adjectivality were characters of the things signified; the adjective coupled some adjectival with some substantival thing, the substantive 'put' the latter 'under' the former (*v.* Prantl, *Geschichte der Logik im Abendlande*, vol. II. Abschn. xv. Anm. 67; vol. III. xvii. 59). So far, the sense of *suppositio* seems to be active; it is defined as *acceptio termini substantivi pro aliquo*; *suppositio* puts the substantive, instead of what it stands for, under what is adjectival; it takes the substantive term for or as representative of something, and predicates about it. But since we do thus *supponere* the substantival term, *suppositio* was said to belong to it, in the sense that not the act of 'supposition' belongs to it, but being the subject of that act; and then it was itself said *supponere pro aliquo*, i.e. to stand for, or be put for (not to put for), something (cf. Prantl, vol. III. xvii. 61, 201: Sanderson's *Compendium Logicae Artis*, Lib. II. c. 2). The same term had different kinds of 'supposition' according to what it 'stood for'; e.g. in 'Homo est animal', *homo* stands for all men, and this is the *suppositio naturalis* of a common term; in 'Homo currit', it stands for

Some logicians have preferred to speak of *names*, rather than terms, or have been ready to apply to a term Hobbes's well-known definition of a name. 'A name', he says, 'is a word taken at pleasure to serve for a mark, which may raise in our minds a thought like to some thought we had before, and which, being pronounced to others, may be a sign to them of what thought the speaker had, or had not, before in his mind'.[1] This definition, if we omit the words 'or had not', expresses fairly well the function of a name; but it is not equally appropriate to define a term; for not all words or phrases which can be predicated of anything would be called names of it, and yet they may all serve as terms. That word is the name of anything which we might give in answer to the question 'What is it called?'—either, if the thing is a concrete individual, a word used to direct our thought just to that individual, irrespectively of *what* it is, or, if our attention is to be directed by a name that signifies *what* that is which we are to think of, a word signifying not some attribute or detail in its being, but its essential or (if one may so say) most constitutive being.[2] Of the first sort

some individual, and this is *suppositio personalis*. Now as a sound having signification, the term was distinguished into the sound as matter, and the signification as form; and when a predication was true of a term as a sound or in respect of its matter, as in 'Homo est disyllabum', it was said to be by *suppositio materialis*: when in respect of what it signified, by *suppositio formalis*. There can be *suppositio materialis* of any part of speech, but *formalis* only of substantives; for only a substantive, or substantival phrase (*haec enim significat rem ut subsistentem et ordinabilem sub alio*, v. Prantl, vol. III. xvii. 60) can have *suppositio formalis*. Cf. p. 157, *infra*.

[1] *Computation, or Logic*, c. ii. § 4. By the words 'at pleasure' Hobbes does not mean that everything about the formation of names is arbitrary, but that there is nothing in a particular sound making it of itself more suited than another to suggest what it stands for; of course this does not apply to names derived from others already significant, but to the formation of underived names it does apply, unless they are 'onomatopoeic'. So Aristotle says that a name is φωνὴ σημαντικὴ κατὰ συνθήκην, 'an articulate sound having signification by convention' (*de Interp.* ii. 16ᵃ 19). The words 'or had not' should go out: a name cannot be a sign of what I am not thinking of, and even a negative judgement does not express the thought I have not in my mind, but the thought which I have, that 'this is not that'. What does Hobbes mean by a thought?—thinking, or the thing thought of? a name makes one think of a thing, or 'raises in my mind the thought' of a thing. My using it is a sign to others that I am thinking of that thing; but it itself is rather a sign of the thing; and when I use names only in my private thinking, they are not signs of my thinking at all, but rather instruments. A name also may consist of more words than one, e.g. Stoke Poges.

[2] The usage of the word 'name' is somewhat uncertain, and the distinction not sharp, because it is often difficult to say whether what a word signifies about that of which it is predicated is its essential being. We should probably agree that we give a screw-wrench its name when we call it a screw-wrench,

are proper names, like *Caesar*, *the Thames*, *Europe*; of the second, the general names of substances, like *man*, *river*, *lead*, and the names of the kinds, attributes, and relations of things, like *humanity* (= *human nature*), *jealousy*, *distance*. But words used of a subject to signify its possession of some attribute or relation, or used of these to signify their presence in a certain subject, or something 'about' them, are not names; 'the Great Commoner' is not a name of Pitt, 'the sin of Adam' not the name of disobedience, 'the needful' not the name of money, nor 'the continuous' of quantity.[1] Ambergris is a valuable substance found in the body of some sperm-whales; 'ambergris' is the name of that substance, 'found in the body of some sperm-whales' is not; but both are terms in that proposition. And there is another reason for distinguishing name and term. There is always a contrast in our minds between a name and what it stands for; but a term is so bound up with its meaning, that we often mean by 'terms' the objects of thought which are subject and predicate, not the words signifying them. Only so could we speak of resolving into its terms a proposition which does not contain the words which we get by our resolution of it. We say too that the subject-term in a proposition is that about which we predicate; but we seldom predicate about the *words*; when the messenger announced to Macbeth 'The Queen, my lord, is dead', it was not of the words that he spoke. To avoid confusion, it is sometimes necessary to indicate whether by the terms of a proposition we mean what is thought of, or the words signifying that; and we might call the former the *terms of thought*, the latter, the *terms verbal*. We shall have to give different definitions of a term accordingly. We may define a *term of thought* as 'whatever can be thought of as the subject or predicate of a proposition':[2] a *term verbal* as 'a

but not a carpenter, when we call him a carpenter; because in the being of a carpenter to be a man is fundamental, to be a carpenter incidental, but in the being of a screw-wrench to be a screw-wrench is fundamental; carpentry however is the name of the carpenter's trade. Aristotle has a formula which can be adapted here. If you can say of a thing called A that it is not something else in order to be A, οὐκ ἄλλο τι ὂν ἐστιν Α, then A is its name. (What he says is that a predicate belongs to A essentially as A, if it is not something else than A in order to have the predicate: v. *Anal. Post. a.* iv. 73^{b} 5–8.)

[1] On the function of a name cf. Lotze, *Mikrokosmos*[4], Bk. V. c. iii. § 5, E. T. vol. I. pp. 627–628.

[2] Or 'of a judgement'. It will be noticed that *subject* and *predicate* are equally ambiguous with *term*; in the one definition they mean what is thought of, in the other the signifying words. Nothing is a term except when it *is* thought of as subject or predicate, or used to signify these; but when we consider terms in isolation, though there is no given judgement, we consider

word or combination of words capable of standing as the subject or predicate of a proposition'.

To avoid ambiguity between terms as words and what they stand for or signify, logicians sometimes give to the latter, when they are not individuals, the name *concepts*. The word 'concept' always signifies something thought of, never the name of it. *Conception* is sometimes used equivalently; indeed in ordinary speech that is the word that would be used, and if a man spoke of the Greek conception of the heavens, he would mean what the Greeks conceived the heavens to be. But 'conception' also means the act of conceiving, as when I say that the conception of an immaterial substance is known to us first in Plato. The ambiguity is common in English with words of this formation; 'narration' may signify either the act of narration or the story narrated, 'composition' either the act of composing or what is composed; we may say that a man is engaged in composition, or that he has sent his composition to the press. The Greek language distinguished the two meanings by different verbal terminations, the act by nouns in *-σις* (like *αἴσθησις* and *νόησις*, *sensatio* and *intellectio*), the object by adjectival words in *-τον* (like *αἰσθητόν* and *νοητόν*, *sensatum* and *intellectum*). As it is important not to confuse the two, it is best to use the word 'conception' to signify conceiving, and 'concept', though it sounds less familiar, to signify what is conceived.

A concept is not the same as a term of thought, because concrete individuals, like the Thames, may be terms of thought, as when I say 'The Thames flows through London' or 'That ship is the Victory'; but they are not concepts, for we may perceive or think of, but not conceive them. Nevertheless many terms of thought are concepts, and it is important to recognize the part they play. The three following paragraphs may throw some light on this, though they belong in other respects more properly to the discussion of the nature of judgement.

It is an old objection to judgement, that since its subject and predicate are different, it cannot be true; for according to the Law of Identity, A is A, and not B.[1] But there can be no thinking unless we allow that the unity of a thing with itself does not preclude variety in what it is. Still the problem of the One and the Many is

their capacity to be terms. Hence I have said '*can be* thought', or '*is capable* of standing', not '*is* thought', or '*stands*'.

[1] Cf. *supra*, p. 13. This puzzle was started by Antisthenes the Cynic in the fourth century B.C. Cf. Lotze, *Logic*[2], Bk. I. c. ii. B. §§ 56–60.

among the chief problems of Logic and Metaphysics; and if thinking expresses itself in the form 'A is B', we must ask what this form means. Now consider the following examples: (1) 'Barkis is willin'', (2) 'the Emperor is captured', (3) 'a bacillus is a vegetable', (4) 'to obey is better than sacrifice', (5) 'to doubt is to think'. In the first, 'Barkis is willin'', the predicate is only one detail in the being of the subject, but the subject is indicated by a name, which does not single out anything else in its being: in the second, 'the Emperor is captured', the predicate again is only one detail in the being of the subject, but the subject is indicated by a word which singles out another detail in its being; in both there is a predicate-concept, in both the subject is a concrete individual, but in the second there is besides the concrete subject a subject-concept; this subject-concept however is but a detail in the being of the concrete subject. In the third, the subject is again a concrete thing, and there is a subject-concept; but this is not a detail in the thing's being, but is its essential or constitutive being, neither is the predicate a detail in its being, but the general being of the subject-concept. Hence while the first ascribes a character to Barkis, viz. willingness, but does not mean that being Barkis is willingness, nor the second that being an Emperor is being taken captive, the third does mean that being a bacillus is being a vegetable. In the fourth, the subject is not a concrete thing, but a concept, i. e. something we conceive; and the predicate is so too; but it is not the general being of the subject-concept, and the proposition does not mean that obeying is superiority to sacrifice. Lastly, in the fifth, as in the fourth, the subject is a concept, but the predicate-concept is its general being, and the proposition does mean that doubting is thinking.

Now the points to which these examples should chiefly direct our attention are these:—(i) concepts are characters (not necessarily sensible) which we find displayed in individuals; (ii) they may be characters which as it were cover the whole being of these individuals —the phrase is Professor Cook Wilson's—or only details in their being; (iii) one character may cover the whole being, or be the general being, of another; (iv) where the predicate-character covers the whole being of the subject, or subject-character, the latter *is* the former essentially, and not only may the things denominated from the subject-character be denominated from the predicate-character ('a *bacillus* is a *vegetable*', 'a *doubter* is a *thinker*'), but the subject-character itself is the predicate-character (being a bacillus

is being a vegetable, doubting is thinking); (v) where the predicate-character is only a detail in the being of the subject, whether individual subject or subject-character, the latter is not thus essentially the former: the predicate-character is incidental to the subject, or coincidental[1] with the subject-character in the same individual subject; and though the subject, or things denominated from the subject-character, may be denominated from the predicate-character, the subject, or the subject-character, *is* not the predicate-character (Barkis is not willingness, being an Emperor is not being taken captive, obeying is not being better than sacrifice).

Thus judgement involves concepts[2] among its terms of thought, but individuals may be terms of thought also; but these terms of thought, whether individuals or concepts, are not in every judgement judged to be related to each other in the same way, though the forms of language do not always bring out these differences in the relation between subject and predicate.

It was said that a concept is a character of something, not an individual thing; neither is an individual sensible quality conceived—e. g. the black colour of this ink; but its general or *universal* character, that of which it is a particular instance, is conceived. It is only by an act of thought that I can apprehend that colour which is the same in black and red and blue. It is also only by an act of thought that I can apprehend blackness as something the same in the black of that ink and of this. Concepts therefore are not sensible. But it would be wrong, because they are not sensible, to suppose that they are not real independently of the conceiving mind: that they are products of the activity of conceiving. Unless what I conceive a thing to be and predicate of it is what the thing is, my thinking is vain, and doomed eternally to defeat itself. Suppose that a study of the literary or other evidence leads a man to judge that Gibraltar belongs to the British Crown. His judgement concerns a rock at the entrance of the Mediterranean and a fact in its present history. The rock exists independently of his thinking about it; but not less does belonging to the British Crown,[3] or his judgement could not be true. Yet belonging to the British Crown is not sensible.[4]

[1] Cf. *infra*, p. 76.

[2] Except where both terms are proper names—e.g. 'Eboracum is York', 'Verulamium is not Colchester'.

[3] The word *exist* is sometimes confined to the concrete individual and its particular sensible qualities, and anything else real is said not to exist but to be.

[4] Idealists of the school of Bishop Berkeley would say that Gibraltar does not exist independently of being perceived or imagined. Most idealists would

[The view that concepts are products of the conceiving mind is as old as Plato, who rejects it in the *Parmenides* 132 b, c; it is expressed by calling them not *νοητά*, things conceived, but *νοήματα*, products of conceiving (as a poem or *ποίημα* is a product of the poet's making or *ποίησις*). Aristotle often countenances it, though perhaps also holding these mental facts, our concepts, to be in a manner the same as the intelligible nature of things, the *νόημα* the same as the *νοητόν*. Others, and among English philosophers notably Locke, have held that the object of conception is altogether mental; that concepts are created by the mind in order through their instrumentality to acquire knowledge about real things, but are not real themselves. This doctrine is known as *Conceptualism*. The objection to it is simple. It holds that concepts render possible a knowledge of real things when they are so formed as to correspond with the nature of the things; but it cannot show how we could be aware of this correspondence without knowing the nature of the things directly, as well as the concepts. If we only know the nature of the things through the concepts, we can no more tell that they correspond, than we could tell that the existing portraits of a man were like him, if we only knew his features through the portraits. And indeed it would be nearer the truth to say that only what is real can be conceived, than that what is conceived is not real. We cannot conceive a square circle, though we can conceive a square and a circle, just because, though circle and square are real, their combination in the same individual figure is unreal and impossible. But there are difficulties also in the way of saying that all that is conceived is real. We may ascribe to the same individual subject a number of attributes, each of which is conceived, and their combination also conceived, and which yet are not really combined in this subject; for example, I might think Gibraltar to be a fortress acquired by treachery; to be a fortress is a real attribute of some subjects, to have been acquired by treachery of others, and their mode of combination is a real mode of combination, exemplified, if not in them, yet in other attributes: nevertheless such a belief would be erroneous. The difficulty here is the difficulty of error. It may be said that other fortresses have been acquired by treachery, and therefore what I think Gibraltar to be is what they were; and so I am conceiving something real, though ascribing it to the wrong subject. But—not to mention other difficulties which this answer does not remove—the elements thought to be combined, or (as it would be expressed) combined in our concept, may be such as

hold that its existence is at least not dependent on the consciousness of this or that finite individual, whatever be the relation of things to mind in the universe as a whole. Without entering upon this question, I am concerned here to urge that what is apprehended in things by thinking, but is not sensible, is not less really in them nor more dependent on the mind than what is apprehended by sense-perception.

[have never been combined in any real subject. Our fathers thought Methuselah to be a man who lived for more than nine hundred years; there are things that have done it, such as some of the big redwoods at Mariposa; there are things that are men; but none that both are men and have done it. Perhaps we ought still to say that what is conceived is something real, but that in these cases (where we are dealing with questions of historical fact) the elements of a complex predicate are conceived, but do not form a real unity, and are not one concept, because we do not see the necessity of their combination. Where we suppose ourselves to see a connection between conceived elements, which nevertheless does not exist—as Descartes thought that '*Vis Viva*' in a body was as the product of the mass into the velocity, not into the square of the velocity—there, when we escape from our error, we realize that we never saw the connection, because it never existed. We may be inclined to say that we conceived what was unreal; but we ought rather to say that we thought we conceived what we did not conceive.

There remains however a further difficulty about the existence, or reality, of objects of conception. We predicate what we conceive of individuals; it was agreed above that a concept must be other than a mere product of our conceiving because we conceive the nature of what exists. Yet we can still conceive it when the individual whose nature we judge it to have been exists no longer. The whole question of the relation of the ultimate reality to its appearance in time is involved here.]

[It has been said that concepts[1] are universal: that what we conceive is the common nature whereof we find instances in individual things. But though we do not conceive the particular instance, our knowledge of it involves conceiving. To hear a sound is not an act of conception; but if thought is at work, and I know it for a sound, I must be aware of what 'this heard' is. I may be determined to action by hearing a sound, without thinking: and hearing words determines me to think of what they signify, without thinking about the words; in this case too I hear sounds but do not think that they are sounds (though of course I do not think that they are not sounds) and so there is no conception of sound. But when I think about what I perceive, and apprehend what it is, the elements of its individual being are known as an instance of that whereof there may be other instances, and that is universal. Conception therefore is involved with my perception. This common nature or universal is not itself perceived, though known in the perception of its instances. But it is to be noted, that in some cases the instances can no more be perceived than can their universal nature. Relations illustrate this. The likeness between my two hands is not the likeness between your two hands, but each is an

[1] On the nature of concepts cf. further, pp. 68–71 *infra*, especially p. 69, n. 1.

[instance of likeness; nevertheless though we can see our hands, we cannot see the likenesses. By and bye it will appear how important this fact is to the theory of induction. The inductive sciences seek to discover causal relations. Now causal relations are displayed in things; the impact of these stones causes Achan's death, of those Stephen's. Yet the particular instances of causality cannot be perceived; otherwise it would be as easy to perceive the cause of a flower's drooping as to see it droop.

The existence of universals is often denied; men are apt to imagine that if they exist one should be able to find them as one finds instances of them. Hence the remark of Antisthenes—*ἵππον μὲν ὁρῶ, ἱππότητα δὲ οὐχ ὁρῶ*, 'I see a horse, but not horseness': to which Plato replied, that it was because, though he had eyes, he had no intelligence.[1] The universal is not one of its own instances, and cannot be found like them. Nevertheless to deny that there are universals is to deny all identity between different individuals, and to do this is to say that we can never, by what we learn of the connection of characters in one individual, infer one from the presence of another in a second individual. We may allow that the relation of an universal to its instances is puzzling; but the puzzle comes partly from trying to describe it in terms of some other relation. The universal is sometimes called a *whole*, or (for distinction) a *logical whole*, and its instances *particulars*, and we ask how they partake of or divide the whole among them; is there in each a part, or is the whole present in each? the first is inconsistent with its unity, the second makes it to be in many places at once.[2] But the question here assumes that the 'logical whole' is like a physical whole or thing in space: that horses share horseness as they do a pottle of hay. If we wish to know the relation of its parts to a physical whole we must consider examples of the quantitative—England and its counties, a day and its hours; so, if we wish to know the relation of its parts to a logical whole, we must consider examples of that in which this relation is exhibited—things of a kind, different instances of the same quality. We find in reflecting on our thoughts about things, that we do think them to be things of a kind, instances of the same. That is why the present discussion is logical; though it is one of the logical problems that concerns also the being of things.

It has been maintained[3] that there are no instances of relations: that the likeness between my hands and the likeness between yours are not two likenesses but the same likeness—not instances of

[1] Cf. Ritter and Preller, *Historia Philosophiae Graecae*[9], § 287. In the story which gives the answer, it is Diogenes who speaks, and a cup and a table take the place of the horse.

[2] Cf. Plato, *Parmenides*, 131.

[3] Cf. B. Russell, *The Problems of Philosophy*, c. ix. (Home University Library), *Principles of Mathematics*, § 55.

[likeness, but numerically one likeness. Without accepting this, it may be granted in regard both to relations and attributes that we are very apt to confuse the instances and the common nature. And we often denote them by the same name; 'colour' means particular colours when I speak of the colours in last night's sunset; it also means colouredness; every distance is a particular distance, but their common nature is called distance also. We do not make this confusion in regard to substances; men and horses are instances of their kinds; and individual men or horses are so much more obviously different from one another than individual likenesses or distances or ultramarines that we cannot overlook in them the distinction between the manifold individuals and the one common nature.[1] But perhaps this distinction is more readily seen in substances because individual substances are something more than instances of their kind. The true instances of human nature are the human natures of individual men; but the human nature of Caesar is *what* Caesar is; and sensible individual substances at any rate we do not seem to discriminate altogether by *what* they are.[2]]

The foregoing consideration of what a term is in general, and of its relation on the one hand to a word and on the other to an object of thought, will have helped to familiarize us with some of the facts determining the main *kinds of terms* that Logic has to recognize. The ordinary classifications of terms are classifications of them as words which signify objects of thought; but the distinctions are based on differences in what we think of, or what in general we think things to be.

In respect of the objects of thought signified, terms are commonly divided first of all into **abstract** and **concrete**: but if we regard also their character as words, or terms verbal,[3] they must be divided into **abstract, concrete,** and **attributive.** A concrete term (verbal) is the name of a person or thing, an abstract term the name of a quality or attribute, or relation; so that the distinction between the thing and its qualities, between substance and attribute or rela-

[1] Yet biologists do not seem always to have asked themselves which they mean when they write about evolution. Do individual men evolve, or is it the human nature which is displayed in them all? and if the latter, and men are descended from animals whose nature was not human nature, but has evolved into human nature, what is the relation of the two, or are human nature and pithecanthropous nature the same common nature? and if so, are there many species or only one?

[2] Cf. *infra*, pp. 54–57. If there are individual substances that are not sensible but purely intelligible, they must be discriminated by the understanding only.

[3] i.e. terms as = the word or words signifying an object of thought.

tion, is the basis of the distinction between concrete and abstract terms. Attributive terms will be explained later.[1]

Our notion of a thing, as has been already indicated, involves two elements, which furnish the basis for a further division of both concrete and abstract terms into those which are **singular** and those which are **common** or **general.**[2] A thing is, first, an individual, having an existence distinct from that of other individuals; the page, for example, on which these lines are printed is a different page from every other in this book. But secondly, a thing has a character, which may be the same in other things; just as other pages in this book, though individually different, are equally pages. This character, which belongs alike to many individuals, is sometimes called, as we saw, an *universal*; and they, as so many different cases or examples of it, are called *particulars*: particulars, as we often say also, of a kind.[3]

Now the various particulars of a kind, so far as they have the same character, may be called by the same name: so far as they are distinct particulars, they will require different names to distinguish them. Their names as *things of a kind* are common or general names: for the name is *common* to all particulars of the kind, or applies *generally* to any; acorn, squirrel, file, metal, are general names. Their names as individuals, if they have any, are singular; like London, Zoroaster, the Matterhorn; such names as these we call **proper** names. A **general** term is thus *one that is predicable of any number of individuals in the same sense*: a **singular** term *one that is predicable of one individual only in the same sense*: and a singular term is a **proper name** if it does not indicate what individual it stands for by reference to any special element in its being. *Smith* for example, as meaning one who works in metal, is a general term, because I mean the same by calling Dick or Thomas a smith; if I use it as a proper name, numerous as are the persons who bear it, I do not mean the same in each use of it. I may refer to the defender of Acre, or to the witty canon of St. Paul's, or to any of a hundred and one others, and in each case my meaning is different;

[1] v. *infra*, p. 36.

[2] That this distinction is applicable also to abstract terms is apt to be overlooked, and I wrongly denied it in the first edition, through not distinguishing abstract terms and names of universals. I owe the correction to Mr. H. A. Prichard.

[3] Strictly, if what was said on the previous page is right, it is the constitutive nature of each concrete individual that is the instance of the kind.

nor is it through referring to anything in particular of what he was that I know, when I hear the name, that Sir Sydney Smith is meant, as it would be if my thought were directed to the same man by calling him 'the defender of Acre'.

We are seldom at a loss for some general term by which a particular thing may be denoted; but comparatively few particulars have singular terms appropriated to them. Many particulars of a kind—for example, new pennies—are not distinguishable at all to our senses, except by each occupying (when we see them together) a different place; these will not have each a different name, for we should never succeed in calling each individual always by its own proper name. In other cases, though the particulars of a kind might be tolerably distinguishable—for example, lumps of chalk of varying shapes and sizes—we have no occasion to refer to them individually, nor to burden our memory with so many names. We are content to employ a common or general name, and to specify the particular object (from among all those that bear the name) to which we wish to refer, by pointing, or the use of a demonstrative or possessive pronoun, or some periphrasis. Thus we say 'the picture there', and point: or 'this year', or 'my great-coat', or 'the bust of Julius Caesar in the British Museum of which Froude used an engraving for the frontispiece of his life of Caesar'. Such expressions are indeed in a manner singular terms, for they serve to designate particular objects; they are not however proper names; they commonly include general terms and are partially descriptive, and they have been conveniently called **designations.**

But where particulars of a kind are distinguishable, and we are interested in them singly and wish to be able to refer individually to them, we give them 'proper names'. Thus every individual man has a name of his own, and every field in the country is named, because the farmer needs to tell his men which particular field to work in; and a railway company for a similar reason names or numbers its various engines and carriages. Though, however, many particular things have no proper names, all which have proper names have general names also; the 'four-acre' is a field, the 'Cornishman' is a train, William the Silent is a man; and on the other hand any particular thing might, if it were worth while, be distinguished by a proper name. The proper name and the common name thus recognize respectively the two elements in our notion of a thing noted above: the proper name recognizes its distinct existence, the

common name its character that it shares with other things: nor could our thought about things express itself fully without concrete terms of these two kinds.

[This has not indeed been always admitted. Thus James Mill in his *Analysis of the Phenomena of the Human Mind* (vol. i. ch. viii. p. 260, London, 1869) writes that it is 'obvious, and certain, that men were led to class solely for the purpose of economizing in the use of names. Could the purposes of naming and discourse have been as conveniently managed by a name for every individual, the names of classes, and the idea of classification, would never have existed. But as the limits of the human memory did not enable men to retain beyond a very limited number of names; and even if it had, as it would have required a most inconvenient portion of time, to run over in discourse as many names of individuals, and of individual qualities, as there is occasion to refer to in discourse, it was necessary to have contrivances of abridgement; that is, to employ names which marked equally a number of individuals, with all their separate properties; and enabled us to speak of multitudes at once'. The position here taken up by Mill is known technically as that of **nominalism,** the doctrine that things called by the same name have only the name in common; a doctrine frequently professed, but not often stated with such uncompromising clearness as in this passage. We do not however really call different individuals by a common name, except because they have or are believed to have a common nature; nor is it conceivable that we could name an individual by a proper name, without at the same time recognizing in it, however vaguely, some character that, as capable of existing equally in other individuals, might be the ground of a general or common name. General names then are no mere means of abbreviating discourse, but their existence is grounded in what we must think the nature of objects of thought to be. Aristotle's distinction[1] between *ὁμώνυμα*, or things called by the same name having only the name in common, and *συνώνυμα*, or things called by the same name having also what is meant by the name in common, may be mentioned here: the distinction is nowadays embodied from the side of names instead of things in that between equivocal and univocal terms (*v. infra*, p. 46). Opposed to nominalism is the doctrine known as **realism,** which maintains the reality of 'universals' or characters the same in more individuals than one—of squareness as well as squares, justice as well as just men and actions, man-ness as well as men. If the common nature be held

[1] Most clearly stated *Cat.* i. 1^a 1–12. The Aristotelian authorship of the *Categories* is disputed; but that the doctrine in it is in the main Aristotelian can be shewn from treatises admittedly his. Cf. for this distinction *Top.* ζ. x. 148^a 24 sq.

[only to exist *in* the various instances, so that there would be no squareness unless there were squares, nor man-ness unless there were men, the doctrine is that of *universalia in re*; if it be held to be eternal, so that with the first existence of squares or men began, and with their disappearance will end, only the manifestation and not the being of squareness or man-ness, it is that of *universalia ante rem*. **Conceptualism** (*v. supra*, p. 25) is an attempt to compromise between the Nominalists and the Realists by saying that different individuals cannot indeed share a common nature, because no common natures but only individuals exist, but that nevertheless we form concepts which somehow correspond with each of a number of individuals, and by their means we are able to have general knowledge, i. e. (on this view) knowledge about an unlimited number of individuals at once. Conceptualism is the doctrine of *universalia post rem*.]

There are thus two kinds of concrete terms, viz. *singular* terms, or names of individuals, and *common* or *general* terms; singular terms can be further distinguished into *proper names*, i. e. names permanently assigned to one individual, and *designations*, i. e. phrases which by a pronoun or what not serve to indicate an individual otherwise than by a name of its own. Now it has not been stated in the last sentence, what general terms are the names of. Are they also the names of individuals, or are they names of the character common to many individuals? The former view seems incomplete, for it does not take account of their difference from singular terms. The latter view is plainly wrong, for *man* is clearly predicated of individual men, not of the nature common to them; and when I say that man is mortal, I mean that men die, not that human nature dies; that is displayed in a succession of individuals who are born and perish, but is not born and does not perish itself.[1] We must then accept the former view. General concrete names are names of individuals, but names of them *in respect of their common nature*. Hence they imply the existence of universals, though they are not the names of these.

Now such universals sometimes have names. It is true their names are not often used in ordinary talk, for our practical interests are in individuals, and only in philosophical reflection are we led to consider the existence of the universal realities whereof they are instances. Still, the nature of man is so interesting to us that we

[1] But what would happen with the death of the last man? Cf. p. 26 *supra*, on the existence of concepts.

have the name *humanity*[1]; and we can form names, like 'horseness' or 'goldness', when we wish to distinguish the common nature of horses or parcels of gold from their instances, or we can use a periphrasis, like 'the nature of gold'. Are we to call such a name concrete or abstract? It would commonly be called abstract, being the name of the common nature of many individuals, considered apart or *in abstraction* from them or from what distinguishes them from one another; though the substantial nature of a thing cannot properly be regarded as a mere attribute of it.

The distinction of individual and universal is not confined to what is concrete. We have seen that attributes and relations also have their instances. The red of one rose is not numerically the same as the red of another, however much their being two depends on their being in different roses, and otherwise they would be indistinguishable.[2] The distance from London to York, even if equal, is not the distance from London to Bideford. But as we can only distinguish the instances of the same attribute or relation by reference to the substances to which they attach, only the latter and not the former have proper names. Hence we are apt to overlook that there are instances of what is abstract. Yet it is clear that the death of Caesar is one of many instances of death, just as Caesar is one of many instances of man; and when it is said that there are so many births and deaths a year in London, *birth* and *death* are as clearly general terms as *house* and *street* in a total of streets and houses. And that means that they are used in the same sense of each birth or death, and that 'the birth of X' or 'the death of Y' is a singular term.

So far the case is the same with abstract and with concrete terms.[3] But men are interested chiefly in the individual instances of what is concrete, and in the general nature of their attributes or relations; and so not only are there no proper names for these, but the general name, besides being used of them, is used also of their general nature, or universal. *Death*, when I speak of Caesar's death or

[1] *Humanity* has of course other meanings, viz. mankind collectively, and also kindliness; in the text it means the human nature common to all men. Cf. also *deity*.

[2] Cf. *supra*, p. 27 *ad fin.*

[3] Hence it is a mistake to say that the plurals of abstract terms are concrete. Deaths, colours, distances are not substances because there are many of them; and a concrete term is the name of a substance. But the plurals of abstract terms often designate not individuals but kinds of attribute or relation.

Alexander's, is a general abstract term, comparable with the general concrete term *man*; when I say that death comes in many forms, it is the name of an universal, comparable not with *man* but with *humanity*. So *colour* is a general abstract term, if I speak of the colours of yesterday's sunset, but the name of an universal—viz. colouredness—when I say that colour has divers species. The fact that many words are used both as general abstract terms and as names of the universals of attributes or relations helps to make us regard the names of the universals of substances as abstract. 'Colour', as predicable not of a coloured thing but of its attribute, is an abstract term; meaning colouredness it is a word of the same sort as 'goldness'; hence we think 'goldness' an abstract term also.[1]

[It will be seen that there are really two antitheses confused together when the division of concrete and abstract is offered as an exhaustive division of all terms of thought, viz. (a) the antithesis of individual and universal; (b) that of substance and attribute or relation. The second member is called abstract in each antithesis —though what belongs to the first member in (a) may belong to the second in (b)—because by abstraction two things are meant, viz. (a) considering the common or universal nature of divers subjects apart from the particular instances; (b) considering some particular element in the nature or being of anything apart from the rest of its nature. The former is what Locke has most prominently in mind when he speaks of the formation of those abstract ideas, which exist, on his view, only in the mind, and do duty instead of any real identity in the various things called univocally by the same common name. The latter is what Aristotle meant when he said that the mathematician considers the subjects of his study ἐν ἀφαιρέσει, in abstraction, i.e. that he demonstrates the properties that belong to what is circular or triangular merely in virtue of being circular or triangular, neglecting—because they are irrelevant—all other characters of those things besides their figure. If we are to avoid confusing the two antitheses we must say that (A) our thought recognizes, and therefore we have names for (i) individuals, (ii) universals; the names of individuals may be either (1) names of them considered as this or that determinate individual, i.e. proper names or designations; these are the *singular terms* of the traditional doctrine; or (2) names of them considered as of a certain sort: these are the *general terms* of the traditional doctrine. (B) Our thought also recognizes, and therefore

[1] Sometimes particular abstracts and their universal nature may be indicated by different words. *Act* is a general abstract name, *action* the name of the common nature of all acts. But 'action' is also used as equivalent to 'act', and we speak of *an* action, and of actions in the plural.

[we have names for (i) substances or things; (ii) their attributes or relations; and the distinction of singular and general applies to the names of both these, since both substances and their attributes and relations are found as instances of a sort; but singular names of attributes or relations are all designations, formed by help of naming the individual substance involved, and not proper names. The distinction of singular and general does not apply to names of universals. Now the traditional doctrine ignores the distinction of individual and universal in regard to attributes and relations, and calls the names either of the instances or of their common nature *abstract terms*; and when names are coined for the common nature of substances, which as a rule in common speech have not got names, it is inclined to count them as abstract also, not having in mind the distinction of individual and universal.[1]

These antitheses, though we certainly make them when we reflect on things, no doubt present difficulties to a closer examination. The nature of relations, and their difference from the terms related, have perplexed many, and have led some philosophers, like Mr. F. H. Bradley, to deny that relations can belong to Reality; it appears to us as a system of things in relation, but transcends this in its own being. And even if we find no difficulty in the existence of relations, we may be perplexed by the distinction between the two kinds of related terms, substances and attributes. The individual substance, we think, exists, and its attributes are elements in its being existing only in it and not apart from it. But that of which they are attributes must be something of a determinate kind, not a mere point of reference for a multitude of attributes. A concrete name denotes such a determinate thing; but on the other hand its concrete nature threatens to break up into a number of distinguishable factors, each of which by itself would be called an attribute. Now they cannot be attributes of each other, nor yet of that which would be left—if anything would be left—if we abstracted them all, a 'something, we know not what, which we feign as a support of qualities', in Locke's phrase. We might say that each is an attribute of the complete thing, of the individual in its whole being: that in fact the so-called attribute is rather an element in the being of that whereof it is called an attribute. But this still leaves it a question whether in the being of the individual substance we rightly distinguish its substantial nature, on the strength of which we call it by a general concrete name, and the attributes called by abstract names, or whether the substantial nature is really but a complex of elements or factors in the thing's being, which, if they were not so numerous, could be named separately, and would then be regarded as so many attributes. On this cf. *infra*, pp. 53–54.]

[1] I owe the outline of this paragraph to Mr. H. A. Prichard.

Abstract terms then are the names of attributes or relations; but we must understand this definition rather widely. It is not only sensible qualities, like flavours or odours, whose names are abstract terms; each element in the being of the individual concrete thing, considered singly and in distinction from, although as qualifying, the thing,[1] is abstract, and its name (where it has any) an abstract term. Moreover, the thing in question need not be a single thing (or person) such as a stone or an elephant: it may be an assemblage of what we regard as distinct things (or persons), like a forest, or an army; but if there are features belonging to this assemblage, though they are not qualities of any one thing in it (as a forest may be extensive and an army skilfully or unskilfully disposed), these features considered in themselves are abstract, and their names, 'extent' or 'disposition', abstract also. Hence discipline, civilization, paternity, are all abstract terms, though it is only by a doubtful extension of language that we could call any of them a quality, like fragrance or sweetness. And we have seen that commonly, though confusedly, terms like 'animality' and 'triangularity' are also called abstract names, that is, not of the distinguishable individual elements in the being of the individual concrete thing, but of the universals whereof either individual concrete things, or the various distinguishable individual elements in their being, are instances.

Besides abstract and concrete terms verbal, there is a kind of terms verbal which cannot well be classed with either—viz. adjectives and adjectival terms. These are called **attributive** terms, e.g. *red*, *beaten*, *insolvent*. They are not the names of qualities, like *redness*, *defeat*, *insolvency*; on the other hand, it is those qualities which furnish their meaning, not the nature of the various kinds of object to which the qualities may belong. Thus cloth may be red and so may silk, but we should not explain what is meant by calling them red if we were to explain the nature either of silk or cloth; and a man may be insolvent and so may a company, but to explain what

[1] It may be objected that whether a colour *is* abstract cannot depend on our considering it in a certain way; if it is not abstract, we are wrong so to consider it; if it is, it is so however we consider it. But if a substance is an unity into whose being various elements enter and combine not in the way in which material things combine into an aggregate, but in the way in which attributes combine into the being of a concrete thing, then to say that these elements considered singly are abstract merely means that they are several and can be distinguished, though only existing in the concrete unities which they form.

is meant by calling them insolvent we must explain the nature not of man, nor of a company, but of insolvency.[1]

J. S. Mill held that adjectives are really concrete, on the ground that 'white' is predicated, or is the name, of snow, milk, or linen, and not of their colour; that it is an army and not a defeat that is beaten.[2] But it is clear that the subjects of which an adjective may be predicated can as well be abstract as concrete; and if the adjective is concrete because it is predicated of a thing, it should equally be abstract because it is predicated of an attribute; so that if we say that cabbages are common, *common* will be concrete; while if we say that indolence is common, it will be abstract. The fact is that the distinction of attributive terms from abstract and concrete corresponds to no further distinction in terms of thought. There are substances, and there are attributes or relations, and the latter qualify the former; but their qualifying them is not a third co-ordinate sort of reality. It is the nature of an attribute to be *of* a subject, as of a relation to be *of* its terms[3]; and when we recognize this in instances, we are said to attribute them to their subjects. But that is an act of judgement, not a term; there is an attributive act, but no third kind of object of thought which we can call attributive. In language however there are words which, though they can be used as predicates, and therefore satisfy the definition of a term verbal, are not properly names either of a substance or of an attribute. Adjectives are such words; but so also are verbs. Verbs however were overlooked by those who placed adjectives

[1] The meaning of attributives may, however, be incapable of explanation without reference to that, in the nature of the subjects whereto the qualities belong, which makes them susceptible of these qualities. Thus neither silk nor cloth could be red unless they had a surface; neither a man nor a company could be insolvent unless capable of having debts. Cf. p. 112, n. 1, *infra*. It may be added that terms like *father* or *musician* are adjectival in sense, and would by some be classed as attributive; for though they are substantives, and are predicated of concrete things, they do not primarily signify the concrete things of which they are predicated; a father must be somewhat else, to be a father. Cf. p. 20, n. 2, *supra*, and pp. 156–158, *infra*. Sometimes indeed an attributive term may signify more of the nature of the subject than the subject term does, e.g. if I say 'the obstacle was human', meaning 'a human being'; for to be a human being is more of the nature of the subject than to be an obstacle.

[2] *System of Logic*, I. ii. 4.

[3] Mr. F. H. Bradley however holds that a relation between two terms must be related to them by a second relation, and so *ad infinitum*, and the impossibility of this infinite process is one reason why he holds that Reality cannot be, though it may appear as, a system of terms in relation. Cf. *Appearance and Reality*, Bk. I. c. ii. The view in the text has the support of Professor Cook Wilson.

among terms. For the terms are the parts into which a proposition is resolved; in them, taken singly, the act of predication is not seen; they are as it were dead members, which could only have been taken apart because the life of judgement had fled and no longer bound them together. But in the verb this life lingers, even if a verb be taken without its subject. Hence logicians, anxious to express a judgement in a way to facilitate its resolution into its terms, have often preferred to sunder, even in language, the word which expresses the predicate from that which expresses its predication: to take the term as it were out of the verb, and say of Lear not, with the doctor,[1] that he 'sleeps still', but that he 'is still sleeping'. Now in such a case the predicate is often adjectival in form; although not always, for the proposition 'He plays cricket' would become, if it were meant that he played habitually, not 'He is playing cricket' but 'He is a cricketer'. Such an adjectival predicate is one of the parts into which the proposition is resolved,[2] whereas the verb belongs rather to the unresolved proposition. The whole question of the separate character of the adjective, or adjectival word, belongs indeed rather to grammar than to logic. But when 'term' means name, or term verbal, as these are either substantival or adjectival, and the concrete and abstract are both substantival, some place is wanted for the adjectival, and so they are classed separately as attributive terms. If their form were to be ignored, and they were to be referred either to concrete or to abstract, they should rather be considered abstract than (as J. S. Mill would have it) concrete; for their invention implies the consideration of some quality or character in the thing in abstraction from the rest of the thing's nature.

A special class of terms is constituted by those which are called **collective.** Like most other distinctions of terms recognized in Logic, this is based on a distinction in things. Individual things or persons may be considered singly: they may also, since there are many of them, be considered in groups; and the names of such groups are collective terms. Thus a group or collection of books forms a library; a group of human beings related in certain ways

[1] *King Lear*, Act iv. 7. l. 13. Cf. p. 17, *supra*.

[2] Adjectives can indeed be used as subjects, e.g. *Beati immaculati in via*, where it is possible to take either term as predicate. In many languages an article is generally necessary in order to make an adjective do duty as a substantive.

forms a family ; related in rather different ways, a tribe ; in other ways yet, an army or a club. Any term that denotes a collection of objects, with certain resemblances or relations among them, is collective. Collective terms may be either singular or general ; for we may wish to refer to a group composed of determinate individuals (as when we say 'the family of King Henry VIII') or simply to a group of individuals, no matter who or what, that is composed in a certain way, such as a family or a regiment : but they are the names of the individuals taken together, and not of the mode of organization among them.[1] A general collective term is said to be used *distributively* of the different groups that it can severally denote, and *collectively* of the individuals in any one group ; thus if we speak of British regiments the term is used distributively of the Coldstream Guards, the 60th Rifles, the Argyll and Sutherland Highlanders, &c., and collectively of the men in each several regiment.[2]

We may sum up what has been so far said of the kinds of terms as follows :—Terms as individual objects of thought are either concrete or abstract ; as names or terms verbal, concrete abstract or attributive ; there are also names of universals, which are commonly classed as abstract : concrete terms (verbal) are either singular, and then either proper names or designations, or else general : abstract terms can only be made singular by help of a singular concrete term, and without this are general ; some concrete (and a few abstract) terms are collective, and some abstract terms denote attributes of a group or aggregate, not of its members. It may be added that attributive terms are obviously general.

We pass now to a fresh division of terms, made from another point of view. As we may give a name to a group of things taken together, which would apply to none of them by itself, so we may give to a thing or quality, when we regard it in its relation to some other thing or quality, a name which would not apply to it considered in itself. Such terms, attributing to one thing or quality some definite relation to another, are called **relative** terms : and in contrast with them, terms that indicate a thing or quality

[1] We may speak collectively of a group of abstracts, as when we say that the natural are more numerous than the theological virtues, or that the Trivium and Quadrivium may be traced back to the fourth century B.C. My attention was called to this by Miss Augusta Klein. But there are no *names* for groups of instances of attributes or relations ; terms indicating them must do so by reference to the individual subjects in which they are displayed.

[2] The frequent division of terms into abstract, concrete, and collective, as if the third were co-alternative with the other two, is therefore a mere blunder.

considered in itself are called **absolute.** It is clear that if one thing or quality stands in relation to another, the latter must also stand in relation to the first; and the name applied to the latter to indicate this reverse relation is 'correlative'; or, since each is correlative to the other, the two together are called **correlatives.** Instances of relative terms are *equal*, *greater*, *subject*, *parent*: with their correlatives *equal*, *less*, *ruler*, *child*; *apple*, *sound*, *man* are absolute terms.

Relative terms are necessarily general,[1] like attributive terms; for the same relation may be exemplified in many particular instances, and therefore many subjects may stand in that relation which the relative term is used of them to indicate. They have this further resemblance to attributive terms, that though meaning a relation, they are applied to a subject standing in that relation: as attributive terms are to a subject possessing the attribute which constitutes their meaning.[2] The existence of attributive terms is grounded in the fact that the various objects of our thought do possess distinguishable attributes; and that of relative terms in the fact that they do stand in distinguishable relations one to another. It has been contended that all terms are really relative, because every object of thought stands in relation to other objects; at least only the totality of existence can be absolute, beyond which there is nothing for it to stand in relation to. But though it is true that everything stands in relation to other things, things are sometimes considered rather in themselves, and receive names accordingly; and sometimes they are considered in definite relations to another thing, and receive names that indicate that particular relation. And this is sufficient ground for the distinction between absolute and relative terms, though there are cases in which it is hard to say whether a given term is one or the other. *Man* is clearly absolute, and *father* relative, though *mountain* might be disputed; for a mountain is so only by its elevation above the plain, and yet in calling it a mountain we have in mind many features besides this relation.

Terms have been further divided into *positive*, *negative*, and *privative*. A **positive** term is said to imply the presence of a quality (or qualities), e. g. *greed*, *greedy*: a **negative** term to imply the absence of a quality, e. g. *colourless*, *unfit*, *unfitness*: a **privative** term to

[1] Except when a relative word is combined with others into a term whose whole meaning is singular: e. g. *first* is general, but *the first Pharaoh* is singular.
[2] Cf. *supra*, p. 37, n. 1.

imply the absence of a quality where it has been or might be expected to be present, e.g. *deaf*, *deafness*, *desiccated*.

There is a certain difficulty in the notion of a negative term, and in the account of it just given; for no term can be purely negative, and imply merely the absence of a quality. The Irishman's receipt for making a gun, to take a hole and pour iron round it, is not more difficult to execute, than it would be to frame a term whose meaning consisted simply in the fact that a particular quality was not meant. A term must have some positive meaning, in order to be a term at all.

It is indeed sometimes said that a negative term includes in its meaning whatever is not meant by the corresponding positive term. According to this view, there is no positive term to which we may not frame a corresponding negative; to *man* there corresponds *not-man*, to *book not-book*, to *square not-square*, to *colour not-colour*; *not-man* is everything which is not man, and includes therefore not only the other animal species, but plants and minerals, books and institutions, birth and immortality; *not-book* includes all these but books, and man besides; and so forth. The two 'contradictory' terms (as they are called) comprise between them all that is; nothing can be conceived, of which one or the other is not predicable; and they divide the universe between them. What the positive term is, does not matter; for whatever it be, the negative term covers everything else; and therefore it may be expressed by a symbol; let A represent any term, and *not-A* its contradictory; we may then say that A and *not-A* between them make up all that is, or that there is nothing of which one or other may not be predicated. 'Everything is either A or not-A.'[1]

[1] This formula, 'Everything is either A or not-A,' is sometimes given as the '*Law of Excluded Middle*'. The 'Law of Excluded Middle' (cf. *supra*, p. 13) is that of two contradictory propositions one or other must be true; they cannot both be false, and therefore any third or middle course between accepting one and accepting the other is excluded. It has been asked whether either of such contradictory propositions as *Virtue is triangular* and *Virtue is not triangular* need be accepted; the former is clearly false, but the latter does not seem true. The answer is that if any one were to assert that virtue is triangular (as the Pythagoreans held justice to have the nature of a square) we should be right to contradict him; but that no one who realizes virtue to be incapable of any spatial character at all would ever put to himself the alternatives, 'is virtue triangular or is it not?' and that to one who, not realizing this, asserted it to be triangular, the proper contradiction is that it has no figure. The case therefore furnishes no exception to the truth of the Law of Excluded Middle, provided the alternatives are not at the outset realized as nonsense; but no one to whom they are nonsense would expect

Such negative terms as these do not really figure in our thought; they are 'mere figments of logic'[1]; Aristotle long ago pointed out that *οὐκ-ἄνθρωπος* was not properly a name at all; and he perhaps extended his countenance too much to it, when he said that, if we were to call it anything, we must call it a 'name indeterminate' (*ὄνομα ἀόριστον*) because, being the name of nothing positive and in particular, it had a purely indeterminate signification; it was applicable equally to things existent and non-existent.[2]

The invention of such terms however is explained when we remember the relation of a term to judgement. The latter, as we have seen, is the primitive and remains the complete act of thought, and terms are got by abstraction from it. Now the affirmative judgement 'All flesh is grass' may be resolved into the terms *flesh* (the subject) and *grass* (the predicate *affirmed* of it); and the negative judgement 'Man is not a fly'[3] into the terms *man* (the subject) and *fly* (the predicate *denied* of it). But since we do therein *affirm* that man is *not* a fly, it seems possible to say that the predicate, *not a fly*, is affirmed of man, as well as that the predicate *fly* is denied of him. This attempt to reduce negative and affirmative judgements to a common affirmative type, by throwing the negative into the predicate, is not really defensible, for the negative term *not a fly* does not signify the nature of anything, and so is not really a term; it should, if it were a general term covering everything except the corresponding positive, be predicable of all subjects except flies in the same sense; but there is no common character in all these which it is intended to signify. Hence, as we should not take the trouble to affirm of man nothing in particular, the only point of the judgement must lie in denying of him something in particular; so that the meaning of the 'infinite' judgement (as it

to test by them the validity of the laws of thought; for talking nonsense is not thinking. The objection to stating the Law of Excluded Middle in the form 'Everything is either A or not-A' is this, that it seems to sanction the formation of nonsensical contradictories, such as we have examined, no less than of contradictories that are rational. Cf. also Bradley, *Principles of Logic*, I. v. §§ 23, 24.

[1] Stock, *Deductive Logic*, § 133.

[2] *de Interpr.* ii. 16[a] 30–33: the technical term in Latin is *nomen infinitum*, whence the English phrase '*infinite term*' is derived: but *infinite* means in this context *indeterminate*; and for the sake of perspicuity, the latter word has been used in the text.

[3] Why hath not man a microscopic eye?
For this plain reason, man is not a fly.
—Pope, *Essay on Man*, i. 193.

is called) 'Man is not-a-fly' lies in the negative judgement 'Man is-not a fly', and it is clear that we have not resolved the negative into the affirmative form, when such affirmative can only be understood by restoration to the negative.[1] But it is out of such attempts that so-called purely negative terms like 'not-fly' have arisen; and it is only by understanding that the term *A* has been the predicate of a negative judgement, that we can understand how the term not-*A* should ever have been formed.

There are however certain negative terms which are not such mere figments of logic as the 'infinite terms' considered above. Where the positive is not a general concrete term but is attributive, there the corresponding negative may be quite legitimate; indeed the distinctions of positive, negative, and privative most properly apply not to all, but only to attributive terms, or to abstract terms founded upon these.[2] For all attributive terms imply a subject of which they may be predicated, and to which they refer that attribute which constitutes their meaning. Therefore even if the term be negative, it still suggests a subject which, lacking the attribute which the negative term excludes, is conceived as having some character instead. And here we have a basis of positive meaning to the negative term; for let *A* be a positive term; then *not-A* will signify what a subject, *which might be A*, will be if it is not *A*. Thus *intemperate* signifies what a man, who might be temperate, will be if he is not that; *uneven* suggests what a line or surface, such as the surface of a road, will be if it is not even; *not-blue* suggests what a thing which might be blue (that is, an object having *some* colour) will be if it has not that colour. The definiteness of the positive meaning which a negative term thus conveys will vary greatly, according to the range of alternative attributes which we conceive possible to a subject that might conceivably have possessed the attribute denied of it; thus *intemperate* has a more definite meaning than *not-blue*, because when temperance is excluded, though there are many degrees of intemperance, yet they have more affinity with one another as opposed to temperance than have the remaining colours as opposed to blue; *unruffled* has a more definite meaning

[1] Cf. Arist. *Metaph.* Δ. vii. 1017ᵃ 18 *οὕτω δὲ λέγεται καὶ τὸ μὴ λευκὸν εἶναι, ὅτι ᾧ συμβέβηκεν, ἐκείνῳ ἐστιν* ('And in this sense the not-white is said to be, because that is which is not white')—i.e. to be not white cannot itself constitute the being of anything, but that may have a positive being of which we can deny that it is white.

[2] Cf. *infra*, p. 45.

still, for a surface which is not in any way ruffled can only be smooth.[1]

It has been alleged that 'not-blue' does not necessarily imply 'coloured in some other way than blue', nor 'not-even' a surface of another kind than even; that it is as true to say of banter that it is not blue as of a buttercup, and that larceny is as much not-even as Lombard Street. But such a contention misinterprets our thought. Just as privative terms imply the absence of an attribute from a subject that possessed or should have possessed it, and therefore must convey a notion of what the subject consequently is without that attribute, so negative terms (at any rate when they are not mere figments of logic) imply the absence of an attribute from a subject that might conceivably have possessed it, and therefore convey a notion of what the subject is instead. The attribute which a negative term excludes belongs to a genus of attributes (as blue belongs to the genus colour, or prudence to the genus feature of human character, or square to the genus figure); and if a subject is unsusceptible of any attribute within that genus, we should not be at pains to deny of it some particular attribute therein; since the soul for example has no figure, we should not say that it is not-square; since furniture has no feature of human character, we should not call a towel-horse imprudent. The negative term is only used of what must have some attribute within its genus; and this genus furnishes a substratum of positive meaning to the negative term; *not-blue* does mean 'coloured not with blue' and *not-even* 'having a surface which is uneven'.[2]

[1] The old Greek proverb will illustrate the point here—*ἐσθλοὶ μὲν γὰρ ἁπλῶς, παντοδαπῶς δὲ κακοί* ('Men are good in one way, but bad in many').

[2] The genus within which any attribute falls, or the subjects susceptible of some attribute within that genus, may be called with de Morgan (*Formal Logic*, p. 41) a 'limited universe'; thus *blue* is a predicate in the universe of colour, or of coloured objects: *prudent* in the universe of human character. A positive term and its corresponding negative (e.g. blue and not-blue) may then be said to divide between them not indeed the whole universe, but the limited universe or whole of things to which they belong; the members of this limited universe have a positive common character, which gives the negative term a positive meaning; whereas if we consider the whole universe, there is no positive character common to all things included in it, except the character of being - which, as Aristotle pointed out, considered in itself and not as realized in some special mode of being, is not a significant term: cf. p. 50 *infra*, and *de Interp.* iii. 16^b 22. Such a 'limited universe' is sometimes called an 'universe of discourse'; but this only means the limited whole which is the subject of discourse, and its limits—e.g. those of the whole within which blue and non-blue fall—are determined by the nature of things, not by our discoursing of it.

Many negative terms indeed are not themselves attributives, but are abstracts which presuppose an attributive; and what has been said of negative attributives is confirmed by the fact that these abstracts—such as *injustice*, *inequality*, *non-intervention*—are very positive in their meaning. 'Injustice' does not mean whatever is not justice (such as 'accidence and adjectives and names of Jewish kings'), but the quality of being unjust; 'inequality' means the relation of being unequal; 'non-intervention' the conduct of the non-intervening. Abstract negative terms like *not-equality* or *not-colour* are as unreal as concrete negative terms like *not-Socrates* or *not-book*.

It may be asked, if all negative terms (and the same is true of privative) have a positive meaning, what is the use of the distinction between them? The answer is as follows. First, with regard to the distinction of positive and privative terms; there are some states which can only be understood as the *privation* of a positive state: deafness would have no meaning, but for our knowing what it is to hear; we cannot think of a body as desiccated, except we think of it as having first contained moisture.[1]

Secondly, with regard to the distinction between positive and negative terms: there is a real difference between a term which signifies one definite attribute, and a term which signifies any attribute within a genus except one; the latter is in most cases[2] comparatively indeterminate and uninstructive; e. g. *vertebrate* signifies a

[1] These two examples are not quite parallel. A notion of deafness can be had by any one who knows what hearing is. A notion of 'desiccated' cannot be had by any one who knows what moisture is, but he must also know what dryness is. 'Desiccated' is a privative term, because it means a dryness due to the withdrawal of moisture previously present; but 'dry' is just as positive a term as 'moist'. It sometimes happens, with two mutually exclusive alternatives like dry and moist, that men dispute whether or not both are positive. Some philosophers have maintained that pain is merely the privation of pleasure, and evil the privation of good; others, that pain and evil are just as positive as good and pleasure. In these cases, it will be also in dispute, whether or not pain and evil are privative terms. But the dispute arises from our uncertainty how to think about the things; and so furnishes another illustration of what has been pointed out in the text, that logical distinctions of terms reflect and are based upon distinctions in the things thought about.

[2] Sometimes, as Miss Augusta Klein has pointed out to me, the positive term may be less determinate in meaning; there are more ways of being coloured than colourless, of being fed than unfed. Here obviously the negative term has a positive meaning; we know the look of a colourless fluid, and an unfed animal is in a very positive state. If it be said that 'unfed' has a meaning also for those who do not know what state an animal is in which has not been fed, we may reply that for them it means 'which has not been fed', and so mere negation is shown to belong to judgement, as stated above.

definite anatomical structure ; *invertebrate* signifies an animal structure which is not vertebrate, but fails to characterize it further. Positive terms are positive directly and precisely, negative terms indirectly and for the most part vaguely. This distinction is important, and we are therefore justified in calling attention to it ; it will be seen for example presently [1] to be one of the rules of definition to state what a thing is, not what it is not ; this is best expressed by the injunction to avoid, as far as possible, negative terms ; and there is no way in which the point of this instruction could be so well conveyed as by the help of the distinction of negative and positive terms.

[The doctrine about negative terms impugned in the foregoing paragraphs furnishes a good example of the dangers that beset a purely formal logic. If we regard only the form of a proposition, '*A* is not *B*' (in which the terms are *A* and *B*), we may 'permute' it to the form '*A* is not-*B*' (in which the terms are *A* and *not-B*) ; and we may formally regard *A*, *B* and *not-B* all equally as terms. But whether *not-B* is a genuine predicate, and the proposition 'A is not-B' really affirms anything, will depend upon the matter of the proposition—upon what kind of a term *B* stands for. In respect of form, *B* has a corresponding negative *not-B* ; but we cannot tell by considering the form alone whether any thought or notion of *not-B* is possible. It may be noted also that the Law of Contradiction should not be formulated symbolically as 'A cannot be both B and not-B', or 'A cannot be not-A', but rather as 'A cannot both be and not be B', or 'A cannot not be A'. For if *not-B* is something positive other than B, or *not-A* than A, what is B or A may have such other positive character besides. If 'to be not-B' is necessarily inconsistent with being B, it is neither more nor less than 'not to be B'.]

We have still to notice the distinction of *univocal*, *equivocal*, and *analogous* terms. **Univocal** terms are terms with only one meaning, so that they are used in the same sense of every subject of which they are used at all : **equivocal** (or **ambiguous**) terms are terms with more than one meaning, so that they may be used of different subjects in different senses—e.g. *fair*, as used of a complexion and of a bargain : **analogous** terms are terms which have more than one meaning, but the meanings have a certain degree of identity or correspondence—e. g. we speak of the foot of a man and the foot of a mountain, meaning different things, but in both cases that on which something stands. We ought in strictness to regard

[1] Cf. *infra*, p. 98.

this distinction as one not in terms but in the use of terms; for *fair* is used univocally of all fair complexions, and is only equivocal when we use it at once in different senses. All proper names belonging to more than one individual are used equivocally of such different individuals.

[The history of the words univocal, equivocal, and analogous will illustrate the tendency to treat Logic from the standpoint of an affair of names. The Aristotelian distinction already alluded to (p. 31) between *συνώνυμα* and *ὁμώνυμα* was one of things. *Univocum* and *equivocum* are merely translations of *συνώνυμον* and *ὁμώνυμον*, and they were defined in the same way (cf. Crackenthorpe's *Logic*, Bk. II. c. i. 'Aequivoca ita describuntur: aequivoca sunt quorum nomen solum est commune, ratio vero illius nominis est alia atque alia.' c. ii. 'Univoca describuntur in hunc modum: univoca sunt res vel entia quorum nomen est commune, et ratio illius nominis est una et eadem in omnibus quibus nomen convenit'). Similarly, it would have been not the word 'foot', but the man's and the mountain's foot that would have been called analogous. In the sense in which terms are not words, but the objects of thought intended by the words, we might still say that equivocal terms are different objects of thought with the same name, rather than the same name with different meanings. But in English usage the distinction of names has really displaced that of things: we do not even (except for the word *analogous*) retain both, like the Latin, when it was said that 'aequivoca' were either 'aequivocantia, ipsae voces aequivocae', or 'aequivocata, res ipsae per illam vocem significatae'. And even in Aristotle, *Rhet.* γ. ii. 2. 1405[a] 1, we find an example of the use which calls words synonymous. Cf. also *Journal of Hellenic Studies*, vol. xxix. pp. 28 and 32, where *συνώνυμον ὄνομα* (=synonymous noun) is reported from a school tablet found in Egypt and belonging apparently to the third century A. D.]

CHAPTER III

OF THE CATEGORIES

The distinctions between terms discussed in the last chapter are not primarily grammatical, like the distinction between substantive and adjective (though here and there, as we saw, the forms of language have affected the mode in which they have been drawn); nor do they belong to any special science, like the use in chemistry of names in *-um* to signify metals, and names in *-ide* to signify compounds. They may be illustrated from all sciences, and are based on certain features that reveal themselves to reflection about any subject whatever; and that is why they belong to Logic. But they involve not only features of thinking, like attribution, affirmation, negation, but also features in what is thought of; and so far they belong to Logic only because the thought which Logic studies is thought about things, and we cannot separate the study of thought from the study of the most general nature of things thought about—such nature as they must have, if they are to be objects of thought at all. It is of special importance to remember this in considering the Aristotelian doctrine of **Categories**, out of which some of the preceding distinctions take their rise. The Categories present a logical, but they present also a real distinction: i. e. a distinction in the nature of the reality about which we think, as well as in our manner of thinking about it.

We saw[1] that reflection on the form of judgement 'A is B' leads us to ask in what sense one thing *is* another; that sometimes it is meant that the predicate character, B-ness, is incidental to the subject A, sometimes that to be A is essentially to be B; thus 'The Emperor is captured' does not mean that to be Emperor is to be captured, but 'Man is an animal' does mean that to be a man is to be an animal. Out of such reflection arose the doctrine of the Categories.[2]

The word category, κατηγορία, means predicate[3]; but its predi-

[1] Cf. *supra*, pp. 22–24.

[2] Cf. Arist. *Metaph.* Δ. vii.

[3] The Latin equivalent is *Praedicamentum*, and Aristotle occasionally writes κατηγόρημα instead of κατηγορία, which means predication as well as predicate: v. Bonitz, *Index Aristot.*, s. vv. κατηγόρημα and κατηγορία.

cate is what any subject *is*; and the categories may be described as a list of predicates, one or other of which declares the mode of its essential being belonging to any subject that exists. In Aristotle's complete list there are ten, viz.

οὐσία	*substantia*	substance
ποσόν	*quantitas*	quantity
ποιόν	*qualitas*	quality
πρός τι	*relatio*	relation
ποῦ	*ubi*	place
ποτέ	*quando*	time
κεῖσθαι	*situs*	situation
ἔχειν	*habitus*	state
ποιεῖν	*actio*	action
πάσχειν	*passio*	passion (being acted on)

These Aristotle calls both 'kinds of predicate', γένη τῶν κατηγοριῶν, and 'kinds of being', γένη τῶν ὄντων. We must examine the latter phrase first, if we wish to understand his doctrine.

In the form of proposition 'A is B', as just observed, the predicate does not seem equally in all cases to declare what the subject is. A man is an animal, and a man is in the kitchen; Tray is a dog, and Tray is happy now; a musician is an artist, and a musician is breaking my hurdy-gurdy: if we look at these judgements, we shall admit that the second does not tell us what a man is so much as the first; that the third is a fuller answer than the fourth to the question 'What is Tray?'; and that the fifth is a fuller answer than the sixth to the question 'What is a musician?'. In Aristotle's phrase the first, third, and fifth of them declare what their respective subjects are καθ' αὑτό, or *per se*: the second, fourth, and sixth what they are κατὰ συμβεβηκός, or *per accidens*. In other words, the predicate is in the one case of the essence of the subject, and 'covers its whole being'[1], and the subject could not exist at all without its being predicable of him; in the other case it is an accident of the subject. What is predicated of a subject καθ' αὑτό tells you what it is necessarily, permanently and constitutively[2]; what is predicated of it κατὰ συμβεβηκός tells you indeed something about it, but something less important, and perhaps unnecessary, to its being—

[1] Cf. *supra*, p. 23.

[2] This is not a complete statement of the meanings in which, according to Aristotle, a predicate may be said to belong to a subject καθ' αὑτό; but it is, I think, a sufficient account of the sense in which the expression is used in this connexion.

something of which it could be divested, and still remain the thing it is, at least something not constitutive of it as such a subject.

The ultimate subject of predication is the concrete individual thing—you, Socrates, Bucephalus, or the stone in your signet-ring[1]; and if you ask of this what essentially it is, you will have to specify in your answer some kind of *substance*[2]; you are a man, Bucephalus is a horse, the stone in your signet-ring is an agate. All these—man, horse, agate—are so many different substances; in saying what you, Bucephalus, or the stone in your signet-ring is essentially, or *per se*, these are the answers I must give; their essential being, therefore, is to be some kind of substance, and the predicates which give their essential being are in the category of substance. But if I ask what is a substance, I cannot find any more general character under which to bring that, as I bring Bucephalus, in declaring what he is, under horse, and horse, in declaring what it is, under substance. Of substance I can say that it is a kind of being; for substances are one kind of things that are; but it is of no use to treat mere being as a genus, of which substances are a species, for to being considered in itself, and not as a determinate way of being (e. g. being a substance), I can attach no meaning.

On the other hand, there are a great many subjects, about which, if asked what essentially they are, I could not possibly say that they are substances. Large, loud, blue, heavier, here, yesterday, fever, horizontal, running, defeat, virtue—each of these is something, or nothing could be said to *be* it: but what are they? Directly or indirectly they all presuppose substances; if there were no animals, there would be no fever: if no fighters, no one could be defeated. But they are something incident to substances, attributes or relations and not things. To say that they are attributes, however, only declares their relation to something else, their dependence; it does not declare what they are in themselves. If we

[1] This is the true meaning of the statement in *Cat.* iii. 1^{b} 10 ὅταν ἕτερον καθ' ἑτέρου κατηγορῆται ὡς καθ' ὑποκειμένου, ὅσα κατὰ τοῦ κατηγορουμένου λέγεται, πάντα καὶ κατὰ τοῦ ὑποκειμένου ῥηθήσεται ('When one thing is predicated of another as of a subject *de quo*, all that is asserted of the predicate will be asserted of the subject as well')—a statement sometimes erroneously quoted as equivalent to the *Dictum de Omni et Nullo*. Cf. *infra*, c. xiv. p. 297 n.

[2] But concrete things sometimes receive names implying their possession of predicates in some other category than that of substance; e. g. a threshold is a concrete thing, but in calling it a threshold I do not give its substance: to do that, I should have to say that it was a stone. It is a threshold because it is a stone in a certain *situation*.

ask that, we shall find ourselves ultimately giving as an answer some one of the other categories.

Thus I may say that 'yesterday was wet': but that does not tell any one the nature of yesterday in itself. But if I say 'yesterday is the day before that on which I am now speaking', I explain what yesterday in itself is. And if next I am asked 'What is that?', I should reply that it is a certain date or *time*; and there I must stop. The kind of being then which belongs to yesterday is not being a substance, but being a time. Similarly blue is a colour, and colour is a *quality*; loud also is a quality, and virtue; so that their being is being qualities; that is what essentially they are. Large is a size, i. e. to be large is to be of a certain *quantity*; to be heavier is to be in a certain *relation*, here is a *place*, fever is a *state* of the body, horizontal a *situation*, running an *action*, defeat a being *acted on*.

There is nothing then, according to Aristotle, that exists or can be thought of, which is not either a substance, or a quality, or a quantity, or in some other of the categories. One or other of them is predicable of everything; and they cannot be further reduced, or brought under any common head.[1] A quality is not a quantity, a time not a place, to do is not to be done to, nor any of these a situation: and so forth. It might be thought that *state* is hardly distinguishable from *quality*, nor *situation* from *place*. But they are not really the same. A state is something which characterizes a whole through the condition of its parts; thus we call a man shod, because he has shoes on his feet; or healthy, because each part of his body is functioning rightly; the healthiness of his body as a whole does not mean that each part of it is qualified alike, nor his being shod that every part of him has shoes on. A quality, on the other hand, is comparatively simple, and if it characterizes

[1] As a matter of fact, however, the category of relation is not equally excluded by the others; and Xenocrates is said to have reduced them all to Substance and Relation. In doing this he would not have effected a real simplification, any more than if they were all reduced to Being; for time, place, action, &c., involve irreducibly different kinds of relation; and mere relation, which is not any definite kind of relation, is almost as barren a concept as mere being. Aristotle probably erected relational predicates into a separate class because they tell us less than others what a subject is (cf. *Metaph.* N. i. 1088^a 23). 'Six feet high' would be in the category of ποσόν: 'taller than his neighbour' in that of πρός τι; it gives more information about what a man is to say that he is six feet high, than that he is taller than his neighbour. The latter predicate may change when his neighbour changes; the former only by a change in the man himself. The former involves relation also; but the latter is more plainly and purely relational.

a whole, does so through being present in the same way in its various parts; if a whole surface is blue, that is because the various parts of it exhibit the same colour, and if a trader's stock is sweet, that is because the things it is composed of are severally sweet. A state, therefore, is more complex than a quality; and so it is with situation and place. 'Upside down', 'horizontal', 'sitting', 'standing', are in the category of situation—predicates which determine not where a thing is, but its 'lie' or position there. Without place there could be no situation; but you do not determine a thing's situation by assigning its place.

The categories, therefore, are a list of predicates, one or other of which must in the last resort be affirmed of any subject, if we ask what in itself it is. They are *γένη τῶν κατηγοριῶν*, kinds of predicate, and equally *γένη τῶν ὄντων*—the kinds of being which we recognize, the kinds (if we may put it so) of what things are.[1] These things, the ultimate subjects of predication, are individual substances, and the categories do not give a classification of these, as is given when things are said to be animal, vegetable, or mineral; they give a classification of the kinds of being displayed in and predicable of them. Those predicates express most fully the being of an individual substance which are in the category of substance, like *man*, *rose*, *gold*; they tell us what essentially it is. But every predicate tells us what in some sense it is, and the kinds of being displayed in what else it is are the other categories beside substance. Thus the distinction between substance and the other categories is a prominent feature of the doctrine; for all the others presuppose and are incidental to substance, since predicates belonging to them are displayed in the being of individual substances. Terms in these other categories may be subjects of predication, as when we say that blue is a colour, and that wisdom is rare; but they exist not independently but in concrete individuals. There is no blue except the blue of the sea or the sky, of a larkspur or a gentian, &c.; no wisdom, except that of the wise. Concrete individual things are substances in the strict and fullest sense. But what is predicated of them is

[1] Cf. Ar. *Met.* Δ. vii, and Apelt, *Beiträge zur Geschichte der griechischen Philosophie*, III. *Die Kategorienlehre des Aristoteles*. In the expression *γένη τῶν κατηγοριῶν*, 'kinds of predicate,' *κατηγορία* refers no doubt to the predicates of things, these predicates falling under the kinds enumerated, not to the heads or most general predicates under which these fall. Hence the concrete individual is not in any category, since it is not what any further subject is (cf. *Cat.* v. 3^a 36 *ἀπὸ μὲν γὰρ τῆς πρώτης οὐσίας οὐδεμία ἐστὶ κατηγορία*, 'for first substances furnish no predicates').

partly in the category of substance, partly in the other categories. We have here that distinction between first and second substances which once occupied so much of the attention of philosophers and theologians.[1]

First substances are individuals like Socrates or Cicero; second substances are the kinds of these, and terms are in the category of substance which, like man, horse, peppermint, parsley, tell what kind of thing an individual is. All else that is said of an individual tells only some quality or state that characterizes him, his activity or situation, his relation to others, &c., and is therefore a predicate in one of the remaining categories.

Undoubtedly it is here that the chief difficulty in Aristotle's conception lies. But the difficulties are not gratuitous; they arise naturally in our reflection upon the nature of things.[2] We naturally incline to think, in considering any concrete individual, that out of all that characterizes it some part is more essential than another, goes more to make it what it is. This we call its kind, and Aristotle called it also its substance; and language contains names that are evidence of this, kind-names like man, horse, gold. It is indeed very hard to say exactly what constitutes the kind; kind-names, as we shall see later, present special obstacles to definition; and a positive account of the substance of an individual seems beyond us. But negatively there is a great deal which we should say does not belong to the substance—the place where the individual is, what it momentarily does or suffers, all in fact that we can refer to other categories. All these we tend to think of as attributes which the individual has, but that it can exist irrespectively of them: whereas, irrespectively of its kind, it would no longer be at all. And yet the kind is universal; it is predicated of more things than one; Socrates, Plato, and millions more are men; the lumps of iron in the world are uncountable. Hence follow two lines of reflection.

First, because the kind, though universal, is at the same time more substantial than the other predicates of an individual are—more concrete, in fact, than they—the kind, or 'second substance', comes to be thought of as having some special claim to independent existence. Other modes of being, other predicates, depend on it; but it is thought of as depending on nothing else for its existence.

[1] This mode of expressing the distinction comes from *Cat.* v. 2^a 11–19.
[2] Cf. *supra*, pp. 28, 35.

True that we only find the kind realized in some concrete individual; nevertheless it is not a mere attribute of the concrete individual, as are predicates in other categories. And some have held that these 'second substances', though displayed in divers individuals, are each not only genuinely one and single, but real, whether there be any concrete individual of their kind or not.[1]

But secondly, because the kind is universal, it is predicated *of* the concrete individual, like predicates in other categories. And as the individual *is* something which *has* them, so it *is* something to which its kind is *attributed*. It cannot be identified with its kind; for then there would be nothing to distinguish one individual from another. Man is predicated equally of Socrates and Plato, and if each as an individual substance were just man, Socrates would be the same as Plato. Therefore we must look elsewhere for what distinguishes them. If we find it in the other predicates of the concrete individual, and say that he is the kind *plus* all his particular attributes, we resolve the individual into an assemblage of universal predicates. If we do not do this, but suppose that his kind and all his particular attributes as well *belong* to the individual, the individual, to which they all belong, becomes a mere uncharacterized *something*. For in saying *what* it is, we should merely assign to it a fresh predicate; whereas we want to get not at its predicates but at that which 'has' them. Thus we should reach a new way of considering the subject of predication. Originally it was the concrete individual, Socrates or Plato; but of what he is, one part was distinguished as what he is essentially, and the rest reduced to be attributes or 'accidents' of him, not necessary to his being, and not to be included in an account of his essence. Now, what he is essentially is also reduced to the position of attribute and mere predicate, and the subject becomes a mere subject of which as such nothing more can be said except that it exists and is unique in each individual. This mere subject of predicates, which cannot in itself be described as specifically of this kind or of that, Aristotle called *matter*.[2] We only know matter in conjunction with form; bricks and timber are the matter or material of which a house is built, but a brick is in turn clay to which a certain form has been given; clay again is matter of a certain form; but matter by itself—

[1] Cf. *supra*, p. 32.
[2] Cf. Ar. *Phys.* a. vii. 191ª 8–12, z. iii. 1029ª 23.

that which is found in various forms, but has no form of its own—is unknowable.[1]

It may be questioned whether Aristotle was justified in his use of the conception of matter. He started by thinking of the material out of which a thing is made. Now the material of anything is always something quite determinate. Economists know in how many ways the products of one industry are 'raw material' to another; but the raw material which is rawest, i. e. which has itself been least worked up, is still matter of a perfectly definite kind. Timber is the raw material of the carpenter, but trees of the lumberman: pig iron of the ironmaster, but iron ore of the smelter; and neither trees nor iron ore are any nearer being formless matter than timber or pig iron. In these cases, the matter (or material) is a concrete thing, in a different state no doubt from that into which it is worked up, but perfectly familiar to us as existing in that state; but in the philosophical antithesis, the matter is not a concrete thing at all, is in no state, is quite unfamiliar and indeed incapable of being known to us as such; and this relation of matter to form has no real analogy with the relation of matter to what is made out of it in the arts.[2] It is true that in using the metaphysical analysis of the concrete individual into matter and form in order to find in different individuals different subjects of the same form, I may not at first sight seem to need the conception of a quite indeterminate matter. The matter of a house, says Aristotle, is stones and timber; the form—what makes the stones and timber the matter of a *house*—is 'to be a shelter for men and goods'. Stones and timber are determinate material, as 'to be a shelter for men and goods' is a determinate form. But suppose two houses built to the same specification; what distinguishes them? We say, that they are built of different materials—different stones and timbers. But what distinguishes these? Not their form, since *ex hypothesi* they are of the same form. We may say that they just *are* different, and leave it at that. But if we are going to use the analysis into matter and form to explain their difference, since they are not distinguished by *what* they are, their predicates, we must find the ground of their difference in the difference of the matter; and this distinguishing matter must be taken as something divested of predi-

[1] ἡ ὕλη ἄγνωστος καθ' αὑτήν, *Met.* Z. x. 1036ᵃ 8. Cf. *supra*, p. 35.

[2] In the foregoing criticism I am particularly indebted to lectures of Professor Cook Wilson.

cates, because in respect of predicates they are the same. The outcome of this line of reflection would seem to be that what makes possible different individuals of the same kind is the indeterminate matter of which *what* they are is predicated; and this at times Aristotle says,[1] and he admits that in one sense matter is substance. But the corollary, that the nature of Socrates, as predicated of this matter, is something that may be common to another, and universal, he does not draw; and it would seem to be his considered doctrine in the *Metaphysics* (however hard to reconcile with some of his other statements) that what makes Socrates Socrates is his form, or *what* he is, and not the matter in which this form is realized.[2] This form is really his substance, or substantial being; and it is neither merely the specific form of men, nor does it include all that can be predicated of him; but we are not told how to distinguish it from predicates in the other categories. We need not pursue the Aristotelian doctrine further; so much has been said in order to illustrate the difficulty of determining what is in the category of Substance. We start with the concrete individual, and draw a distinction, among all that can be predicated of him, between that which declares what he is essentially, and is his substance, or in the category of substance, and that which declares about him something not essential, and belonging to one of the other categories. But a *predicate* in the category of substance seems universal, as in any other; and if it belongs to several individuals, these must be distinguished otherwise than by it; hence the tendency to say that what individualizes is *material* substance, not universal, nor capable of figuring as predicate. But then the kind, what is predicated of individuals in the category of substance, ceases to be essential to them, for they would still be, and be individually different, without it. Thus the attempt to distinguish what is from what is not essential to the individual must either be abandoned in a doctrine of individual forms—for if we suppose that there is something *about* Socrates which makes him Socrates, we have no principle on which to select this from among the sum total of all his predicates; or else it leads us to distinguish the individual both from his essential and from his non-essential attributes, and then he is individualized by

[1] Cf. *Met.* Z. viii. 1034ª 5–8; and v. Bonitz, *Index Arist. s. v.* ὕλη, 786ª 52–58. But individuality cannot be explained by difference of mere matter: cf. *infra*, p. 90.

[2] Cf. *Met.* Z. x. 1035ᵇ 27–1036ª 9, xiii. 1038ᵇ 8–15; H. i. 1042ª 28–9. But one cannot really support any statement on the point except by reference to his whole discussion.

neither, and neither is essential to his being the very individual he is. The ' first substance ' is at the outset the whole concrete individual. We try to distinguish within what it is what is essential to it, and we only really find what is essential to its being *of a certain kind*. Taking this as what is essential to *it*, we regard it as constituting the individual, and so as possessing a substantiality of its own and being a sort of ' second substance '. But then we find that a second substance will not individualize.

We shall be met later with the same difficulty, when we consider the doctrine of the Predicables, and the problem of definition. The metaphysical issue raised is fundamental. But for the present it is enough to have called attention to it. Logical and metaphysical problems have a common root. We cannot reflect upon the being which is asserted in all predication, without asking how things can be conceived to exist. And it may readily be shown, with regard to the different categories in particular, that we could not use predicates in them, except so far as we conceived subjects to exist in certain ways. Thus no predicates in the category of quantity can be used of the mind, because the mind is not extended ; if it were, it might have a capacity of 3 or 30 cubic feet, and an area and maximum diameter ; since it is not, we cannot apply such epithets to it at all ; and it is only because the existence of material things *is* existence in space, that we can call them large or small, three feet square or four feet long. In the same way, if it were not for the fact that the world is spatial, there could be no predicates in the category of place ; and space also renders possible predication in the category of situation ; for it contains the distinctions of up and down, front and back, right and left ; and it allows the parts of a body to alter their relations to certain fixed points above and below, behind and before, to the left and right of them, while the whole body remains within the same limits. This is what happens when a man lies on the sofa where he was formerly sitting, or when an hour-glass is inverted on the table. And a perfectly homogeneous sphere, though it may change its place, can be situated only in one way ; and if we are to distinguish a right and wrong way up in it, we must mark or single out some point in the circumference, whereby it ceases to be perfectly homogeneous ; and this again illustrates how the distinction of categories arises out of the distinguishable modes of being in things. For it is because it is a figure of a certain kind, that such a sphere does not admit of the same varieties of situation

as a cylinder; and because it does not admit of these, they cannot be predicated of it; and if nothing could be perceived or imagined to admit of them, predicates in the category of situation, and therefore the category of situation, would not exist. Again, there are predicates in ποιεῖν and πάσχειν because things act one on another; and the two categories are distinguishable because there are two terms, agent and patient, in all causal interaction. And the different tenses of verbs, which make a difference to a predication in time, though it remains in the same category of ποιεῖν or πάσχειν, ἔχειν or κεῖσθαι,[1] presuppose that things exist in time; otherwise, how could we distinguish the meanings of ὑγιαίνει and ὑγίανεν, *vapulat* and *vapulabit*, *vivit* and *vixit*, *sits* and *sat*? Of that which had no continuous existence through differences of time, predication would be possible only for a moment in the present. But reciprocally, as we could not predicate in these categories unless things existed in certain ways—as substances, with qualities, extended in space, persisting in time, &c.—so we cannot predicate about things except in one or other category; in other words, if we think of anything, we must think it to be determined in one or another of these ways.[2] That which was not conceived as a substance, or a quality, or a state, and so forth, would not be conceived at all; and a concrete thing that was no substance, had no quality or state, and so forth, would be just nothing. And therefore the consideration of these distinctions belongs to logic, since the thought of them is involved in our thought about objects in general; and though logic is not interested in the indefinite variety of existing qualities—blue, green, sour, shrill, soft, &c.—(because a substance, in order to be a substance, need not have any one of these qualities in particular, but only one or other) yet it is interested in the *category* of quality, or in noticing that a substance must have some quality or other: in the *category* of relation, or in noticing that it must stand in relations to other things: and so on.

[1] i.e. action or being acted on, state or situation. It is to be observed that the predicate of the same proposition may determine its subject in more than one category. In the proposition 'The other disciple did outrun Peter' the predicate is in the category of time, for the past is a time, and the event is referred to the past: and of action, for running is an activity: and of relation, for 'faster than Peter' is a relation. But of course, if we distinguish these different elements in the predicate, we can refer them, considered separately, to different categories.

[2] It is not necessary, however, to hold that Aristotle's list of categories is complete.

The problem underlying Aristotle's doctrine of Categories may be expressed thus—to discover the forms of existence which must be realized in some specific way in the actual existence of anything whatsoever. His classification may exhibit defects, but the importance of his undertaking must be admitted. And many of the distinctions between terms insisted on by those who attach least importance to the Aristotelian doctrine of Categories express an attempt to solve part of the problem which he was attacking, and are derived from his doctrine. Those distinctions, as was pointed out in the last chapter, rest upon certain fundamental features of the existence which we conceive the objects of our thought to have. The distinction between singular and general concrete terms corresponds in the main to that between *πρώτη οὐσία*, the concrete individual, and predicates in the category of substance; for the most noticeable of general concrete terms are in the category of substance, as *man*, *stone*, or *beast*, though some (which might be called substantives of an attributive kind) are in other categories, as, for instance, *officer* and *organist*. The distinction between concrete and abstract terms corresponds roughly to the distinction between substance and the other categories. That relative terms are predicates in the category of relation is plain. The attention paid to collective terms reminds us that we can consider not only things severally, but what they are in certain groupings or combinations; and the distinction between quality and state involves the same fact.[1] The logical divisions of terms rest on differences apprehended in the being of things; this is apt to be overlooked when the subject is approached from the side of names; Aristotle's doctrine of Categories has this advantage, that throughout it fixes our attention on things.

[The Aristotelian doctrine of Categories bulks large in the history of Logic; such conceptions are instruments of thought; the instruments forged by one generation are handed on to the next, and affect subsequent thinking. On that account alone therefore it is fair to give some attention to it; but it is still valuable as serving to express and distinguish certain important features recognized by our thought about things. That a quality is not a quantity is a truth which those overlook who think that sound can *be* a wave-length in the vibration of the air; they forget that it is not possible to define terms of one category by another.[2] Moreover a conception

[1] It is not meant that collective terms are in the category of State.

[2] Except as terms in a derivative category involve terms in those from which it is derived.

[of categories not very far removed from that of Aristotle has, through Kant and Hegel, become one of the chief doctrines of modern metaphysics.

These admissions do not bind us to consider Aristotle's list as perfect. One important remark on it would perhaps hardly have been regarded by him as a criticism. The different categories are not all equally distinct or ultimate. Thus the distinction between *ποῦ*[1] and *ποτε*[2] is far more fundamental than that between *ποιεῖν*[3] and *πάσχειν*.[4] A thing need not have a place because it has duration, nor can any one doubt under which category such predicates as 'at home' and 'belated' respectively fall. But to be acted on implies something acting; indeed, if action and reaction are equal and opposite, for a thing to be acted on implies that it acts itself: and it is often difficult to say to which of these categories a predicate is to be referred. A ship travels: are we to attribute the motion to the ship, and say that she acts, or to the engines, and say that she is acted on? or shall we say that the engines in turn are acted on by steam? Aristotle in a measure recognized the mutual implication of these two categories, for in one place he includes them together under the single term *κινησις*.[5] Language bears traces of it also, in deponent verbs, which have a passive form with an active meaning, and neuter verbs, which have an active form with sometimes a passive meaning. We cannot admit, as Trendelenburg and others have maintained, that the distinctions of categories were derived by Aristotle from the grammatical distinctions between parts of speech; but undoubtedly they are reflected (though in an imperfect way) in grammatical forms. Again, as we have seen, the notions of *ἔχειν*[6] and *κεῖσθαι*[7] are derivative: *state* presupposes the distinction of whole and part, which, in material things at least, implies the category of *ποσόν*,[8] and it presupposes also the categories of *ποιεῖν*[3] and *πάσχειν*[4], and of *ποιόν*[9]; for a whole is in a certain state through the interaction of parts having certain qualities, as when the body is well or ill; or through something done to certain parts of it, as when the body is shod or clad; a *situation* presupposes the distinction of whole and part also (a point can have place, but no 'situation'), as well as the categories of *που*[1] and *πρός τι*[10]; for when a thing changes its situation, some part that was formerly above another comes to be below it, and so on. On these two derivative categories Aristotle lays least stress; they are only twice

[1] Place. [2] Time. [3] Action. [4] Being acted on.
[5] Movement, or change: v. *Met.* Z. iv. 1029^b 25. See for a conspectus of the lists of the categories found in different parts of the Aristotelian *corpus* O. Apelt, *Beiträge zur Geschichte der griechischen Philosophie, Kategorienlehre*, pp. 140-141.
[6] State. [7] Situation. [8] Quantity.
[9] Quality. [10] Relation.

[included in his enumeration. But though derivative, they are peculiar, and contain something not in the notions from which they are derived; it is quite impossible to treat a state like health as being of the same nature with a quality like sweetness, or place with situation in that place. Kant made it a ground of complaint against Aristotle that he had included derivative conceptions in his list along with pure or underivative; but it would probably be a fairer criticism, that he had not taken account of all the derivative conceptions which call for recognition.

A word may be added upon Kant's doctrine of Categories, and its relation to that of Aristotle, though it is very difficult to put the matter at once briefly and intelligibly in an elementary treatise. Aristotle had sought to enumerate the kinds of being found in the different things that are; Kant was interested rather in the question how there come to be objects of our experience having these diverse modes of being. He maintained that in the apprehension of them we are not merely receptive and passive; on the contrary, all apprehension involves that the mind relates to one another in various ways the elements of what is apprehended; if the elements were not so related they would not be elements of one object; and they cannot be related except the mind at the same time relates them; since relation exists only for a mind. Kant called this work of relating a function of synthesis; and he desired to determine what different functions of synthesis are exhibited in the apprehension, and equally in the existence, of objects; for the objects in question are not *Dinge an sich*, things by themselves, existing out of relation to the perceiving and thinking mind; of these, just because they are out of relation to it, the mind can know no more than *that* they are, not *what* they are; the objects in question are objects of experience, and their being is bound up with the being of experience of them. He maintained in the first place, that the mere perception of anything as extended, or as having duration, involved certain peculiar ways of relating together in one whole the distinguishable parts of what is extended or has duration. These modes of synthesis we call space and time. As to time, I know that I am the same in the succession of past, present, and future; I could not do this unless I distinguished as different the moments in which I am (as I realize) the same; I could not distinguish them except by the differences of what I apprehend in them; but unless these differences were conceived as differences in the being of something persistent and identical, I could not hold them together; hence through my function of synthesis there come to be objects combining manifold successive states into the unity of one and the same thing. It is the same with any spatial whole. I must be aware at once of its parts as distinct in place, and yet related together in space; space is a system of relations in which what is extended stands; but the relations are the work of the mind that apprehends that manifold

[together. But these two modes of connecting in an unity the parts of what is manifold Kant attributed to *sense*, for reasons which we need not now consider; thinking, the use of general conceptions, did not enter into them; and therefore he did not include them in his list of categories, which were to be the most general conceptions by which in understanding we connect into an unity the manifold parts of an object, and so make it an object for ourselves. The *perception* of an object involved space and time; but perception was not enough. We think of it in certain ways, or *conceive* it, in apprehending it as an object. Now this conception of an object involved, according to him, four things: (1) its having *quality*: and quality can only exist in degrees, each of which is distinguished from and related to the other degrees of the same quality; heat only exists at a definite temperature and blue must be of a definite shade and saturation: (2) its having *quantity*, or being a whole composed of parts: (3) that it should be a *substance having attributes*, one or permanent through its changing and successive states, and that its changes should be determined according to *laws* by its relation to other substances with which it stood in *interaction*: (4) that every such object conceived to exist should be conceived as connected with every other existing object in a way that knowledge could apprehend, and express in the form of necessary inference. The various peculiar relations involved in these requirements Kant called Categories; and he pointed out that, in all the sensible diversity of concrete objects as we know them, these categories or forms of relation exemplify themselves. Let something be presented to me; if there is nothing which I can call it, or regard it as being (for the question is one of thought and not of names), it is so far nothing for me; but if I call it sky-blue, I am thinking of it as qualified; I am 'taking it in' by help of that conception of quality (realized in a specific mode of quality, sky-blue) which is one of the notions by which I relate together all that is sensible in what objects are. Of course it might have a colour unlike any colour I had seen hitherto, which I had no name to indicate; but I should still be apprehending it as coloured in a certain way, though I could not name the colour, and therein I should be using the conception of quality. If I call it a sky-blue tassel, I am using in a specific form the notion of a whole of parts; for to one who could not connect distinguishable parts in one whole a tassel would not be apprehensible as one thing; I am also using the conception of substance and attribute, when I regard it as a thing, one of whose qualities it is to be sky-blue. I cannot call it woollen, without connecting its existence by causality in a definite way with the life of a sheep; and so forth: the forms of space and time being presupposed in my apprehension of it throughout. It is not meant that these notions or categories are abstractly grasped, and consciously applied as guides in our apprehension and description of

[objects, as a doctor who had recognized that height, weight, chest measurement, and state of the teeth were important characters in determining the health of children at a given age might use these headings in a statistical description of the health of children in London schools. We only become aware of the part which these notions play in our apprehension of objects by reflection upon the use we have unconsciously made of them ; just as we become aware in the abstract of using certain forms of inference, by reflecting upon the inferences we have drawn in divers fields. But as there would be no men if there were no animals, and no circles if there were no figures, so we should not judge anything to be coloured if we could not conceive quality ; we should never think that a horse pulled a cart, if we could not conceive a substance to have attributes and to determine changes in another substance ; we should never call the movement of the cart necessary, if we could not think of the different real things in the world as so connected that we could infer one thing from another. And in all these different ways, we are relating, or distinguishing and connecting, features and parts of what we apprehend : what is merely sensible is not the work of the mind ; but the mind effects a synthesis in what would otherwise be a mere chaos or confusion of manifold sensations or *sensibilia*.

Now it has been seen that Aristotle also noted that what, by making them subjects of predication, we recognize as existing are sometimes substances with attributes, sometimes attributes of various kinds ; we recognize the existence of qualities ; of quantities in things that are wholes or parts of such and such a size ; of relations and positions in place and time ; of what things do and have done to them ; of their states and situations. But Aristotle approached the matter from the side of the object ; he asked what modes of being we can distinguish in that which we recognize to be. Kant approached it from the side of the knowing subject, and asked what were the modes of synthesis on the part of our mind, through which objects are apprehensible by us as the sort of objects they are. If Kant is right in thinking that there could be no objects known to us, except through the mind's activity in relating according to certain principles their manifold differences, then we should expect that when we reflect upon the modes of being which these objects exhibit, we should find just those which the mind by its synthetic or relating activity makes possible for them. Hence the two lists of categories should correspond ; and in the main they do ; and the differences between them can be readily explained. Aristotle's list we have seen. Kant recognized four classes of category, those of Quality, Quantity, Relation and Modality. Now Quality and Quantity appear in Aristotle's list as well (though in Kant's they are each analysed into three aspects, or 'moments', which here need not concern us). But in Kant the category of Relation covers the three relations of Substance and Attribute, Cause and Effect,

[and Interaction (which last really involves the other two); the distinction of substance and attribute is present in Aristotle's doctrine, when he says that the rest presuppose Substance, and in ποιεῖν[1] and πάσχειν[2] we have the recognition of the relation of cause and effect; but there is nothing in Kant corresponding to the Aristotelian category of πρός τι[3]. The reason of this is that all predicates in the category of πρός τι[3] really involve some other category as well; *larger* involves ποσόν[4], *earlier* ποτέ[5], *slave* πάσχειν[2], *farthest* ποῦ[6], and *loudest* ποιόν[7]; reciprocally, all categories involve relation, and Kant's whole point is that the relational functions involved are different. For Kant, who was interested in distinguishing these functions specifically, it would have been absurd to treat predicates in which relating, no matter how, is especially prominent, as involving a special kind of relating[10]; or to suppose that there was any other kind of relation involved when I say that Socrates was more scrupulous than Crito, or taller than Tom Thumb, than when I say he was scrupulous or four cubits high. All scrupulousness must be of some degree, and all height of some quantity, so that as far as the function of relating in the way of quantity or degree is concerned, it is equally present whether my term is positive or comparative. But from the side of the object, there are predicates which relate it particularly to some definite other object: and these Aristotle placed under the category of πρός τι[3]. It might perhaps be objected to him that all predicates in the category of πρός τι[3] were also in ποῦ[6] or ποτέ[5], ποιόν[7] or ποσόν[4], ποιεῖν[1] or πάσχειν[2], ἔχειν[8] or κεῖσθαι[9]; but he would have replied that they were referred to the category of relation not because they involved qualitative or quantitative, spatial, temporal, or causal relations, but because they determined a thing as standing in some relation (of any one of these kinds) to some other thing, and they were predicated of it not so much in itself as in relation to something else[11]. Again, terms in

[1] Action. [2] Passion. [3] Relation.
[4] Quantity. [5] Time. [6] Place.
[7] Quality. [8] State. [9] Situation.

[10] The reason why Kant gave the name of Relation to the three syntheses of Substance and Attribute, Cause and Effect, and Interaction was historical. He quite recognized that all his categories were really modes of relating a manifold.

[11] Τὰ πρός τι are defined first in *Cat.* vii. 6[a] 36 as 'what are said to be that which they are *of* another'—ὅσα αὐτὰ ἅπερ ἐστὶν ἑτέρων εἶναι λέγεται, and more closely later in 8[a] 32 as that 'for which to be is the same as to be related in some way to another'—οἷς τὸ εἶναι ταὐτόν ἐστι τῷ πρός τί πως ἔχειν. The implication of πρός τι with some other category is recognized in particular cases, but not stated generally; cf. vii. 6[b] 11, ix. 11[a] 20–38, and esp. 37–38 ἔτι εἰ τυγχάνοι τὸ αὐτὸ πρός τι καὶ ποιόν ὄν, οὐδὲν ἄτοπον ἐν ἀμφοτέροις τοὺς γένεσιν αὐτὸ καταριθμεῖσθαι ('besides, if the same thing happen to be both related and of such a quality, there is nothing strange in its being counted in both kinds'). Cf. *Met.* N. i. 1088[a] 21–25, where it is said that relation presupposes quality and quantity.

[ποσόν[1], like 'three-foot' or 'year-long', involve space or time as well as the relation of whole and part; and Kant thought right to distinguish the perceptual syntheses of space and time from the conceptual synthesis of whole and part; hence also he objected to the presence of ποῦ[2] and ποτέ[3] in the Aristotelian list at all. But Aristotle cared only to notice the modes of being that were to be found, the kinds of predicate that concrete things had, and was not interested to distinguish the parts which sense and thought respectively play in rendering the apprehension of them possible. Once more, Aristotle included the 'derived' notions of ἔχειν[4] and κεῖσθαι[5] with the rest, because they certainly are different modes of being; Kant, who thought them to involve only the co-operation of functions of synthesis already recognized, gave no place to them. The most considerable difference between the two doctrines is the absence from Aristotle's of anything at all corresponding to the Kantian categories of modality, i. e. to the notions of actual, possible, and necessary as determinations of our thought about things; but their absence will not surprise us if we consider that to the question, what essentially a subject is, no one would ever answer that it was actual, possible, or necessary. Speaking generally, however, we may put the relation of the two doctrines in this way, that whereas Aristotle had classified the products, Kant distinguished the processes of that synthesis or relating, through which (as he held) objects in all their manifold variety, however much they may materially or sensibly differ one from another, are all alike objects of knowledge and so far formally the same. Merely to be, said Aristotle, is not possible: ὄν is not a significant predicate[6]; what is must be in a particular way, and its being thereby fall under one or other of the γένη τῶν κατηγοριῶν, the kinds of predicate, which he enumerated; and all the modes of being characterize in the last resort some concrete individual thing, which exists in and through them. An object, said Kant, cannot be an object of experience, and therefore cannot exist in the world of our experience, except through being perceived and thought in certain ways: the general ways in which an object is perceived or thought, the forms of perception and conception involved (one or another of them) in every predicate through which an object is known, are the 'forms of the sensibility'—viz. space and time—and the 'categories of the understanding'.[7]]

[1] Quantity. [2] Place. [3] Time. [4] State. [5] Situation.

[6] Unless in the sense of οὐσία or Substance; but that is one of the categories.

[7] Kant may have been wrong (as Mr. H. A. Prichard has powerfully argued in his *Kant's Theory of Knowledge*) in supposing that the 'formal' characters which belong to all objects of possible experience are not merely apprehended in them by the mind, but are there to be apprehended through the mind's activity. Nevertheless what has been said above will still express the relation which, on his doctrine, subsists between Aristotle's categories and his own.

CHAPTER IV

OF THE PREDICABLES

The distinctions to which our attention was directed in the last chapter are distinctions of terms according to the nature of their meaning; and if we understand what a term means, we may know to what category to refer it, without waiting to learn the subject of which it is predicated; *large*, for example, is in the category of quantity, whether it be predicated of a triangle or of a gooseberry, and *just* in the category of quality, whether it be predicated of Aristides or his actions. Such difficulty as may exist in determining the category to which a term is to be referred arises through defect in the list of categories (i. e. of the conceptions under which we are to classify all possible predicates), or through the complexity of meaning in the term itself, whereby it involves more than one category at once, like a verb with tense; but not through the fact that we are considering the term by itself and without reference to the subject of which in a particular proposition it may be affirmed or denied. And the treatise called the *Categories* indicates this when it puts forward the list of ten categories as a division of *terms out of construction*.[1]

In the present chapter we have to consider another division of terms, based upon the relation in which a predicate may stand to the subject of which it is predicated. Aristotle recognizes four such relations, and one of them he subdivides, obtaining five in all; later logicians give five, but their list is in one important respect different. According to Aristotle, in every judgement the predicate must be either the *definition* (ὅρος), the *genus* (γένος), the *differentia* (διαφορά), a *property* (ἴδιον), or an *accident* (συμβεβηκός) of the subject. The later list,[2] losing sight of the principle on which the division was

[1] Τῶν κατὰ μηδεμίαν συμπλοκὴν λεγομένων ἕκαστον ἤτοι οὐσίαν σημαίνει ἢ ποσὸν ἢ ποιὸν ἢ πρός τι ἢ ποῦ ἢ ποτὲ ἢ κεῖσθαι ἢ ἔχειν ἢ ποιεῖν ἢ πάσχειν, *Cat.* iv. 1^{b} 25 ('what is said out of construction signifies either substance or quantity or quality or relation or where or when or situation or state or action or being acted on').

[2] The Aristotelian list is given in the Topics, a. iv. 101^{b} 17–25. At the outset Aristotle names γένος, ἴδιον and συμβεβηκός; he then says that διαφορά

made, omits *definition*, and includes instead *species* (εἶδος), running therefore as follows—*genus, species, differentia, proprium, accidens.*

The distinctions are known as the Five Predicables, or more strictly as the Five Heads of Predicables. The words have passed into the language of science and of ordinary conversation; we ask how to define virtue, momentum, air, or a triangle; we say that the pansy is a species of viola, limited monarchy a species of constitution; that one genus contains more species than another; that the crab and the lobster are generically different; that man is differentiated from the lower animals by the possession of reason; that quinine is a medicine with many valuable properties; that the jury brought in a verdict of accidental death; and so forth. The fact that the employment of the words is not confined to any special science suggests that the consideration of them may belong to Logic, as expressing something recognized in our thought about all kinds of subject.

'Predicable' here means a predicable character, i.e. not an individual substance, but *what* it is; all kinds, qualities, states, relations, &c.; and these may be exemplified in and belong to more than one individual subject, and so we may say that they are universal.[1] All terms, therefore, except proper names may be brought under one of these five heads of predicables in relation to the subject of which they are predicated; but proper names are not included [2]; they may indeed be predicates in a proposition (in Aristotle's view only improperly); but they stand for individuals, and an individual is not the character of anything. The Parthenon, for example, is not the genus or species of anything; nor is it that which differentiates any species from another species; nor is it a property or accident of anything. It is a particular building; and the name denotes that building, with all that it is—

may be ranked with γένος, ὡς οὖταν γενικήν, i.e. presumably, as being a modification of that; and he distinguishes ἴδιον, as what is common and peculiar to the subject, into ὅρος, which gives the essence, and ἴδιον *sensu strictiore*, which does not. In c. viii he offers a proof that the five-fold division is exhaustive. The later list passed into modern Europe from a little work by Porphyry (b. A. D. 233), the Εἰσαγωγή, or Introduction to the *Categories*, through the medium of a Latin version and commentary by Boetius, who lived in the last quarter of the fifth and first quarter of the sixth century A.D.

[1] Except when they are what no second subject can be; e.g. there can be only one Omnipotent, and only one superlative in any kind. Professor Cook Wilson has called attention in an unpublished paper to the fact that there may be universals with only one instance.

[2] Nor designations, though what is general in a designation may be.

a temple, Doric, of Pentelic marble, beautiful by the simplicity of its lines and the magnificence of its sculptures, the work of Pheidias and his assistants, the glory of Athens. All these things are predicable about it, and they are universals; for might not another building be a temple, in the same style, of Pentelic marble, and so forth? It, however, is not predicable; nothing else can be the Parthenon. We may ask what kind of thing is the Parthenon, but not of what things is it the kind. The distinctions which we have to consider, therefore, do not afford a classification of things, but of concepts[1]: and (unlike the categories) of concepts considered not in themselves but in their relation one to another.

But things are known to us through these concepts; and an enquiry into the relation of concepts is an enquiry into the nature of things. There is indeed another sense of knowing. It has been frequently pointed out that the English language uses only the one verb, 'know,' to represent two different acts, which in some languages are distinguished by different verbs[2]: the knowledge of acquaintance with a thing, and the knowledge about it. In Latin, the former is signified by *cognoscere*, the latter by *scire*; French uses respectively the cognate words *connaître* and *savoir*; German the words *kennen* and *wissen*. Knowledge of acquaintance does not come barely through conceiving; however much may be told me about Napoleon, and however clearly I may have succeeded in conceiving the features of his character, I never knew him, and never shall know him, in the sense of being acquainted with him: such knowledge comes only by personal intercourse, and separate intercourse is needed with each individual that is to be known. But knowledge about a thing comes by concepts; and this too is necessary to real acquaintance,[3] though it does not by itself amount to acquaintance. I may know a great deal about a man, without having ever met him: but I may in fact once have met him, without knowing who he was or

[1] To use a phrase of Mr. F. H. Bradley's, it is the 'what' and not the 'that' of things which we have to consider.

[2] Cf. e.g. J. Grote, *Exploratio Philosophica*, Pt. I, p. 60—a work and by an author less known than they deserve to be; the expressions 'knowledge of acquaintance' and 'knowledge about' are borrowed thence.

[3] Though not to such familiar recognition as a dog may show of its master, or a baby of its mother. The less developed mind acts in ways very difficult to describe, because it does not shew completely what mind is; but it is wrong in principle to 'interpret the more developed by the less developed', as Herbert Spencer would have us do.

anything about him; and I am no more acquainted with him in the latter case than in the former.

Now most of our knowledge is knowledge about things; things are useful and important to us for the most part not because they are such particular individuals but because of what they are; this is not equally the case with persons; and yet with persons too it is very largely the case. 'Wanted, a good coat-hand': it is not Smith, who is taken on, that is wanted, but only the coat-hand: the master-tailor is satisfied to know that he has engaged a coat-hand, and very often does not desire his acquaintance: if he knows about Smith, he can regulate his business accordingly, without knowing Smith.

Through concepts, then, i.e. through what we conceive of their being, we are not acquainted with things individually, but we know and think and reason about them thereby. And a concept may be said to differ from a thing in being universal, not individual: an object of thought and not of sense: fixed and not changing: completely knowable and not partially[1]. Take, for example, the concept of a timepiece: a timepiece is a machine in which the movement of wheels is so stimulated and regulated as to cause a hand or

[1] The characters recognized and named in things are often imperfectly understood; but they might be understood completely, whereas the individual thing cannot be. Hence we may say that a concept is completely knowable, though not completely known. About the unchangeableness of a concept certain difficulties arise. (1) It is said that men's concepts change as their knowledge increases, e.g. there are now timepieces indicating the time by cards on which the hour and minute are printed, and which displace each other in proper succession; and therefore we must modify our concept of a timepiece. But this only means that we must change the meaning of a name. What was conceived does not alter; it is still displayed in the instruments to which the name was hitherto given; now, when the name is also given to instruments which effect their purpose in a different way, something different is conceived when the name is used (cf. *infra*, c. vi). So, if we arrange a row of books according to height, we may say that the height increases along the shelf; but no book is getting higher. (2) But we may conceive a changing character; and here, what is conceived is not unchanging. In a body moving with an acceleration, the velocity changes. Cannot we then conceive velocity? In the growth of an organism, perhaps we ought to say that the specific form changes; yet this, one would say, is only known by conceiving. We must remember here the distinction between an universal and its instances. The velocity of this bullet may change; but velocity is one in all these momentary velocities. When it is said that we know things through concepts, that means, through what they are; but what they are is an instance of an universal nature. Between instances of these universals relations hold which do not hold between universals; Juvenal's indignation may cause his activity in verse-making, but one universal does not cause another. So in the instances there may be change, but not in the universal.

hands to move at an uniform rate (usually twice in twenty-four hours) round a dial, and by pointing to the divisions marked upon the dial to indicate the time of day. That is the concept of a timepiece: it is clearly universal, for it applies to all timepieces; it is an object of thought, and cannot be seen or felt, like the watch in my pocket; it is fixed and unchanging, while my watch wears out or gets broken; and it is completely knowable or intelligible, whereas there is a great deal about my watch which I do not know or understand: where the metals of which it is made were quarried, and by what series of events they came into the hands of the maker: why it loses 10″ to-day and gains 13″ to-morrow, and so forth. No one knows the whole history and idiosyncrasy of any particular timepiece, but he may conceive its general nature satisfactorily for all that.

It has been asked, as we noticed above,[1] is a concept merely an object of thought, with no existence in things (as it is put, outside our minds)? or does it exist in things[2]? Much ink, and even much blood, have been spilt in disputing over this question. An elementary treatise must be content to be brief and dogmatic. Concepts, we maintained, have existence in things, as well as in our minds. The thing which I can pull out of my pocket, and see and feel, and hear ticking, is itself a machine wherein the movement of wheels causes hands to tell the time of day as set forth in stating the concept of a timepiece. What I conceive a timepiece to be, that (if my concept is a right concept) every particular timepiece is; what I know about things is the nature of the things; nor would it otherwise be they wherewith my knowledge dealt. But though the features of things exist in the things, besides being conceived by our minds, the manner of their existence is different in an important respect from that of our conceiving them. In our minds,[3] each is to some extent isolated; my knowledge of an individual thing is expressed piecemeal in many predicates about it; each predicate expressing a different concept, or a different feature in the nature of the thing. But in the thing these features are not isolated. The individual thing is at once and together all that can be predicated of it separately and successively (except indeed as far as

[1] *Supra*, pp. 25, 31–32.

[2] Or does it (as some have held) exist apart at once from particular things and from our minds? Cf. *supra*, *loc. cit.*

[3] What is conceived by the mind is sometimes said to be in the mind. To be in the mind means to be the object of a conceiving, thinking, remembering, or imagining mind: not of course to be in the brain, or inside the skull.

predicates are true of it successively—a man, e. g., is successively awake and asleep). Thus in thinking of my watch I may think of it as a timepiece, as an heirloom, as being two inches in diameter, and so on: between these concepts there is no connexion thought of; they are as it were separate from one another; but they and much besides are united in the thing.[1] The individual thing is all that can be predicated of it (and there is no end to what might be predicated, if we knew its whole nature and history); but one thing that can be predicated of it is not another.

An object comes into the room, which I call Tray: what is Tray? it is a dog, an animal, yelping, at my feet, mine; Tray is all these: but is a dog all these? A dog (that is, any dog) is an animal, and a dog yelps; but I cannot say that a dog (meaning any dog) is mine, or at my feet; and though a dog is an animal it is not equally true that an animal is a dog, or that what is at my feet is mine, or that what is mine is at my feet.

What, then, is the relation of those various concepts to one another, which can all be predicated of the same individual? Are they united in it like stones in a heap, where the stones together are the heap? or like almonds in a stewed pippin, where the pippin is not the almonds? or like links in a coat of mail, where the links indeed are the coat, but only because they are peculiarly looped one into another? It is easily seen that none of these analogies is appropriate. According to Aristotle they are related in one of five ways. Take any proposition, '*A* is *B*,' where the subject *A* is not a proper name, but a general concrete term, or an abstract term. The predicate *B* must be either definition, genus, differentia, property or accident [2] of *A*: one or other of these relations must subsist between the two concepts *A* and *B*, in any individual characterized by them.

The statement just advanced clearly concerns our thought about subjects generally: the technical terms have yet to be explained, but it is the actual procedure of our thought which they profess to indicate. Logic invented the terms, but it discovered the relations denoted by them.

[1] The word thing here is used first of the concrete subject of predication, then of the character predicated. It has been used already in both these senses. The English idiom allows both uses—we may say, for example, 'about that thing I know nothing'; and it may be worth while to use the word closely together in both senses, in order to direct notice to the ambiguity.

[2] But cf. pp. 76, n. 1, 104, n. 1, *infra*. The Porphyrian list of predicables will be considered later.

If we take any term that is general, and not singular, and make it the subject of a proposition, then the predicate must be either commensurate with the subject, or not. One term is said to be commensurate with another, when each can be predicated of everything whereof the other can be predicated[1]; *equilateral triangle* and *equiangular triangle* are commensurate terms, because every equilateral triangle is equiangular, and every equiangular triangle equilateral; but the term *equiangular* is not commensurate with *equilateral*, for there are figures equilateral which are not equiangular. It may be pointed out (for it is important to bear in mind that we have to deal now with the relation between the different 'universals' predicable of the same individual, and not the relation between them and the individual of which they are predicated—with the relation of 'animal' and 'mine', &c., to 'dog', and not with the relation of these terms to Tray)—it may be pointed out that when the subject of a proposition is singular, the predicate is hardly ever commensurate[2]: for the predicate is an universal, and so commonly predicable of other subjects besides this individual: *mine* is predicable, for example, of other subjects than Tray; whereas this individual is predicable of none of those: nothing else that I can call mine is Tray. Now where the predicate of a proposition is commensurate with the subject, there it is either the Definition or a Property of it: where it is not commensurate, there it is either part of the Definition, i. e. Genus or Differentia[3], or an Accident.

The **definition** of anything is the statement of its essence[4]: what makes it that, and not something else. In the following propositions, the predicate claims to be the definition of the subject: 'An organism is a material body, of which the parts are reciprocally ends and means'; 'a church is a building devoted to the service

[1] And therefore, of course, neither of anything of which the other cannot be predicated. Here and in some later passages I put *triangle* as equivalent to *rectilinear triangle*. Spherical and other triangles are ignored for the sake of simplicity.

[2] Only if it is a predicate which from its nature can belong to no more than one individual: cf. *supra*, p. 67, n. 1.

[3] But sometimes a differentia is commensurate: v. p. 74.

[4] Ὁρισμὸς μὲν γὰρ τοῦ τί ἐστι καὶ οὐσίας, Ar. *Anal. Post.* β. iii. 90^{b} 30. We may ask the question τί ἐστι;—what is it?—of an attribute (like momentum) as well as a substance (like a man or a lobster); and the answer will be a definition. In strictness we can define the οὐσία of an individual, if at all, only as meaning the kind to which it belongs; cf. the previous ch., pp. 53–57, and also p. 28.

of God according to the principles of the Christian religion'; 'momentum is quantity of motion'; 'wealth is that which has value in exchange'; 'a triangle is a three-sided rectilinear figure'; 'a line is the limit of a superficies'. The predicate states what it is that makes anything an organism, a church, a line, a triangle: what constitutes momentum or wealth, as distinguished from everything else, such as apathy or architecture. In these judgements it is clear that the predicate, in claiming to be a definition, claims to be commensurate with its subject; if an organism is a material body of which the parts are reciprocally ends and means, then my dog Tray, being an organism, must be that, and whatever is that must be an organism: for to be such a body is to be an organism. If wealth is that which has value in exchange, then gold, having value in exchange, is wealth, and so forth.

The **genus** is that part of the essence of anything which is predicable also of other things[1] differing from it in kind.[2] Each of the definitions above given begins by declaring the subject something, which other and different subjects are besides; an organism is a material body—so is a machine, or a block of stone; a church is a building—so is a stable; a triangle is a rectilinear figure—so is a square; a line is a limit—so is a point, but of a line; wealth is that which has value—so is honesty, but not in exchange, for you cannot transfer it[3]; momentum is quantity—of motion, but not of matter. These (building, rectilinear figure, limit, &c.) are the genus, in each case; and the genus, being predicable of other subjects, is clearly not commensurate.[4] Genus is sometimes explained as a larger class including the class defined within it; *figure*, for example, as a class including triangles, squares, cones

[1] 'Thing' here again does not mean only a concrete thing.

[2] *Γένος δ' ἐστὶ τὸ κατὰ πλειόνων καὶ διαφερόντων τῷ εἴδει ἐν τῷ τί ἐστι κατηγορούμενον*, Ar. *Top.* a. v. 102[a] 31. The notion of a kind is here presupposed. Some discussion of it will be found below, pp. 91–103. In botanical and zoological classification, genus is not merely correlative to species, but marks a certain degree of affinity, lower than specific, higher than that of families, orders, &c. Hence a genus, and even a family, may contain only one species, if that diverges as far from the species nearest it as do the species of different genera or families; *Homo Sapiens* is in the zoological genus *Homo* and family *Hominidae*, and is alone in them. (I borrow the latter part of this note from Miss Augusta Klein.)

[3] The honest man, however, commands in many situations a higher price, and so far some economists would reckon honesty as wealth.

[4] This must be received subject to modification from what is said below as to the genus being in itself indeterminate, and actually different in each of its species. Cf. pp. 83–88, 138.

and many other subordinate classes besides: *building* as a class including churches, stables, barracks, and so forth. This explanation cannot be considered a good one, for reasons to be presently stated[1]; but it may put some into the way of grasping a better.

The **differentia** is that part of the essence of anything—or, as we may say, of any species—which distinguishes it from other species in the same genus; it is the differentia of an organism that its parts are reciprocally ends and means—in this it differs from other material bodies; it is the differentia of a church, to be for the service of God according to the principles of the Christian religion—in this it differs from other buildings; and so forth. The genus and differentia (or differentiae[2]) between them constitute the species, or make up the essence of that which is defined. The differentia, like the genus, need not be commensurate with its subject. The Book of Common Prayer is for the service of God in accordance with the principles of the Christian religion, but not being a building, it is not a church. On the other hand the differentia is commensurate with the subject of which it is predicated in cases where no genus except that to which the subject belongs is susceptible of the particular attribute which serves as differentia; thus a vertebrate is an animal of a particular structure which cannot exist except in an animal, so that the differentia of vertebrate is commensurate with it. And it is only where this is the case that the ideal of definition is attained, because only there is it precisely the common genus which is shewn to be realized in the several species.

Those who speak of the genus as a larger class containing the species or smaller class within it sometimes explain the differentia as the attribute, the possession of which marks off the smaller from the rest of the larger class. If squares and rhomboids, triangles and pentagons, &c., are all placed in the class of plane rectilinear figures because they have that character in common, triangles, on the other hand, are differentiated from the remaining classes included within that of plane rectilinear figure by possessing the attribute of being three-sided. Provided it is not supposed that the differentia is *added* to the common character of the

[1] v. *infra*, pp. 83–84.

[2] In the plural if the genus has divers determinable points, some or all of which have to be specified differently in the different species. Cf. *infra*, pp. 100-101. In the rest of the paragraph, the singular must be taken as covering a complex of differentiae.

'larger class' in the same extraneous way that sugar is added to tea, there is no fresh harm in this mode of expressing oneself.

A **property** is an attribute common and peculiar to a subject[1] (and therefore obviously commensurate with it), but not part of its essence, and so not included in the definition of it. This is Aristotle's original account of a property, though we shall see that he also used the term with a less restricted meaning.[2] An organism, for example, is contractile, irritable, assimilates food, reproduces itself after its kind: these are attributes of every organism, and of nothing else, and therefore common and peculiar to the subject organism; but they are not in its definition. A triangle, again, has its interior angles equal to two right angles, and its area half that of the parallelogram on the same base and between the same parallels; a line is either straight or curved (here the alternatives together are common and peculiar); and so forth.

All other attributes of any subject are **accidents.** An accident is defined as a non-commensurate predicate not included in the essence: or as an attribute which equally may and may not belong to a subject. The latter is the better definition, because it tells us what an accident is, whereas the former only tells us what it is not.[3] It is an accident of an organism to be used for food; for it may be so used, but need not. It is an accident of a church to be a cathedral; some churches are cathedrals, and some are not. It is an accident that a contractor should be an honest man, and an accident that he should be a rogue; for roguery and honesty are both compatible with being a contractor.

The doctrine just illustrated presents many points for consideration, of which the following are perhaps the most important:—

1. the antithesis between accident on the one hand and all the other heads of predicables on the other;

[1] The subject being indicated, it must be remembered, by a common, not a singular term. I cannot speak of yelping as an attribute common to Tray, but I can speak of it as an attribute common to the dog—i.e. belonging to every instance of dog. Aristotle sometimes spoke of an attribute peculiar to an individual, and not to a kind or universal, as a property; and also of attributes peculiar to one out of a certain definite number of kinds, and therefore serving to distinguish it from the rest (though found perhaps again outside their number) as *relatively* properties; thus it is a property of man relatively to any quadruped to go on two legs; but so also does a bird. He recognized that this use of the term 'property' was not the same as that given in the text, and not (in his view) so proper a use. Cf. *Top.* ϵ. i.

[2] Cf. *infra*, pp. 80–81, 104.

[3] Cf. Ar. *Top.* a. v. 102^b 4–14. Cf. *Top.* ϵ. i. The former also includes generic properties: cf. *infra*, p. 104, n. 1.

2. how to understand the analysis of a definition into genus and differentia ;

3. the ground of the distinction between the essence of anything and its properties.

(1) When we classify the members of a genus or class, we sometimes, after specifying as many distinct species as we can think of, add another to include anything that does not fall within any of these ; I may classify my books, for example, according to subject into historical, philosophical, philological, scientific, and miscellaneous—the last division being merely added in order to receive any book which does not fall within the others, though the miscellaneous books have no common character that distinguishes them all alike from the rest. Now Accident is a head of predicables which includes any predicate that is neither definition, genus, differentia, nor property of its subject[1] ; but it is not a heading like ' miscellaneous ' ; there is a very definite and important difference between the relation of those predicates to their subject which are classed as accidents, and that of those which fall under the other heads ; the latter belong to their subject necessarily and universally, the former do not.

Of any individual, as we have seen, an infinity of predicates may be asserted. Some of them are seen to be connected, or (as we may express it) have a *conceptual* connexion ; i.e. if we rightly conceive one predicate, we see how it involves another. Tray, for example, is a dog and an animal ; and these predicates are conceptually connected, because the concept of a dog involves that of animal. My watch has hands, and there is a conceptual connexion between having hands and being a watch, since without hands a watch could not fulfil the task of telling the time, which is part of the concept of it as a timepiece. But there are also many predicates which *coincide*[2] in one and the same individual, without being conceptually connected. Besides being a dog, Tray is mine, and was born at Bishop Auckland ; now there is no reason in the nature or the concept of a dog, why it should belong to me, nor in a thing being mine, why it should be born at Bishop Auckland, nor in being born at Bishop Auckland, why it should be mine, or be a dog. No doubt in the case of this particular dog Tray, there

[1] Συμβεβηκὸς δέ ἐστιν ὃ μηδὲν μέν τούτων ἐστί, μήτε ὅρος μήτε ἴδιον μήτε γένος, Ar. *Top.* a. v. 102b 4.

[2] Cf. *supra*, p. 24. *Coincident* is really a better translation of συμβεβηκός than accident.

is a reason why he is mine and a reason why he was born at Bishop Auckland ; but the reason for the first fact (which may be that he was given me) has nothing to do with the reason for the second (which is that his mother was there at the time) ; nor has the reason for either anything to do with his being a dog ; he would have been a dog still, if he had never been given to me, or if he had been born at Bishop's Lydeard.

Of course with more knowledge the coincidence of attributes in an individual may often be explained ; but the explanation will always be largely historical, connecting the coincident attributes severally according to laws with other facts which are found conjoined but not seen to be connected. We have here the great difference between science and history. In science we seek to ascertain the connexion of universals. Sometimes we can only do this inductively ; by noticing how attributes are historically found conjoined or disjoined in divers individuals we determine which must be supposed to be connected[1] ; but having established these ' laws ', we trace out by mere thinking their consequences in divers situations of fact. Sometimes, without the appeal to experience which induction makes, we can, as in geometry, trace necessary connexions between one character and another in things. But history is interested in individuals in whose total being we find characters coincident, the conjunction whereof we can never wholly see to be necessary. Even where they are so far of a kind that we know how they must behave in a given situation, yet each situation presents different conjunctions. No doubt the scientific and historic interests interpenetrate. Some sciences, like geology, are largely occupied in applying what they know of the connexion of universals to the elucidation of the history of individual things, or aggregates, if we hesitate to call a mountain range or a coal formation one thing. And the historian attempts to trace connexions among the events that make the history of individuals, or groups of individuals, and so far to be scientific. Perhaps, even if we started with complete historical knowledge of the conjunction of individuals at a given time, the subsequent course of history could never be wholly explained this way ; it may be that the nature of individuals cannot be exhaustively given in terms of universal characters, but that there is in each something unique.

[1] The illustration of this forms a considerable part of what is called Inductive Logic ; we shall find that many connexions are inductively established whose necessity remains unconceived.

But anyhow there will always be the bare conjunction of facts in the historic situation, which cannot be deduced except from the previous conjunction in another historic situation.

That the accidental should be opposed to what is necessary and universal conforms to the usage of common speech. Sir Robert Peel was killed by a fall from his horse, and we say his death was accidental. Why ? he was a man, and for a man it is necessary to die, and for any one who falls in that particular way it may be necessary to die ; but it is not necessary that a man should fall in that way ; that is not predicable universally of man. We sometimes dispute whether there is such a thing as chance in the world, or whether everything has a cause, and happens necessarily. Few people really believe that anything happens without a cause ; but chance is not the negation of cause ; it is the coincidence of attributes in one individual, or events in the same moment, when each has its cause, but not the same cause, and neither helps to account for the other.

If we bear in mind this fundamental contrast between the accidental and the necessary, we shall not be inclined to think that Aristotle was engaged in a trivial pursuit when he attempted to classify the various relations in which a predicate might stand to its subject. Discussions as to what we mean by cause occupy much space in many modern treatises. Now the causal relation is also grounded in the nature of universals : Tray yelps not because he is this individual Tray, but because he is a dog, and unless any dog yelped, it would not be because he is a dog that Tray does so. But when we call this the cause of that, the relation intended is not always the same ; just as when we say that A is B, the relation of B to A is not always the same. It might be supposed that if one thing X is the cause of another Y, then you could not have X without Y, nor Y without having had X. And yet we say that molecular motion is the cause of heat, that the heat of the sun is the cause of growth, that starvation is sometimes the cause of death, that jealousy is a frequent cause of crime. We should in the first case maintain that the cause and effect are reciprocally necessary ; no heat without molecular motion, and no molecular motion without heat. In the second, the effect cannot exist without the cause, but the cause may exist without the effect ; for the sun shines on the moon, but nothing grows there. In the third, the cause cannot exist without the effect, for starvation

must produce death, but the effect may exist without the cause, since death need not have been produced by starvation. In the fourth case, we can have the cause without the effect, and also the effect without the cause ; for jealousy may exist without producing crime, and crime may occur without the motive of jealousy. It is plain, then, that we do not always mean the same by our words, when we say that two things are related as cause and effect ; and any one who would classify and name the various modes in which two things may be causally related would do a great service to clear thinking. Now that is the sort of service that Aristotle attempted in distinguishing the heads of predicables. Many predicates are asserted of the subject *A*. Those of them are accidents, whose cause does not lie in its nature as *A*, or which, when they belong to any individual of the kind *A*, do not belong to it because it is *A*. The rest are in some way or another connected causally with *A*, and are predicable of any individual because it is *A*. Whether Aristotle's account of the different modes of connexion between a subject and a predicate is satisfactory is another question, involved principally in that of the value of his account of ' property '. But that the theory of predicables is closely akin to the question of the various senses in which one thing can be the cause of another may be seen by this : whenever science tries to find the cause not of a particular event, such as the French Revolution (whose cause must be as unique as that event itself is), but of an event of a kind, such as revolution, or consumption, it looks in the last resort for a *commensurate* cause. What is that exact state or condition of the body, given which it must and without which it cannot be in a consumption ? What are those conditions in a political society, given which there must and without which there cannot be a revolution ?

The kindred nature of the two enquiries will be further seen, by looking at certain cases where it is disputable whether a predicate should be called an accident of its subject or not ; for an exactly parallel difficulty may arise in determining whether one thing shall be called the cause (or effect) of another or not. An accident is a predicate of a subject *A*, the ground for whose existence in that subject does not lie in its nature as *A*. Hodge drives a plough ; and a full knowledge of his history would show me why he drives a plough, and the ground for it therefore lies in the history of the subject Hodge ; it is not of him that driving the

plough is predicated as an accident. But a man drives a plough. That is an accident; for the subject now is not Hodge wholly, but a man, and it is not in his nature as a man that the ground or reason of his driving a plough lies; else should we all be at the plough-tail. And yet no animal but man can drive a plough: so that it is partly because he is a man that Hodge drives it; and therefore, when it is said that a man may drive a plough, the relation of the predicate to the subject seems not completely accidental. Contrast the statement that a cow may be knocked down by a locomotive. There the nature of the subject, as a cow, contributes nothing; it is in no wise necessary to be a cow, in order to be knocked down by a locomotive [1]; and the relation is purely accidental.

If we consider these two examples, we see that our account of an accident, just given, may be interpreted in two ways. A predicate may belong accidentally to the subject of which it is predicated either

(i) when the ground for its existence in the subject does not lie completely in the subject-concept,[2] or

(ii) when the ground for its existence in the subject does not lie at all in the subject-concept.[2]

The first interpretation would rank as accidents of a subject [3] all predicates that are not either part of its definition, or else common and peculiar to that subject, i.e. properties in the strictest sense; and such, if we take him at his word, is Aristotle's view. But we are then required to say that it is an accident of money to be

[1] So far as a cow is a body, and only a body can be knocked down, it must be allowed that the nature of a cow contributes something to the accident; but the second sentence will stand without qualification.

[2] When a general term is subject of a proposition, though the proposition concerns individuals (designated individuals or not, according as the general term is or is not combined with a demonstrative), yet these are characterized only by the general term. The character by which they are thus distinguished is the subject-concept. If I say that a cow was knocked down by a locomotive, the subject is an individual cow; it is distinguished in my proposition from other obstacles by *being a cow*; this being a cow, or cowness, is the subject-concept. What is knocked down is a cow, not cowness; but being knocked down is accidental to cowness in the cow; and I can therefore say that the relation of accident lies between universals, though exhibited between the instances of them in this cow. It would of course be absurd to say that the particular cow contributed nothing to the accident, since it could not have been knocked down if it had not been there. Students of Professor Cook Wilson's lectures will remember this distinction between subject and subject-concept.

[3] i.e. of the subject as distinguished by the subject-concept.

valuable, since it would have no value if there were nothing to buy with it: or of coal to burn, since it would not burn in a vacuum. The second interpretation would refuse the name of accident to anything that could be said about a subject, however rare and unconnected the conjunction of circumstances through which it came about, where the nature of the subject contributed anything at all to the result. Thus we could hardly call it an accident that an animal should die of overeating itself, since it must be an animal in order to eat. In practice we make a compromise between these extreme interpretations. We call it a property rather than an accident of belladonna to dilate the pupil, though the result depends as much upon the nature of the muscles as on that of belladonna; we call it an accident rather than a property of the plough to be a favourite sign for country inns, though its necessary familiarity to countrymen accounts for its selection. The further pursuit of these difficulties does not concern us now; but it remains to be shown that they arise also in regard to the relation of cause and effect. Is the cause of an effect that, given which and without anything besides, the effect follows? in other words, must it contain the whole ground of the effect? then a spark is never the cause of an explosion, for it will produce no explosion without powder. Is the cause anything, however slight, without which the effect could not have occurred? in other words, is that the cause which contributes anything whatever to the effect? then are cooks the cause of health, since there would be little health without them.

(2) The antithesis between accident and the other heads of predicables needs perhaps no further illustration. We may pass to the second of the three points enumerated on pp. 75–76, viz. how to understand the analysis of definition into genus and differentia.

It should first be noticed that definition is never of an individual, but always of what is universal, predicable of individuals—whether it be what we call their 'kind', or some state or attribute of them, or relation in which they stand. For what is defined is thereby marked off and fixed in our thought as a determinate concept; but the individual is made the individual he (or it) is by an infinity of attributes; he is as it were the perpetual meeting-place of concepts; we can neither exhaust what is to be said of him, nor make a selection, and declare that this is essential to him, and that unessential. Moreover, even if we could, we should still only have settled what he in fact is, but a second person also might be; for

every concept is universal. What makes him *this* individual and not another we should not have defined, nor could we; for there is something which makes me me over and above what can be predicated of me; else, what makes me me might also make you you; for what can be predicated of me might be predicable of another, you for example; and then why does the same character make me me and you you, and not rather make me you and you me, or each of us both?

We can only define then what is universal, or a concept. But we have already said that concepts are the natures of things; and therefore in defining concepts, we may define things, *so far as they are of a kind, but not as individuals.* It is sometimes maintained that definitions are not of things, but only of names[1]: that they set forth the meaning (or, as it is also phrased, the connotation[2]) of a name, but not the nature of a thing. Yet the names are only used to convey information about things; and to explain what the name means, is to explain what the thing is said to be. Definitions then are not really of names; but we shall see later the difficulties which drove men into saying so.

Now when we define we analyse; and the elements into which we analyse that which is defined are called, as we saw, genus and differentia. These might be called attributes of the subject: it might be said, for example, that *rectilinear figure* and *three-sided* are attributes of a triangle. But the expression is not quite appropriate; for an attribute implies a subject beyond itself, to which it belongs; but the parts of a definition themselves make a whole, and coalesce into the unity to which they belong. This may be best explained by a contrast. We may take any attributes we like—say far, sour, pink, soft and circular—and we may give one name to the aggregate of these. But they do not form *one* notion; they remain obstinately five. If we took a single name to signify the possession of these attributes, we could explain the name as meaning that assemblage, but we should feel that in so doing we were merely explaining a name, and not defining any unity. But when we analyse into genus and differentia, this is otherwise; then we feel that the two together really make a single notion. They have such a connexion in their own nature as makes one fit the other, so that they constitute the essence of *one* thing, or state,

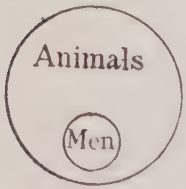
[1] e.g. Mill, *System of Logic*, I. viii. 5.
[2] On 'connotation' cf. *infra*, c. vi.

or action, or quality, or relation. And the reason for the parts of a definition being one[1] is this: that they are not attributes independent but coincident, but the genus is the general type or plan, the differentia the 'specific' mode in which that is realized or developed. Take again the definition of a (rectilinear) triangle. It is a rectilinear figure; but to be merely that is impossible, because incomplete. There cannot be a rectilinear figure without a definite number of sides, though *any* definite number above two will do; and if the number in a triangle is three, then three-sidedness is the specific mode in which the general plan, or as we may say the potentialities, of rectilinear figure are realized in the triangle. We may say that the genus and differentia are one, because they were never really two. Three-sidedness can only be realized in a figure, rectilinear figure can only be realized in a definite number of sides. The genus therefore never could exist independently of a differentia, as soft may of sour: nor the differentia of the genus. It may be said perhaps that though three-sidedness can only exist as the form of a figure, rectilinear figurehood exists independently of three-sidedness in the square, the pentagon, &c. But it is not quite the same thing in the square or pentagon as it is in the triangle. So intimately one are the differentia and the genus, that though we refer different species to the same genus, yet the genus is not quite the same in each; it is only by abstraction, by ignoring their differences, that we can call it the same. Triangle and square and pentagon are all rectilinear figures; but in the sense in which they actually *are* such, rectilinear figure is not the same in them all. Thus the differentia modifies the genus. And the genus also modifies the differentia. It might be said that three-sidedness is not confined to the genus figure; for a triangle is a three-sided figure, and N is a three-sided letter. And doubtless, so far as the genus is the same in two species, the differentia may be the same in the species of two genera. But three-sidedness is plainly different in the figure, where the sides enclose a space, and in the letter, where they do not; and the genus as it were fuses with the differentia, so that each infects the other through and through.

For this reason the genus is not well described as a larger class including the smaller class or species within it. For the word *class*

[1] That the parts of a definition are one is a thing on which Aristotle frequently insists, and says that the main problem about definition is to show how that can be. Cf. e.g. *Met.* Z. xii, H. vi.

suggests a collection, whereas the genus of any species is not a collection to which it belongs but a scheme which it realizes, an unity connecting it with things different from itself. It may seem at first plain-speaking, without any metaphysical nonsense, to say that a genus is a class of things that all have certain features in common; and that its species is a smaller class composed of some of those things, which all possess not only the features common to the whole genus, but others not belonging to the other members of it. But what is really meant by being included in a class? The phrase is sometimes put forward as if it were simple, and presented no difficulty; but such is not the case. The words 'to be within', or 'to be included in', have many meanings, and we must know what meaning they bear in the phrase 'to be included in a class', before we can know what that phrase signifies. We may distinguish in particular two meanings, which are quite inapplicable to the relation between a genus and its species; but they are more easy to grasp than the meaning in which the species *can* be said to be included in the genus, because they can be in a manner represented to the senses; whereas the relation of genus to species can never be represented to the senses, but only apprehended by thinking. Because one of these inapplicable meanings is readily suggested to the mind, when we are told that the genus of a thing is a class in which it is included, we fancy that the expression helps us to understand what a genus is; for these inapplicable meanings are easily understood. But as they are inapplicable, they help us not to understand but to misunderstand the logical relation of genus and species.[1]



In the first place, one thing may be included in another as a letter is included or enclosed in an envelope, or as Mr. Pickwick and the wheelbarrow were enclosed in the pound. In this case, all that is included may be removed, yet that in which it was included will be left. Such is clearly not the sense in which species are included in a genus; for there would be no genus left if the species vanished. Yet the logical relation is often represented by a diagram, which inevitably suggests this sense. Two circles are drawn, one

[1] Though the relation of a species to individuals is not the same with that of genus to species in all respects, yet what is said here upon the vice of calling the genus a class in which species are included applies also to the habit of calling the species a class including individuals.

enclosing the other; the genus being represented by the outer and the species by the inner circle. It is not impossible to use such diagrams without being influenced by their obvious suggestions; yet their obvious suggestions are false, and to avoid them is difficult.

Secondly, a thing may be included in an aggregate, which is constituted by that and all the other things included along with it. In this sense a cannon-ball is included in a heap, and a particular letter in the pile on my table. We do actually use the word class on some occasions to indicate a total formed in this way; in a school, for example, a class is a certain number of boys taught together, and when a boy is moved from one class to another, he is sent to do his work with a different set of boys. Here we have a notion which is so far nearer the logical notion,[1] as that the class would disappear upon the disappearance of what is included in it. But a little reflection will show that the logical relation of genus to species is no more like that of an aggregate to its members than it is like that of an envelope to its contents.

If Tom Smith is in the first class in his school, I should look for him among the boys in a particular class-room; but if a triangle is in the class figure, or a Red Admiral in the class lepidoptera, that does not mean that I should look for either in a collection of figures or of lepidoptera; it is true that a collection of these objects would include specimens of the triangle or the Red Admiral; but they do not belong to their respective genera because they are in the collection; specimens of them are placed in the collection because they belong to the genera. Were it otherwise, I could not say that a triangle *is* a figure, or that a Red Admiral *is* a lepidopteron, any more than I can say that Tom Smith *is* the first class; I could only say that as Tom Smith *is in* the first class, so a triangle *is in* the class figure, and a Red Admiral *in* the class lepidoptera; whereas it is characteristic of this to *be* a lepidopteron, and of that to *be* a figure.

The 'class' to which species (or individuals) are referred is apt not to be thought of as something realized in its various members in a particular way; but the genus is something realized in every species (or, if it is preferred, in the individuals of every species)

[1] i.e. the notion which the phrase 'to be included in a class' must bear in logic, if it is to be used in any applicable sense at all. And note that even a class at school is not a chance collection, but a collection of boys supposed to share the same level of attainments.

belonging to them, only realized in each in a special way. The differentia carries out as it were and completes the genus. Individuals are not included in one genus because agreeing in certain attributes, and then in one species within the genus because agreeing in certain other attributes that have no connexion with the first; as you might include in one island all men who had red hair, and then rail off separately within it those of them who had wooden legs; *wooden-legged* could not be a differentia of the genus *red-haired*; it must be some modification of red-hairedness itself, and not of the men having it, which could serve as a differentia to that genus. It is therefore a phrase that may mislead, to say that the differentia *added* to the genus makes the species, or makes up the definition. For adding suggests the arbitrary juxtaposition of independent units; but the differentia is not extraneously attached to the genus; it is a particular mode in which the genus may exist. And hence, when we distinguish the various species of one genus, in what is called a logical division,[1] assigning to every species the differentia that marks it off from the rest, our several differentiae must be themselves homogeneous, variations, as it were, upon one theme and, because each cognate with the same genus, therefore cognate with one another. If rectilinear triangle, for example, is regarded as a genus, and one species of it is the equilateral, the others will be the isosceles and the scalene: where each differentia specifies certain relations in the length of the sides; if one species is the right-angled, the others will be the obtuse- and the acute-angled: where each differentia specifies certain relations in the magnitude of the angles. The principle that the differentiae must be thus cognate is technically expressed by saying that there must be one *fundamentum divisionis*; this, however, has its proper place of discussion in the next chapter.

To define anything then *per genus et differentiam* is to put forward first a relatively vague notion and as it were the rough plan of the thing, and then to render this definite by stating in what way the rough plan is realized or worked out. And the differentiae are of the essence of the things, because they belong to the working out of this rough plan. In the definition of organic species (inorganic kinds we will consider later) this is what we aim at doing. We start with the general notion of a living body, and classify its various forms in such a manner as to show how this scheme is

[1] Cf. *infra*, c. v. p. 115.

realized in successively more complex ways. Our first division is into unicellular and multicellular organisms (protozoa and metazoa): the former obviously admit of no composite cellular structure; in a multicellular organism there must be a method of constructing the system of parts. Hence we proceed to differentiate these according to the principal modes of structure which they exhibit; on this basis is founded for example the division of the metazoa in the animal kingdom into coelentera and coelomata; of coelomata into a number of 'phyla' (φῦλα, tribes), the platyhelmia or flat-worms, annelida or worms, arthropoda, mollusca, echinodermata and chordata; of chordata, according to the form which the noto-chord assumes, into hemichorda, urochorda, cephalochorda and craniata; and of craniates, according to the different forms which the general principle of craniate structure may assume, into fish, dipnoi, amphibia, reptiles, birds and mammals.[1] When it is said that we start with the general notion of an animal body, it is not of course meant that historically we conceive that, before becoming acquainted with individuals. We first become acquainted with individual plants and animals. But the use of general names shews that some apprehension of their common nature comes to us from the beginning along with our experience of individuals; only we may long remain unable, or not endeavour, to formulate it. This also applies, at a higher level, to the common nature of various species—horse, dog and fox, oak, elm and apple—with which we have become familiar; we may detect that there is such an identity, before we know what it is, and call them all by a generic name, like animal or tree. The genus is that with which, when we have acquired an insight into the nature of these various kinds, we then start; it is first in the order of our thought

[1] The extent to which, in subordinating species and genera to a superior genus, a common type or plan can be definitely traced through them all, may vary at different stages of a classification. The same functions of animal life are diversely provided for in protozoa and metazoa; and within the comparative complexity of metazoa, in coelentera and coelomata; but it would be impossible to give any one diagrammatic representation of the structure of all these, or even of all metazoa. Such representations are given for coelentera in general, and coelomata in general; yet they are a mere outline, in which even the principal organs of many important types are sacrificed. On the other hand, for each separate phylum among the coelomata zoologists can give a representation, in which a place is found for every principal organ that all the species of that phylum, though with manifold variation of development, at some stage of life or other alike exhibit; and for the subdivisions of the craniata this can be done more adequately than for the subdivisions of the chordata.

about them when we understand them, not in the order of our acquaintance with them when we perceive them. According to the Aristotelian formula, it is *φύσει πρότερον*, or *λόγῳ πρότερον*, not *ἡμῖν πρότερον*: first or fundamental in the nature of the thing, or in an account of it, but not what strikes us first. And Aristotle also expressed its function by saying that the genus is, as it were, the matter, *ὕλη*, of the species or kind.

In saying that a genus is related to its species as matter to form, the relation of matter to form is conceived as that of the less developed to the more developed, the potential to the actual. A word of caution is necessary here. We often compare two particular objects, say a 'bone-shaker' and a modern bicycle, and observing that one carries out more completely certain features imperfectly present in the other, call them respectively more and less developed. The same thing may be observed in the arrangement of a picture gallery, where the pictures are placed in such an order as will exhibit the gradual development of an artist's style, or of the style of some school of artists: and in a museum, where the development of the art of making flint implements is illustrated by a succession of specimens each more perfect than the last. Now in all these cases, the more and the less developed specimens are all of them concrete individuals: each has an actual existence in space and time. But with genus and species it is otherwise. They are not individuals, but universals; the genus does not exist side by side with the species, as the bone-shaker exists side by side with the best bicycle of the present day; and you cannot exhibit genus and species separately to the senses. It is our thought which identifies and apprehends the generic type, say of mammal, in the different species, man and horse and ox; and in thinking of them, we may say that the single type is developed in so many divers ways; but genus and species do not exist in local or temporal succession, the less developed first, and the more developed later, like the specimens which illustrate the development of a type or style. Obvious as these remarks may seem, they are not superfluous, if they help to guard against the idea that a genus is something independent of its species.

[It would be travelling too far beyond the limits of an elementary work to enquire into the meaning of arranging individuals in an order of development: whether (like plants and animals) they proceed one from another in a true genealogical series, or are manu-

[factured independently, like bicycles or arrowheads. A criticism of the conception of development is however of great importance; for the complacent application of the notion to disparate subjects, under the influence of the biological theory of evolution, by writers like Herbert Spencer has diffused many fallacies. Perhaps it may be suggested that, if we wish to know what we mean when we apply the conception of greater and less development to the relation between individual things, we should first examine what we mean by the conception in the relation of genus and species. We cannot throw any light on the relation of genus and species by comparing it with what subsists between individuals at different stages of 'evolution'; but we may get some light upon the conception of evolution from reflection on our conception of the relation of genus to species. For the 'evolution of species' is generally supposed to be not mere change, but development; yet it is often supposed also to involve nothing of the nature of purpose, or design. Now unless we find, in considering individual things, that there is a character or form *suggested* to us in what we call the less developed, but not adequately exhibited there as we conceive it, and that this same character or form is more adequately exhibited in what we call the more developed thing, we have no right to call them *more and less developed* at all. The relation therefore is not between the things as individual, but between their characters; we cannot identify with the less developed individual the character or form which is less developed in it; there is the same at different levels of development in each individual; and the evolutionary history of the series of individuals must be a manifestation of such a character or form in them, unless we are to say that there is no real development, but only change, and that to call this change development is to read into things a fancy of our own. The example of such development best known to us is in the activity of the intelligence.]

[In the first chapter, the antithesis of form and matter was employed in explaining how a common character might belong to divers things. Two shillings, we saw, may be said to be of the same form, while the matter in them is different: and two propositions to be of the same form, so far as each asserts a predicate of a subject, while their matter varies with the difference of subject and predicate. But in saying that genus is related to species as matter to form, it is implied, as between two species, that their common genus, the 'matter', is that in which they agree: while the specific form assumed by this matter in either is the basis of the distinction between them. Indeed, the phrase '*specific* differences' implies that their differences constitute their *form*. It may seem strange that whereas in one sense matter is that which is different

[in things of the same form, in another it is that which is the same in things of different form.

A little consideration will show that the common notion in both these uses of the term *matter* is the notion of something undeveloped. With regard to the phrase that calls the genus the matter of the species, this point has already been illustrated. And when we contrast, in a shilling, the matter (silver) with the form, it is still so. We regard a shilling as an object having a certain form (that might also be stamped in gold or copper) impressed upon a certain matter, silver: and say that both are necessary to its being a shilling. But the *material* which the minter takes has a shape as much as a shilling has, though one geometrically less simple; whereas the *matter* which the metaphysician contrasts with form is really silver *as of no shape*, or without regard to shape (cf. pp. 55–56 *supra*). Now in thinking of silver in abstraction from any shape, our thought of it is incomplete. As the genus only exists in the species, so the matter, silver, only exists in some form. It is however true that there is no special relevance between the nature of silver and the shape of a shilling, whereas the specific form of man can only be realized in the genus mammal; and hence the conception of development applies more closely to the relation of genus and species, than to the relation of matter and form in a concrete thing.

Many controversies have been waged over what is called the *principium individuationis*. What is it that makes one individual distinct from another individual of the same species? Some of the schoolmen held that, being of the same species or form, they were distinct in virtue of their matter; and it followed, since angels have no matter, that every angel is of a different species: except their species, there is nothing by which they can be distinguished from each other. We may be less ready to dogmatize with confidence about angels than were the schoolmen; but the fashion of deriding their speculations because they were exercised in solving that kind of questions is fortunately in diminished vogue. The problem of the *principium individuationis* is a serious philosophical problem.

It may throw some further light on what has been said of the antithesis between matter and form, to point out that matter cannot really be the *principium individuationis*. Two shillings which have the same form are said to be of different matter. Now their matter is silver: but it is not because it is made of silver that one shilling is different from another shilling. In that respect all shillings agree; it is because they are made of *different masses or pieces* of silver that they are different shillings. But if so, it follows that to be of silver is a character common to both pieces (quite apart from their being from the same die); and though we say they differ in matter, we mean that though of the same matter, they are different

[pieces of it. The problem of the *principium individuationis* is not therefore solved by the distinction of matter and form ; the shillings are different, though of the same form, because in each that form is stamped upon a different piece of silver ; but the pieces of silver themselves present the same problem, of a common form (the nature of silver) in different individuals. Matter is indeed, strictly speaking, not a particular thing or an aggregate of particular things, but a generic concept. We recognize various species of it, which we call elements : the elements are different forms of matter ; and in calling them so, we imply something common to them all, as we imply something common to man and ox in calling them both animals ; though we are less able in the former case than in the latter to conceive the common or generic character in abstraction from its specific differences.]

It hardly needs now to be pointed out, that where the predicate of a proposition defines the subject, it is related to its subject far otherwise than where it is an accident. We realize (or we should realize, if our definitions were what we aim to make them) that the genus, modified or developed in the way conceived, *is* the subject ; the definition and that which is defined are not two but one. Of course, when a green thing is square, the same particular thing is both square and green ; the green thing and the square thing are one thing ; but here the subject is not an universal, and we have only to recognize the coincidence of attributes in the same individual. Being green and being square are not one, as being a square and being a four-sided rectangular and rectilinear figure are[1] ; there is a conceptual unity between these ; between those only an accidental.

It follows that there is a conceptual connexion between any subject and its genus or differentia ; he who understands the nature of the subject sees that it must be what is predicated of it as its genus or its differentia. What belongs to the essence of anything *must* belong to it ; for else it would not be that kind of thing, but something different.

(3) We may now take up the last of the points raised on pp. 75–76, viz. the ground of the distinction between essence and property ; since the last paragraph suggests the question, What do we mean by the essence ? If the essence of anything be what makes it what

[1] Aristotle would express this by saying that τὸ χλωρόν may be τετράγωνον, but τὸ χλωρῷ εἶναι is not τὸ τετραγώνῳ εἶναι—the green is square, but greenness is not squareness ; whereas triangularity is three-sided-rectilinear-figurehood. Cf. *supra*, pp. 15, n. 1, and 22–23.

it is, of course it would be something different, were any element in its essence wanting; but what makes it what it is?

Those who hold the view, already mentioned, that definition is of names only and not of things, have an answer ready here, agreeable to that view. They say that we cannot tell what makes anything what it is, but only what makes it to be called by a certain name; and that the world might have been spared much useless controversy, if men had realized that by the essence of anything they meant no more than the attributes which they agreed should be signified by a general name: or, as Locke called it[1], the *nominal* essence. The essence is on this view determined arbitrarily, i.e. by human convention, though doubtless not without regard to human convenience—in particular, the convenience of conforming our nomenclature to what experience shews us of the grouping of qualities in things. The view is readily suggested by a consideration of material things. If we were to regard only the definitions of geometry, it would appear paradoxical to maintain, that men determined arbitrarily what to include in the definition of circle or rectilinear triangle, and what to omit. Manifestly you declare better what a rectilinear triangle is by saying that it is a three-sided rectilinear figure than by saying it is a rectilinear figure whose angles are equal to two right angles; or a circle, by saying that it is the figure generated by the revolution of a straight line in one plane round one of its extremities remaining fixed, than by saying that it is a plane figure having a larger area than any other of equal perimeter. What has led men to suppose that definition is a matter of fixing the meaning of names is chiefly the difficulty found in defining natural kinds, i.e. the various species of animal, plant, or inorganic element; in despair they have looked to the signification of the name for the only meaning of the essence of the thing. Our procedure with abstract notions like wealth or crime or liberty has lent support to the same view. In these cases, the subject defined cannot be presented to the senses in an example, as can gold, or the holm-oak, or the buffalo; we cannot be sure therefore that different men intend to define the same thing, when they offer definitions of such notions; and instead of settling first by its appearance that a given act is a crime, or an object wealth, or a state one of liberty, and then arguing to its nature from our definition, we have rather to determine whether

[1] v. *Essay concerning Human Understanding*, Bk. III. c. iii. § 15.

it is to be called a crime, or wealth, or a state of liberty by considering whether its nature is such as mankind, or particular writers, have agreed to signify by those names. Hence it might appear that in the case of abstract terms [1] at any rate, convention settles what the essence of them shall be; in the main it is not really so, even with them; for the understanding of facts would not then be facilitated as it is by the substitution of 'better' for 'worse' definitions of abstract terms; but the plausibility of the view here adds weight to the arguments which are drawn, in the manner we must now proceed to show, from the definition of natural kinds.

Suppose that we wish to define the natural substance dog, or gold. The forms of language recognize a difference between a substance and its attributes; for we say that Gelert is a dog, but not that he is a faithful; and speak of a piece of gold, but not of a piece of heavy. Yet when we define a substance we can only enumerate its qualities or attributes,[2] and leave out of account what *it* is that has them. What attributes of Gelert then are we to enumerate, to explain what we mean by calling him a dog? or what attributes of a wedding-ring, to explain what we mean by calling it gold? In each case a certain fixed nucleus, as it were, of attributes, holding together in repeated instances and through great varieties of circumstance, is included in our concept of a thing called by such a general concrete name. But which attributes are to form this nucleus, and on what principle are we to make our selection? If it be said that we are to include every attribute common to all dogs, or all gold, two difficulties arise. The first is, that we should include in our notion of dog or of gold all the *properties*, as well as the attributes that are to constitute the *definition*: for the properties

[1] Such complex abstract notions were called by Locke 'mixed modes'; which he said we could define, because we had first made them by putting together simple notions (or in his language, simple ideas) with which we were perfectly acquainted. The expression 'mixed mode' has not established itself; perhaps because the words are not well adapted to convey the meaning which Locke intended by their combination; but it would be useful to have an appropriate expression to indicate what he meant. Cf. *Essay*, Bk. II. c. xxii.

[2] We have, however, seen, in discussing genus and differentia, that these cannot well be called attributes. But it might be urged, that although they cannot be attributed to any other 'universal' as qualifying it, they must be attributed to something which in any individual is what has the substantial character, in virtue of which we call it a dog or gold, as well as having such other attributes as mangy or fine-drawn; cf., however, pp. 54-57, *supra*.

of a kind are the predicates common and peculiar to all the individuals of that kind; and hence we should still lack a principle upon which to discriminate between property and essence. The second difficulty is more serious. We are to include in our definition of a kind every attribute common to all individuals of that kind; but until we have defined the kind, how can we tell whether a particular individual belongs to this kind or another? Let the definition of gold be framed by collecting and examining every piece of gold, and noting down the attributes common to them all; the task is impossible in practice, but that might be overlooked; it is, however, vicious in theory; for it implies that we already know what gold is, or what makes a particular thing a piece of gold, and can by that knowledge select the things which are to be examined, as specimens of gold, in order to determine the nature of that substance. Thus we seem to be moving in a circle; what is gold we are to settle by an examination of the things that are gold; what things are of gold, by knowing what gold is.

Hence our selection must be arbitrary; for we have no principle on which to make it. We may take a particular atomic weight, the power to resist corrosion by air, ductility, malleability, and solubility in aqua regia; and say these constitute gold, and are its essence. And in that case its colour is a property, or for all we can tell, an accident; for we can see no necessary connexion between a yellow colour and all or any of those attributes, and if we found a white metal with those five attributes we should have to call it gold. But if we chose to include yellow colour with them in our definition, then nothing could be gold that was not yellow; yellow would be of the essence of gold; but only because we had decided to give the name to no metal of another colour; it would be the meaning of the name that fixed the essence, and the essence would be only 'nominal'.

It has been assumed in the above that the attributes included in the definition may be not only arbitrarily selected, but without any perceivable connexion among themselves; so that any attribute omitted from the definition should drop at once into the rank of accident; the essence is only a collection of attributes comprised in the signification of the same name, and there are no properties at all. And some logicians have maintained that we can never see any necessary connexion between different attributes; and that when we speak of them as universally connected, we really mean no more

than that they have been very frequently found accompanying one another. Without for a moment agreeing with this opinion (which denies any sense in the distinction between a connexion that is necessary and universal, and a conjunction that is accidental) it may be admitted that we often regard attributes as necessarily and universally connected, because we believe that with fuller knowledge we *might* see into the necessity of the connexion, when as yet we cannot actually do so. This is markedly the case with the various properties of an inorganic substance; and the kinds of plant and animal also present us with many instances where different peculiarities in a species are inferred to be 'correlated', because the same conditions seem to affect them both, or because within our experience they are uniformly present and absent together, without our being able to understand the connexion between them.

The difficulty of determining what attributes are essential to a substance, and therefore of discriminating between essence and property, does not however arise entirely from the seeming disconnexion among the attributes of a kind. It arises also, at least in the organic, from the great variation to which a species is liable in divers individuals. Extreme instances of such variation are sometimes known as border varieties, or border specimens; and these border varieties give great trouble to naturalists, when they endeavour to arrange all individuals in a number of mutually exclusive species. For a long time the doctrine of the fixity of species, supported as well by the authority of Aristotle and of Genesis, as by the lack of evidence for any other theory, encouraged men to hope that there was a stable character common to all members of a species, and untouched by variation; and the strangest deviations from the type, excluded under the title of monstrosities or sports or unnatural births, were not allowed to disturb the symmetry of theory. Moreover, a working test by which to determine whether individuals were of different species, or only of different varieties within the same species, was furnished, as is well known, by the fertility of offspring; it being assumed that a cross between different species would always be infertile, like the mule, and that when the cross was uniformly infertile, the species of the parents were different. But now that the theory of organic evolution has reduced the distinction between varietal and specific difference to one of degree, the task of settling what is

the essence of a species becomes theoretically impossible. It is possible to describe a type ; but there will be hundreds of characteristics typical of every species. Who is to determine what degree of deviation in how many of these characteristics will make a specimen essentially or specifically different ? Will it not have to be decided arbitrarily at the last ? so that here again our use of names will settle what is essential to the species. Everything will be essential that we require in a specimen in order to call it by a certain specific name.

Such are the reasons for saying that the essence of anything is settled by the meaning that we give to names, and if the essence is thus arbitrary or fixed by convention, the distinction between essence and property is similarly infected. But that distinction is obnoxious to another objection, already noticed on p. 93 : that if the property is common and peculiar to the kind, it ought to be included in the essence, because connected with it universally and necessarily. It is as little possible for a rectilinear triangle not to contain angles equal to two right angles, as not to have three sides ; as little possible for a line not to be straight or curved, as not to be the limit of a superficies. If the property of a subject is grounded in the nature of that subject alone, why is it not regarded as a part of its nature ? if it is grounded partly in the nature of the subject, partly in conditions extraneous to the subject, then the subject only possesses it in a certain conjunction, and it ought to be called an accident.[1]

Having thus presented our difficulties, we must endeavour their solution.

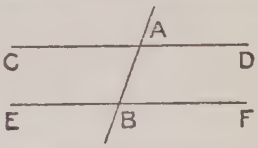
The inexpugnable basis of truth in the theory of the predicables lies first in the distinction between the necessary and the accidental : secondly, in the analysis of definition into genus and differentia. The first underlies all inference ; the second, all classification. But the notion of essence, and the distinction between essence and property, are not applicable in the same way to every subject.

They present at first sight no difficulty in geometry. The essence of any species of figure includes so much as need be stated in order to set the figure as it were before us : whatever can be proved of such a figure universally is a property. Thus the definition is assumed, the properties are demonstrated ; and

[1] Cf. *supra*, pp. 80–81.

that is the true Aristotelian distinction between essence and property.

But how are the properties demonstrated? Only by assuming a great deal else besides the definition of the figure of which they are demonstrated. We assume, for example, the postulates; and that means that we see that we always can produce a straight line indefinitely in either direction, or join any two points, or rotate a straight line in one plane about its extremity. We assume the axioms; and that means that we see, e. g., that any two right angles must be equal; and that if a straight line *AB* falling on two other straight lines *CD*, *EF* makes the sum of the angles *CAB*, *EBA* equal to the sum of the angles *DAB*, *FBA*, *CD* and *EF* must be parallel, and if not, not; we assume also in one proposition all that we have already proved in others. It is seldom from considering merely the definition of the figure which we contemplate that the perception of its properties follows; we must set the figure into space-relations with other lines and figures, by an act of *construction*; and the truth of our conclusion involves not solely the essence of the figure as set out in its definition, but that taken together with the nature of space; for it is really the nature of space which we apprehend when we realize that the sum of the interior angles made by two particular parallel straight lines with a line that cuts them is the same on either side of it, or that a given straight line can be produced to meet another with which it is not parallel. Another point must be noticed. It was said that whereas the properties are demonstrated, the definitions are assumed; but that does not mean that they are arbitrarily taken for granted. They are assumed, because they are what we start with. But they are not arbitrarily taken for granted, because it is self-evident to us that the existence of a figure as defined is possible; and this is self-evident, because in the process of defining we realize in an actual or imaginary example that such a figure can be constructed. We know that three straight lines are enough to make a figure, because we make it of them in imagination; we know that a figure may have five sides, because we see the pentagon before us. It is this power which geometry possesses of creating instances of the objects of its own study that distinguishes it from the non-mathematical sciences. And it creates its objects by constructing them—i. e. by drawing lines and surfaces; and in this possesses



a natural principle upon which to distinguish between property and essence. For though commonly, in geometry, properties are commensurate with their subjects, and may be reciprocally demonstrated, yet everything depends upon the power mentally to see the lines and surfaces; thus the angles of a triangle determine the position of its lines as much as the position of the lines determines its angles; but it is only through dividing space by lines that the angles can be realized. The visible figure is therefore our necessary starting-point. A definition which fails to determine that waits for application until the figure can be pictured. Let a circle be a plane figure having a larger area than any other of equal perimeter; that does not set a circle before us; an infinity of figures can, we see, be made by a line that returns upon itself and is flexible at will; and the property specified will not, previously to demonstration, afford us any means of selecting the figure intended. But say that a circle is the plane figure generated by the revolution of a straight line about one of its extremities remaining fixed, and then we have it before us; then we understand what it is about which the property of having a larger area than any other figure of equal perimeter is affirmed. Once again, in geometry there are no happenings, no conjunctures. It is true that in order to geometrize we have, actually or in thought, to draw the figures: but our process of drawing only renders visible space-relations which we conceive are eternally present everywhere in space. Therefore the circle or the triangle is not subject to mutation on different occasions; there is nothing to prevent it at one place or time from being the same as at another; and the conditions under which it exists do not vary; the general nature of the space in which it is is uniform and constant. Hence the properties of any geometrical figure, though, as we have seen, we must take the general nature of space into account, as well as the definition of the figure, in order to realize their necessity, may yet without risk of any false deduction be regarded as if they were grounded in the essence of that figure alone. For the general nature of space is a 'constant'; it is everywhere the same, and conditions every figure alike; it is not because that ever changes, that different figures have different properties, but because the figures are different.[1]

Geometry therefore deals with subjects capable of definition:

[1] Some deny that we *know* Euclid's axioms; they are only the most convenient assumptions. Even on this view, though we shall have demonstrated the properties which in Euclidean geometry are demonstrated of the

in which the definition serves to set the subject before us: and in which the distinction between essence and property, though from one point of view questionable, is from another sound. It is questionable, so far as the properties of a figure are as necessary to it as its definition, and do not really any more depend on the definition than the definition on them. But it is sound, so far as the essence is that with which we must start, in order to have the figure before us, and say anything about it, while the properties are what we can demonstrate. The process of demonstration may require that we should make a further construction than what the figure itself demands; but this further construction is not necessary in order that we may apprehend the figure itself; and hence the definition, which as it were constructs the figure, gives us what is essential, the demonstration what is necessarily bound up therewith.[1]

Now the science of geometry, both in Aristotle's day and since, has been apt to seem the model of what a science should be; and that deservedly, so far as its certainty and self-evidence go. But though we may desire an equal certainty and self-evidence in other sciences, we must not ignore the differences between their subject-matter and that of geometry; nor must we assume that the distinction of essence and property will have the same applicability to concrete bodies as to figures in space. The subjects which we study in chemistry, in botany, or in zoology, are not constructed by us; they are complex, and for all we know may differ much in different instances; and they exist under conditions which are not constant (like space) but infinitely various. Hence in them we cannot expect to find the determination of the essence, and the separation between that and its properties, as soluble a task as in geometry.

Let us consider first the definition of inorganic kinds. Here, since a compound may be defined by specifying its composition, our problem deals with the elements. It will be instructive to look for a moment at the Greek treatment of this question. There were

figures defined only subject to these assumptions, the definitions of the figures will have the function stated above. So far as non-Euclidean geometry deals with what cannot be constructed or imagined, the above statement of the distinction between essence and property will for it have to be qualified. In analytical geometry, the distinction between essence and property is harder to draw; yet it must be remembered that unless we could envisage the figure, there would be nothing to analyse. Cf. also *infra*, p. 333.

[1] Yet where there are alternative modes of constructing a figure (e.g. an ellipse) it will be arbitrary which of them we select to define it by; we can only say that the definition must enable us to construct the figure.

two main attempts to define the famous four elements of Empedocles, earth, air, fire, and water. Plato supposed that they differed in the geometrical construction of their particles, those of earth being cubic, of air octohedral, of fire tetrahedral, and of water eicosihedral. If these were their differentiae, what was their genus? We can only reply, solid.[1] They were *something filling space*, of different figures. In assuming the concrete things which he defined to fill space, Plato did what every one who defines a natural substance does. We do not always mention this character in our definition; we might define a scabius, for example, as a certain kind of composita; but to be a composita involves it; and it is necessary if the definition is to furnish the conception of a material thing at all. In taking geometrical figures as his differentiae, he attempted to gain in physics the advantages which geometry derives from our power of constructing its objects; but he failed to show how the sensible properties of the different elements were connected with their respective figures. Aristotle preferred the method of those who distinguished the elements not by the figure of their particles, but by the mode in which they combined certain fundamental sensible qualities, heat, cold, moisture, and dryness. Fire he thought was the hot and dry substance, water the cold and moist, earth the cold and dry, air the hot and moist. These definitions have the disadvantage of using terms that possess no very precise signification. How hot is unmixed fire, and how moist is pure water?

Modern science recognizes in each element a whole legion of common and peculiar attributes. Some of these, such as its atomic weight, are conceived to be constant or to characterize the element in all conjunctures; others it only exhibits upon occasion; this is the case, for example, with its reactions towards other bodies. We have very little insight into the inter-connexion of the various attributes thus characterizing each element; but unless we are to regard everything in nature as accidental, we are bound to believe them interconnected.[2] It is impossible to include in its definition all that is known to be characteristic of an element; and for the mere purpose of identification, many of the attributes of an element would serve equally well. But we prefer to select as differentiae, and include in the definition, such attributes as appear, in some

[1] Or perhaps, regular solid.

[2] On what kind of evidence particular attributes are held to be connected, it is the business of the theory of the inductive sciences to show.

form or another, in all or a large number of elements; because we are thus able to exhibit the divers elements as related to one another upon a scheme, or in other words to classify them. Thus the atomic weight of a substance is more suitable for defining it than some peculiar reaction which it exhibits, although perhaps less useful for identifying it; because all elements must have some atomic weight, but no other need exhibit the same sort of reaction. If, however, a reaction is common to a number of substances, it may serve as a ground for collecting those into one class, like the acids: the common reaction being a generic character; especially when for any reason, such as the number of attributes that are commensurate with it (i. e. are found where it is found, and not where it is absent), such reaction seems to count for much in the being of the substances to which it belongs.

Such considerations may guide us in choosing what to include in our definition; and we shall also *ceteris paribus* prefer for differentiae those attributes that are continuously exhibited to those that an element only exhibits in a rare conjuncture. Nevertheless it is plain that our procedure is in great measure arbitrary; and the distinction between essence and property is not applicable as it was in geometry. For among the constant attributes of an element we cannot start with some and demonstrate the remainder; and those which it exhibits only in particular circumstances are not properties in the full sense. We may indeed call it the property of an element to exhibit a certain reaction in certain circumstances [1]; but whereas the 'circumstances' under which geometrical figures exist and possess their properties are in every case the same (being their existence in space), the circumstances relevant to the manifestation of the several properties of an element are different; hence we cannot afford to omit the statement of them in stating its properties; and since they are often very numerous and complex, and involve many other substances, it may be more natural to refer the property to a compound, than to one element. Nevertheless, since causal connexion is fundamental in the notion property, we rightly regard these attributes as properties rather than accidents. For although the subjection of an element to any particular conditions rather than others is strictly speaking accidental, since it depends upon

[1] Cf. Ar. *Top.* ε. i. 128^{b} 16 *ἀποδίδοται δὲ τὸ ἴδιον ἢ καθ' αὑτὸ καὶ ἀεὶ ἢ πρὸς ἕτερον καὶ ποτέ* ('a property is ascribed to a subject either *per se* and always or in a particular relation and time').

historical causes that are independent of the nature of that element, yet its behaviour when subject to those conditions is not accidental: so that it is fairly called a property of gold to be soluble in aqua regia, though very little gold be so dissolved. On the other hand, we call it an accident of gold to lie in the cellars of the Bank of England; for though it is not accidental that it should lie where it is placed, but its doing so is connected with other features in the nature of gold, yet that the particular place should be the cellars of the Bank of England no more illustrates a general principle, than that the aqua regia in which it is dissolved should have been bought in Cheapside. No reasonings that apply to gold universally, but only historical reasons, will show that certain parcels of gold must be lying there.

The use of the singular without the article (as in a proper name) when we say that gold is malleable, or iron rusts, or silver tarnishes, is worth remark. It implies that we think of gold, or silver, or iron as one and the same thing always: that we are looking to the unity of kind, and not the particular specimens. Different parcels of the same element may be found in divers states, solid, liquid or gaseous, crystallized or uncrystallized, in molecules of different numbers of atoms, and so forth. But we conceive that any one sample is capable of all states whereof any other sample is capable; they have no 'individuality'. Even when we investigate the properties of a compound, so far as the composition is really known with accuracy, we have the same confidence in attributing to that compound universally the properties discovered in a particular sample. But in organic kinds, though we may know the chemical composition of the parts, we cannot know with the same accuracy the composition of the heterogeneous parts into the whole. Hence we do not know how far different individuals are capable of the same behaviour. And if an organism has a real unity, the differences between one and another individual of the same kind will never be fully explicable from their composition.

Indeed the problem of distinguishing between essence and property in regard to organic kinds may be declared insoluble. If species were fixed: if there were in each a certain nucleus of characters, that must belong to the members of any species either not at all or all in all: if it were only upon condition of exhibiting at least such a specific nucleus of characters that the functions of life could go on in the individual at all; then this nucleus would form the essence

of the kind. But such is not the case. The conformity of an individual to the type of a particular species depends on the fulfilment of an infinity of conditions, and implies the exhibition of an infinity of correlated peculiarities, structural and functional, many of which, so far as we can see (like keenness of scent and the property of perspiring through the tongue in dogs), have no connexion one with another. There may be deviation from the type, to a greater or less degree, in endless directions ; and we cannot fix by any hard-and-fast rule the amount of deviation consistent with being of the species, nor can we enumerate all the points, of function or structure, that in reality enter into the determination of a thing's kind. Hence for definition, such as we have it in geometry, we must substitute classification ; and for the demonstration of properties, the discovery of laws. A classification attempts to establish types ; it selects some particular characteristics as determining the type of any species ; these characteristics should be (*a*) of the same general kind for each type within one genus, or, as it was expressed on p. 86, variations upon the same theme, in order to exhibit the mutual relations of agreement and divergence among the various types : (*b*) important, or, as one might say, pervasive : that is, they should connect themselves in as many ways as possible with the other characters of the species. It will be the description of the type, drawn up on such principles as these, that will serve for definition. It is avowedly a mere extract from all that would need to be said, if we were to define (upon the supposition that we could define) any species of plant or animal completely.

The full nature of an organic species is so complex, and subject to so much variation in different individuals, that even if it could be comprised in a definition, the task of science would hardly consist in demonstrating its properties. To discover the properties of kinds belongs to the empirical rather than to the scientific stage of botany or zoology. Science proceeds to ask what it is in any kind on which a particular property belonging to it depends. Herein we break up or analyse the complex character of the kind, in order to determine what we call the *laws* of organic life. If a species, for example, is keen-scented, that must depend upon conditions that are but a small part of what would be included in a complete account of its nature. In order to find the commensurate subject of which a property is predicable, we must abstract from all in the species which is not relevant to that one property ;

and our subject will not be the concrete kind, but one determined by a set of conditions in the abstract. The property whose conditions we have found is of course the property not of those conditions, but of anything that fulfils those conditions; keen-scentedness, for example, is not a property of a particular construction of the olfactory organ (though we should call it an effect of this), but of an animal in whom the olfactory organ is thus constructed; the laws of organic life suppose of course that there exist organisms in which they are exhibited. We may still speak therefore of properties of kinds; but the demonstration of them considers the nature of the kind only so far forth as it concerns the property in question. The property is not common and peculiar to the kind, if other kinds, as may well be the case, agree with it in those respects on which the property depends; or if it depends on conditions which cannot be fulfilled except in an individual of that kind, but are not fulfilled in every individual thereof.

Such reflections led the schoolmen to distinguish four senses of the term property—

1. *id quod pertinet omni sed non soli*: thus it is a property of the cow to give milk; but other animals do the same; and to give milk is the commensurate property not of a cow but of a mammal; being causally connected with a feature which though present in a cow is present in other species besides.[1]

2. *id quod pertinet soli sed non omni*: thus it is a property of man to write poetry, but not universally; for the writing of poetry requires powers which no creature but man possesses, but which also one may not possess and yet be a man.

3. *id quod pertinet omni et soli, sed non semper*: in this sense it is a property of the male egret to grow a certain kind of feather, much used by ladies in their hats; but only to grow it at the pairing season.

4. *id quod pertinet omni et soli et semper*: in this sense it is a property of a rectilinear triangle to have its angles equal to two right angles; but it is difficult to find an example of such a property among organic kinds, for a feature so constant and universal would be regarded as part of the essence: unless like the schoolmen we call it a property in this sense to be capable of exhibiting a property

[1] If all the subjects possessing the property are in one genus, it is called a *generic* property. Aristotle's definition of property as a non-commensurate predicate not included in the essence places a generic property under the head of accident. Cf. p. 126, *infra*.

in sense 3; they often gave it as an illustration of property in the third sense that man laughs; and in the fourth sense, that he is capable of laughter; for the capacity is permanent, but the exercise of it occasional.

In all these uses of the term *property* the notion of a necessary or causal connexion is retained; but commensurateness with the subject is not insisted on in all. No doubt a commensurate subject for every predicate is to be found; but only by specifying the precise conditions (in an organism or in whatever it may be) on which the property depends; but the concrete thing is the subject about which we naturally make propositions, naming it after its kind; and kinds being complex may agree together in some points while differing in others with intricate variety; so that when we have distinguished the species to which things conform, and the attributes which they possess, we cannot divide the latter among the former without overlapping.

Many general and abstract terms, which form the subjects of propositions, designate neither natural substances nor mathematical entities. There are names of qualities and states of things, like *softness* or *putrefaction*: or psychical states and processes, like *pleasure*, *anger*, *volition*: of the material products of human or animal skill, like *pump*, *umbrella*, *bridge* or *nest*: of natural features of the earth's surface, like *beach* or *valley*: of determinate parts of an organism, like *cell* or *sympathetic nerve*: of forms of human association, like *army*, *university*, *democracy*, *bank*. It would be tedious to proceed further with such an enumeration. About all of these terms it is to be observed that the notion of them involves a certain abstraction. *Bridge* and *pump* are concrete terms, but they are names given to material things because they serve a certain purpose, or exhibit a certain structure; and all else in the nature of the thing is disregarded, in considering whether it is a bridge, or whether it is a pump. In attempting to define an element on the other hand, or an organic species, we have to wait upon discovery, in order to know the nature that a thing must possess as gold, or as a crab; the whole nature of the concrete thing forms the subject of our enquiry. It is the abstract character of the terms which we are now considering, or the limited extent of their signification, that renders them more capable of satisfactory definition; they are least definable, where that which they denote is most complex; thus it is easier to define *army*

than *democracy*, and *rigidity* than *putrefaction*. The more complex any subject, the less is it possible to exhaust its nature in any brief compendium of words, and the greater also are its capacities of various behaviour under varying conditions; all these are part of the notion of it, and no definition will really be worth much to any one who cannot realize how different the thing defined would be in different circumstances. Thus a definition of democracy means most to him whose mind is most fully stored with a knowledge of history and of institutions and of human life; he can realize what government of the people by the people for the people (if that were our definition) really involves. But comparatively little knowledge is needed in order that the definition of a bridge may be fully understood. It will be readily seen, that what has been said of the difficulty of determining either property or essence in regard to natural kinds applies also to such terms as we are now considering in proportion to the complexity of the notion to be defined; the more complex the subject, and the greater the range and variation of the modes in which it manifests itself, according to the conditions under which it exists, the more arbitrary becomes our choice of characters to be included in the definition, and the less can properties be commensurate attributes.

We have now reviewed the theory of predicables as it was first propounded; we have seen that the scheme of knowledge which it implies cannot be realized upon all subjects; that it is best exemplified in mathematics, and in other sciences which deal with abstractions. But we have also seen that it contains distinctions of great value and importance. These are

1. the antithesis between an accidental conjunction (or coincidence) and a necessary or conceptual connexion;

2. the conception of the relation of genus and differentia, and of the unity of genus and differentia in a single notion;

3. the resting the distinction of essence and property upon the distinction between that which we start with and that which we demonstrate therefrom; though this use of the term property cannot always be adhered to in practice.

It remains to say a few words upon the Porphyrian doctrine.

It differs to appearance in one point alone; the Porphyrian list of predicables substitutes *Species* for *Definition*. But that difference implies a change in the point of view. It implies that we are to find the meaning of these five terms—Genus, Species, Differentia,

Property, Accident—in the relations which its predicates bear to an individual subject not as an individual *of a certain sort*, but barely *as that individual*; for it is of individuals as individuals, not as of a certain sort, that their species (such as man, or horse, or parrot-tulip) are predicated.[1] And various inconveniences arise from this change. First and foremost we have to determine what is a true species, and what only a genus within a wider genus.[2] Do I predicate his species of Cetewayo when I call him a man, or when I call him a Zulu? if Zulu be a species, man is a genus, though included within the wider genus of mammal, craniate, or animal; but if man is the species, Zulu is an accident. The question thus raised is really insoluble; for species, as is now believed, arise gradually out of varieties. It gave rise to many great controversies, as to whether a species were something one and eternal, independent of individuals, or on the other hand no more than a name. These opposite views were indeed older than Porphyry or the mediaeval thinkers who discussed them so earnestly; nor can any philosophy refuse to face the controversy between them. But it was a misfortune that the theory of predicables should have got involved in the controversy; partly because it led to a mode of stating the fundamental issue which is not the best: partly because the true value of the theory of predicables, as a classification of the relations between universals predicated one of another, was lost sight of in the dust of the dispute between the realists and the nominalists.

A second inconvenience in the Porphyrian doctrine is that while beginning by distinguishing the relations of its predicates to an individual, it cannot continue true to this standpoint. Species is properly predicated of an individual; we ask what is the species not of man, but of Cetewayo; and if the species can be analysed into genus and differentia, it is possible to regard these as predicated of

[1] There is a suggestion in Aristotle's *Topics* of this point of view, for he allows that ἴδιον may mean a peculiarity that distinguishes an individual from others; cf. the passage quoted, p. 101, n. 1, *supra*, and *e.* i. 129ᵃ 3–5. But his doctrine as a whole implies that the subject term is general.

[2] In technical language, what is an *infima species* and what a *species subalterna*; it was said that a species subalterna 'praedicatur de differentibus specie', an infima species 'de differentibus numero tantum'. But it is clear that this does not help us to solve the problem: how are we to determine whether men differ in number only and not in kind? It is no easier than to determine whether man or Zulu is the infima species; being in fact the same problem restated. Looked at from the other side, the species subalterna can of course be called the *genus subalternum*: cf. Crackenthorpe's *Logic*, Bk. I. c. iv.

the individual belonging to the species. Nevertheless they are his genus and differentia not as this individual, but as an individual of this species. And similarly with property and accident: a property is necessary to its subject, either absolutely or under definite conditions, i. e. it belongs to a subject of a certain sort because it is of this sort, or of this sort under these conditions; an accident is not thus necessary; it belongs in a given instance to a subject of this sort, but not because it is of this sort, and so need not belong in a second instance. But of a subject indicated by a proper name [1]—of an individual as this individual—we cannot thus distinguish the predicates. A predicate which is connected with one character in the being of an individual is merely coincident with another; but a proper name does not signify one character to the exclusion of the rest. Without such selection, we cannot say whether a predicate is property or accident. If it is asked whether it is a property of Cetewayo to talk, or fight, or be remembered, we must demand, of Cetewayo considered as what? Considered as a man, it is a property of him to talk; considered as an animal perhaps it is a property of him to fight; but considered as a man, or as an animal, it is an accident that he should be remembered, though perhaps a property considered as a barbarian who destroyed a British force. So long as we consider him as Cetewayo, we can only say that all these attributes are predicable of him. They all help to constitute his being as Cetewayo, though not all as a barbarian who destroyed a British force.

Thirdly, the Porphyrian doctrine gave rise to a division of accidents into separable and inseparable which, if a singular term be the subject, is confused, if a general, self-contradictory.[2] An inseparable accident of an individual is an accident of the species

[1] Or by a designation, unless we regard only the general terms in the designation, and not the demonstrative which makes it singular. 'The king' is a designation; if I say that it is a property of the king to be exempt from prosecution, I mean of a king, and therefore of George V.

[2] Ἰδίως δὲ διαφερειν λέγεται ἕτερον ἑτέρου, ὅταν ἀχωριστῳ συμβεβηκότι τὸ ἕτερον τοῦ ἑτέρου διαφέρει. ἀχώριστον δὲ συμβεβηκὸς οἷον γλαυκότης ἢ γρυπότης ἢ οὐλὴ ἐκ τραύματος ἐνσκιρρωθεῖσα, Porph. *Isag.* c. iii, *init.* ('One thing is said to differ peculiarly from another when it differs by an inseparable accident. And an inseparable accident is such as greyness of the eye, hook-nosedness, or the scar of a wound.') Porphyry indeed says that accidents in general subsist primarily in individuals—καὶ τὰ μὲν συμβεβηκότα ἐπὶ τῶν ἀτόμων προηγουμένως ὑφίσταται, *ib.* c. x; and also that they are predicated primarily of individuals—ἀλλὰ προηγουμένως μὲν τῶν ἀτόμων (sc. κατηγορεῖται, from the context) and secondarily of the species containing these, κατὰ δεύτερον δὲ λόγον καὶ τῶν

under which he is considered, but inseparable in fact from him. Thus it is an inseparable accident of a man to be born in England, but a separable accident to wear long hair ; because he can cut his hair short, but cannot alter his birthplace. Now this notion of an inseparable accident is confused, because the attribute is called an accident in relation to the species, but inseparable in relation to the individual ; the whole phrase therefore involves two standpoints at once. And the distinction between separable and inseparable accidents thus understood has really nothing to do with the doctrine of the predicables as a classification of *conceptual* relations between a subject and its predicates. There are, properly speaking, no accidents of an individual as the complete concrete individual. The Old Pretender might have been born elsewhere than in England, and might have cut his hair shorter : regarding him as the son of James II, each of these things is an accident ; but regarding him completely as the man he was, there was reason for each, and neither could have been otherwise without certain historical circumstances being different, though history does not usually concern itself with tonsorial incidents in the lives even of princes. That one thing was alterable while he lived and the other unalterable leaves them equally accidents from one standpoint, and equally little accidents from the other. If however the subject of which a predicate is said to be an inseparable accident be a general term, then the expression is self-contradictory. Porphyry said that blackness is an inseparable accident of the crow. But if it is an accident at all, then it is a mere coincidence that all crows are black, and there is nothing in the fact that a bird is a crow requiring it to

περιεχόντων τὰ ἄτομα, *ib.* c. vi. But he does not seem to see that it is not from their relation to the individual that they are called accidents. For his account of the distinction between separable and inseparable accidents, cf. c. v συμβεβηκὸς δέ ἐστιν ὃ γίνεται καὶ ἀπογίνεται χωρὶς τῆς τοῦ ὑποκειμένου φθορᾶς. διαιρεῖται δὲ εἰς δύο· τὸ μὲν γὰρ αὐτοῦ χωριστόν ἐστι, τὸ δὲ ἀχώριστον. τὸ μὲν οὖν καθεύδειν χωριστὸν συμβεβηκός, τὸ δὲ μέλαν εἶναι ἀχωρίστως τῷ κόρακι καὶ τῷ Αἰθίοπι συμβέβηκε, δύναται δὲ ἐπινοηθῆναι καὶ κόραξ λευκὸς καὶ Αἰθίωψ ἀποβαλὼν τὴν χροιὰν χωρὶς φθορᾶς τοῦ ὑποκειμένου. ('Accident is what comes and goes without the destruction of the subject. It is of two kinds, separable and inseparable. To sleep is a separable accident, to be black is an inseparable accident of a crow or an Ethiopian ; a crow can be conceived to be white or an Ethiopian to have lost his colour without the destruction of the subject.') That he regarded inseparable accidents as predicated both of species and of individuals as subject is clear from c. vi τὸ δὲ μέλαν τοῦ τε εἴδους τῶν κοράκων καὶ τῶν κατὰ μέρος (sc. κατηγορεῖται), συμβεβηκὸς ὂν ἀχώριστον, καὶ τὸ κινεῖσθαι ἀνθρώπου τε καὶ ἵππου, χωριστὸν ὂν συμβεβηκός. ('To be black is predicated both of the species of crows and of crows severally, being an inseparable accident, and to move of man and horse, being a separable accident.')

be black; it cannot therefore be inseparable, however constant in our experiences the conjunction may have been. *Per contra*, if it is inseparable, that must be because the nature of a crow as such requires it, and then it cannot be an accident. The so-called inseparable accident of a species is really an attribute which we find to characterize a species so far as our experience extends, without knowing whether its presence depends on conditions necessary to the being of the species, or partly on conditions in the absence of which the species may still exist. That amounts to saying that we do not know whether it is an accident or a property; and so a phrase is adopted which implies that it is both.

It would be well therefore to abandon the division of accidents into separable and inseparable; and it would be well to abandon the Porphyrian list of predicables in favour of the Aristotelian.[1] Either list raises very difficult questions; but those which have been discussed in this chapter are questions that must be raised, whether we attach little value or much to the use of the terms Genus, Species, Differentia, Property, and Accident. The attempt to think out the connexions between one thing and another is so vital a feature of our thought about the world, that Logic may not ignore the consideration of it. Abstract terms, and general concrete terms, signify not individuals as such, but as *of a kind*. We do regard attributes as connected with one another, and with the kind of a thing, sometimes necessarily and universally, sometimes through a conjuncture of circumstances in the history of an individual. We need a terminology in which to express these differences. We do conceive substances, attributes and states, that cannot be analysed into mere assemblages of simple qualities, but only *per genus et differentiam*. These are the facts which justify this somewhat difficult part of logical theory.

[1] Mr. C. C. J. Webb has called my attention to the following interesting passage in John of Salisbury, *Metalogicon*, iii. 5 'Proinde quid genus aut diffinitio, quid accidens sit aut proprium, docet [Aristoteles] longe commodius his qui in Porphirio aut Categoriis explanandis singuli volumina multa et magna conscribunt. In consilium illorum non veniat anima mea, nec aliquis amicorum meorum praeceptoribus his utatur.'

CHAPTER V

THE RULES OF DEFINITION AND DIVISION: CLASSIFICATION AND DICHOTOMY

In the last chapter the nature of Definition was discussed at some length; but nothing was said of the technical rules in which the requirements of a good definition have been embodied. The process of dividing a genus into species was also mentioned, but neither were the rules given which should be observed in that. It seemed better to defer to a separate discussion these and one or two cognate matters. Treated first, they would have been less intelligible. But what has been said about the relation of genus and differentia, the practical difficulties that lie in the way of adequately defining certain—indeed most—terms, and the homogeneity which ought to characterize the differentiae of the several species in one genus, should serve to render the present chapter easily intelligible.

The rules of definition are as follows :—

1. *A definition must give the essence of that which is to be defined.*

The essence of anything is that in virtue of which it is such a thing. It is in virtue of being a three-sided rectilinear figure that anything is a rectilinear triangle : in virtue of being an institution for the education of the young, that anything is a school: in virtue of having value in exchange, that anything is wealth. We have seen, however, that in the case of natural kinds, and in some degree of highly complex abstract notions, the essence cannot be comprised in the compass of a definition, or distinguished very sharply from the properties of the subject. In these cases one must be content to do the best he can : remembering—

(*a*) That the attributes included in the definition should always be such as are the ground of others rather than the consequences. Thus the higher species of mammal are better defined by the character of their dentition than of their habitual food ; since the kind of food that an animal can eat depends on the formation of its teeth, and not vice versa.

(*b*) That we must not give only some comparatively isolated

attributes of the subject, but also indicate the kind of subject which these attributes qualify. This is done by giving its genus,[1] and hence our second rule is :

2. *A definition must be per genus*[2] *et differentiam* (sive *differentias*).

The better the definition, the more completely will the differentia be something that can only be conceived as a modification of the genus : and the less appropriately therefore will it be called a mere attribute of the subject defined. Thus a lintel is a bar placed to form the top of a doorway ; it can hardly be called an attribute of a lintel that it forms the top of a doorway, for that implies that having already conceived a lintel, I notice this further as a characteristic of it ; whereas really, until I have taken this into account, I have not conceived a lintel. On the other hand, if sodium be defined as an element exhibiting line *D* in the spectrum, the differentia here may fairly be called an attribute. For one may have a pretty definite notion of sodium without knowing that it exhibits this line in the spectrum. The complexity of the subject under definition is in this case such that whatever be taken to serve as differentia can be only a small part of its whole nature ; we have a pretty substantive concept (if the phrase may be allowed) without the differentia ; and therefore this appears as a further characteristic, which is really selected because it is *diagnostic*, i.e. it is a feature by which instances of the subject can be readily identified.

3. *A definition must be commensurate with that which is to be defined* : i.e. be applicable to everything included in the species defined, and to nothing else.

4. *A definition must not, directly or indirectly, define the subject by itself.*

A subject is defined by itself directly, if the term itself or some synonym of it enters into the definition. The sun might, for example, be thus defined as a star emitting sunlight ; or a bishop as a member of the episcopate. Such error is a little gross ; but in the indirect form it is not uncommon. It arises with correlative terms, and with counter-alternatives,[3] where one is used to define

[1] Cf. Ar. *Top.* ζ. v. 142[b] 22–29. But properties, according to Aristotle (*An. Post.* β. x), are defined *per causam et subjectum*, i.e. by specifying the subjects in which they inhere, and the cause of their inherence in their subjects.

[2] Where there is a series of terms in subordination, *per proximum genus.*

[3] Where a subject occurs in two forms, and every instance must exhibit either one or other, then these forms may be called counter-alternatives. Thus in number, the counter-alternatives are odd and even ; in a line, straight and curved ; in sex, male and female ; in property, real and per-

the other. A cause, for example, is ill defined as that which produces an effect, or an effect as the product of a cause ; for correlatives must be defined together, and it is the relation between them that really needs to be defined ; this is the ground of applying both the correlative terms, and in defining this, we define them. The objection to defining a term by help of its counter-alternative is that the latter may with equal right be defined by it. If an odd number is a number one more than an even number, the even is similarly that which is one more than the odd. It sometimes happens, however, that counter-alternatives cannot really be defined at all ; if a man does not immediately understand from examples that a categorical proposition either affirms or denies, there is no other knowledge to which we can appeal in order to explain to him the nature of the distinction, for it is unique ; and in the same way there is no defining the difference between straight and curved. In such cases, to explain one counter-alternative by the other, though not definition, is sometimes the best course we can adopt ; for their mutual contrast may help a man to apprehend them both, and he may be more familiar with one than with the other.

There are subtler modes of defining a thing indirectly by itself. We may use a term into whose definition that which we profess to be defining enters. Aristotle illustrates this by a definition of the sun, as a star that shines by day ; for day is the period during which the sun is shining.[1] J. S. Mill's[2] definition of a cause as the invariable and unconditional antecedent of a phenomenon errs in this particular ; for *unconditional* cannot really be explained without presupposing the conception of cause.

It should be noticed that where the thing defined is designated by a compound word, it may be legitimate to employ in its definition the words that form parts of the compound. Thus a ball-race is the hollow way between the axle and the wheel in which the balls run that are used to take the thrust of one against the other. The term ball, used in this definition, is not of course what had to be defined.

5. *A definition must not be in negative where it can be in positive terms.*

The propriety of this rule is obvious. A definition should tell us

sonal, &c. Contraries, and opposites generally, may be wrongly used to define one another in the same way.

[1] *Top.* ζ. iv. 142ᵃ 34. [2] *System of Logic*. III. v. § 6; cf. *infra*, p. 405.

what the thing defined *is*, not what it *is not*. This it must do up to a point in naming the genus; but unless the species is distinguished by lacking altogether some character which, in one form or another, other species possess, it should continue doing so in naming the differentia. An acute-angled triangle, for example, should be defined, not as one containing neither a right angle nor an obtuse angle, but as one containing three acute angles. In this case it is true that a very little knowledge of geometry would enable any one to extract from the negative information of the former definition the positive characterization of the latter. But the negative differentia is in itself inadequate, and such would in most cases leave us quite uncertain what the subject positively is. If real property were defined as property that cannot be transferred from place to place, we should not necessarily gather that it was property in land. If anger be defined as an impulse not directed to obtaining for oneself a pleasure, who is to understand that it is an impulse to repay a hurt? But for the reason indicated in the exception above, a definition with negative differentia is not always faulty. In defining a privative or negative concept it is inevitable. A bachelor is an unmarried man; and the very meaning of the term is to deny the married state. Injustice, said Hobbes, is the not keeping of covenant. A stool is a seat for one without a back to it.[1] And short of this, definition by a negative differentia is justifiable, in defining a species which is distinguished from other species in its genus by lacking what they possess.[2] Thus *Amoeba proteus* is an amoeba without a nucleus; the melancholy thistle (*Carduus heterophyllus*) is differentiated by the absence of prickles. But it must not be assumed that because a term is negative in form it need be negatively defined; intemperance is the excessive indulgence in strong drink.

6. *A definition should not be expressed in obscure or figurative language.*

The use of obscure words where plain and familiar words are available is a fault in definition, because it militates against the

[1] From Watts's *Logic*. In the definition of injustice, the genus, conduct, is not stated.

[2] My attention has been called to this class of cases by Miss Augusta Klein, from whom I borrow the illustrations; such definitions are diagnostic. The subject so defined exhibits the generic character as determinately as other species. But the definition, instead of stating in what ways that character is positively determined, names a part or feature whose absence makes a notable difference. For positive and negative terms cf. *supra*, c. ii, pp. 40–46.

object of definition—viz. that one may understand the nature of the thing defined. The use of figurative, or metaphorical, language is a graver fault, because metaphors, where they are intended to do more than merely to embellish speech, may suggest or lead up to a right understanding of a subject, but do not directly express it. Memory, for example, is ill defined as the tablet of the mind; for though knowledge is preserved in memory, so that we can recover it again, and writing is preserved in tablets for future reference, yet the two things are very different, and the actual nature of what we call memory is as little like that of a tablet as possible.

It must be remembered that language is not necessarily obscure because it is technical. Every science is bound to use 'terms of art' which will be obscure to the layman, but may express the matters belonging to that science clearly and precisely. The obscurity forbidden is that which would be acknowledged by those acquainted with the field of study to which the definition belongs.

In the process of Definition, we take some species, or other concept, and distinguish in it its genus and differentia. Thus wealth is that which has value in exchange. There may be things which have value, but not in exchange—the air, for example, which has value in use; these are not wealth, and with them, in defining wealth, we are not concerned; though they belong to the same genus. But we might be interested in distinguishing the different species which all belong to one genus; and this process of distinguishing, or of breaking up a genus into, the species that belong to it is called **Logical Division.**

Logical Division is a process of great importance in science. Things belonging to one genus will be studied together; and the aim of our study will be to discover all the general propositions that can be made about them. But though there may be some statements that will apply to everything contained within the genus, others will only be true of a portion. And the better our division of the genus into its species, the larger will be the number of general propositions that can be made about its species or parts.

Division [1] is closely allied to **Classification**; and both to Definition. The difference between Division and Classification seems to

[1] In Logic, if Division is spoken of without any qualification, Logical Division is meant; though there are other operations of thought, to be mentioned later (pp. 132–133), to which the name Division is also applied.

be principally this, that we divide the genus, but classify the particulars belonging to it. In other words, Division moves downwards from the more general to the more special, Classification upwards from the particulars through the more special to the more general. This, at least, is the difference which one would intend to indicate if he contrasted the two operations; but in actual practice our thought moves in both directions at once; and the process of dividing a genus is at the same time one of classifying the things in the genus. If, for example, one were asked to divide the genus *novel*, he might suggest a division into the novel of adventure, of character, and of plot; but he would at the same time run over in thought the novels that he had read, and ask himself if they could be classed satisfactorily under these three heads.

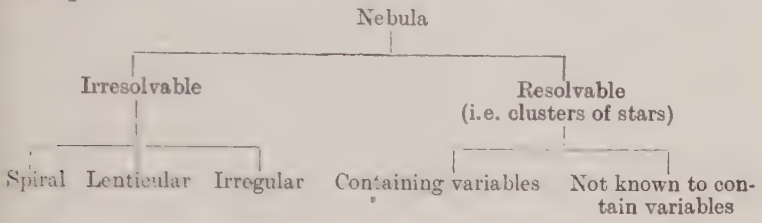
The close connexion between Division or Classification and Definition is obvious. If we divide a genus into species, it must be by the help of differentiae, which serve to define the species we are forming. If the genus *rectilinear figure*, for example, be divided according to the number of a figure's sides into those with three, with four, and with more than four sides, we obtain the definitions of *triangle*, *quadrilateral*, and *polygon*. In a classification also, the classes established must be distinguished by characters that will serve to define them.

A division may be carried through several stages, i.e. the species into which a genus is first of all divided may themselves be subdivided into species; and this may be continued until the species reached no longer require subdivision. The species with which a division stops are called **infimae species**; the genus with which it starts, the **summum genus**; and the intermediate species, **subaltern genera**, i.e. genera (for they are genera in respect of the species next below them) subordinated to another genus.[1] The **proximum genus** of any species is that next above it in the series; and the words *superordinate*, *subordinate*, and *co-ordinate* are used to indicate respectively the relation of any genus to those below it, above it, or standing on the same level with it (i.e. having the same proximum genus). These terms are also used in reference to a classification; for a classification when completed may be regarded as a division and vice versa. The co-ordinate species into which a genus is

[1] Cf. p. 107, n. 2, *supra*. According to one doctrine, nature has determined where division should stop, and infimae species are fixed by nature; according to the other, they are fixed by us with reference to our purpose or convenience. Cf. p. 95, *supra*.

divided are sometimes called its *constituent species*,[1] as together composing or making up the genus.

A division, or a classification, may be set out in a scheme, somewhat after the manner of a genealogical tree. The following is an example :—



The following are the *rules* which should be observed in a logical division :—

1. *A division must be exhaustive* : i. e. there must be a place for everything belonging to the genus in one or other of the constituent species into which it is divided. This rule may also be expressed by saying that the constituent species must be together equal to the ' totum divisum '.

The necessity of this rule hardly needs indicating. The aim of division is to set out in orderly relation whatever is included within a certain genus ; and if the division is not exhaustive, this is not done. Suppose that an income-tax is introduced ; it is necessary that the Act imposing it should state what forms of wealth are to be regarded as income, and taxed accordingly. The rent of land and houses is clearly a form of income, and would be included in the division of that genus ; but if the owner of a house lives in it instead of letting it, he receives no rent. Nevertheless, he enjoys an income, in the shape of the annual value of the house he lives in, just as truly as if he had let that house, and received for it a sum of money sufficient to hire himself another ; and he ought to be taxed if he lives in his own house as much as if he lets it. But if the income-tax Act omitted to include among the species of income the annual value of houses occupied by their owners, he would escape payment on that head altogether. Such is the practical importance of making a division exhaustive.

2. *The constituent species of the genus must exclude each other.*

Unless we secure this, we do not properly *divide* ; for the parts of that which one divides must be separate from each other.

[1] In Latin, *membra dividentia*, as the species are conceived to share the genus amongst them.

There are two ways in which a breach of this rule may come about. We may co-ordinate with a species another which ought properly to be subordinated to it; as Dr. Johnson is said to have divided the inhabitants of the country north of the Tweed into Scotchmen and Damned Scotchmen; or as the proverb distinguishes 'fish, flesh, fowl and good red herring'. In these instances the logical error points a sarcasm; but in itself it is comparable to the procedure of the philosopher, who cut two holes in his door, a large one for the cat and a small one for the kitten.

The second mode in which this rule is broken is by a cross-division; the nature of this will be explained in connexion with the rule now following.

3. *A division must proceed at every stage, and so far as possible through all its stages,*[1] *upon one principle, or* fundamentum divisionis.

The **fundamentum divisionis,** the principle or **basis** of a division, is that character of the genus, in respect of which the species are differentiated.[2] Let the genus be soldier; in a soldier we may look to the mode in which he fights, the military rank which he holds, or the conditions of service by which he is bound. Proceeding upon the first basis, we should divide into artillery, cavalry, infantry, engineers, and flying corps; perhaps staff and commissariat ought to be added. Proceeding upon the second, we should divide into officer and private, officer being again divided into commissioned officer and non-commissioned. Proceeding upon the third, into regulars, reserve, and territorials. When the division is carried further than one stage, the same *fundamentum divisionis* should be retained in the later stages which was used in the first. If the division of soldier into artillery, cavalry, infantry, engineers, and flying corps be prolonged, we might divide artillery into horse-artillery, field-artillery, garrison-artillery, and mountain-battery; cavalry into light and heavy dragoons, lancers, and hussars; infantry into mounted and unmounted. But it would not be proper, unless we wish to distinguish our species by combinations of differentiae, after beginning with the mode of fighting as our *fundamentum divisionis*, to proceed with that of military rank, and divide artillery into officers and privates; for that is a division of soldier generally, and not of artillery any more than of cavalry, infantry, or engineers; so that if it is applied to one of these species, it must equally be applied to the others.

[1] Cf. *infra*, p. 131.

[2] Cf. *supra*, c. iv. pp. 86, 101.

A division which proceeds on more than one *fundamentum divisionis* at once is called a cross-division; as if one were to divide soldier into artillery, cavalry, privates, and territorials. It is called a cross-division, because the grouping required by one basis cuts across that required by another; in distinguishing privates, for example, from other soldiers, we disregard the distinction of cavalry and artillery, taking all members of both those arms who are not officers. A cross-division is worse than useless; for instead of assisting to an orderly arrangement of things in thought, it introduces confusion.

It is plain that in a cross-division the constituent species will not exclude each other. The only security for their being mutually exclusive lies in their being formed upon one basis; for then they are distinguished by the different modes in which they exhibit the same general character. But if different characters A and B are taken, both of them belonging to the genus, everything within the genus will exhibit some mode of both these characters; and the same individuals which are included in a species that is constituted by the particular mode a' in which it exhibits the character A may also be included in a species constituted by the particular mode b' in which it exhibits the character B; hence a' and b' may not exclude each other.

There are two apparent exceptions to be considered to the statement that, where more than one *fundamentum divisionis* is employed, the resulting species do not exclude each other.

The ancient division of matter into the four elements, already alluded to as having been adopted by Aristotle,[1] proceeds (or appears to proceed) upon a double basis, of temperature and of humidity. Matter is either hot or cold; matter is either moist or dry; and hence four species were established, the hot and dry (fire), the hot and moist (air), the cold and dry (earth), the cold and moist (water). But there is not really a cross-division here. We do not, while professing to divide upon the basis of temperature, at the same time introduce species founded upon the basis of humidity (as if we were to distinguish the hot, cold, and moist elements); our real basis is neither humidity nor temperature, but the combination of the modes of temperature with the modes of humidity. And such a basis offers a peculiarly favourable opportunity for a good division. For given a certain number of characters in a genus, each found in so many different modes, and granted that every member of the genus

[1] Cf. *supra*, c. iv. p. 100.

must exhibit each character in some mode, and no character in more modes than one, then the possible alternative combinations are discoverable with mathematical precision. But it is only where the *combination* of certain characters happens to be of primary importance, that such a basis of division can be profitably adopted. There would be no advantage in applying the method in such a case as the division of the genus soldier, where, if we took the three bases of mode of fighting, military rank, and conditions of service together, assuming five alternatives under the first head, three under the second, and three under the third, we should obtain a division into forty-five members. These would be mutually exclusive; yet such a result would for most purposes be valueless; for the three bases of division are not such as it is useful to attend to together; though in a particular connexion, as, for example, in drawing up a scale of rates of pay, it might be advisable to proceed thus.[1]

In the above case, a cross-division seemed to be employed when it was not; in the next it might seem not to be employed when it is. It may happen that in respect of the individuals belonging to them, the constituent species into which a genus is divided upon one basis coincide respectively with those into which it is divided upon another. Thus angiosperms, or plants whose seed is contained in a pericarp, may be divided according to the method in which they form new wood into exogenous and endogenous; and according to their mode of germination in the seed into dicotyledonous and monocotyledonous. It happens that all the exogena are dicotyledonous, and all the endogena monocotyledonous; so that if the genus were divided into exogena and monocotyledons, there would not in fact be any plant that fell within both members. Nevertheless, the division is *logically* a cross-division, for there is nothing that we can see to prevent the existence of such a plant, and we can imagine endogena which are dicotyledonous; and therefore that our con-

[1] Dr. Venn, *Empirical Logic*, c. xiii. pp. 318–321, points out the part played by this method in Symbolic Logic. Suppose a class S, whose members are characterized by the presence or absence of each of the attributes X, Y, Z; but not all combinations are found. Then we may work out mathematically the *class-compartments* determined by the different possible combinations of differentiae; and if we symbolize the absence of X by X', there will be XYZ, $X'YZ$, $XY'Z$, XYZ', and so on. Then the statement that whatever is X and Y is Z is equivalent to saying that the class-compartment XYZ' is not 'occupied', and can be written symbolically '$XYZ' = 0$'. Such methods of symbolization may facilitate the working out of the implications of a number of propositions relating to the same genus. But they do not express the common character of all reasoning.

stituent species do not overlap must be regarded as our good fortune, whereas it ought to arise out of the necessity of the method on which our division proceeds. And even if we came to understand the connexion between these differences in mode of wood-formation and of germination, such a division would still be vicious; for it would not *exhibit* our species as necessarily excluding each other; and this because (what is more important) it would not exhibit them as alternative developments of a single, or common, notion.[1]

There is a form of division called **Dichotomy**, which is of necessity exhaustive, and the species yielded by it of necessity exclude each other; for it divides the genus at every stage into two members (as the name implies), which respectively do and do not possess the same differentia; everything in the genus must therefore belong to one side of the division or the other, and nothing can possibly fall into both. Animal, for example, may be divided into vertebrate and invertebrate, body into animate and inanimate, substance into corporeal and incorporeal; each of these divisions is exhaustive, and its members mutually exclusive.

Some logicians have held that in order to secure these advantages all divisions ought to proceed by dichotomy. But the truth seems rather, that when a division is undertaken with the view of classifying or arranging all that is contained in the genus, dichotomy should not be used. Its use is in analysing or defining some one subordinate species. It may, however, sometimes be used to

[1] A cross-division is in fact a defect of principle, which is not removed because practical inconveniences are avoided. H. Sidgwick, in his *Methods of Ethics*, holds that it is reasonable for a man to seek his greatest private happiness, and also to seek the greatest happiness of the greatest number; and he admits that, so far as happiness in this life is concerned, these principles would conflict in their application to many situations. He thinks however (v. Concluding Chapter) that this 'fundamental contradiction' would be removed, if the Deity by a system of rewards and punishments hereafter made it for the greatest happiness of the individual to promote the greatest happiness of the greatest number. But the theoretical difficulty, that reasonable action is conceived in two ways, between which we see not only no necessary connexion, but possible collision, would still remain. So in the division of angiosperms into endogenous and dicotyledonous, the specification proceeds disparately, and the absence of collision is an 'uncovenanted mercy'. If a genus were merely 'items of connotation', to which differentiae were added as further items (cf. Venn, *op. cit.*, c. xii. p. 310), such procedure in dividing it would have no impropriety: angiosperms X ($= abc$) could be divided into Xd and Xe. Thus we see the impropriety is evidence that we do regard the relation of genus and differentia in the way described in the previous chapter: that the alternative species of a genus are so many ways in which the same nature is realized or carried out.

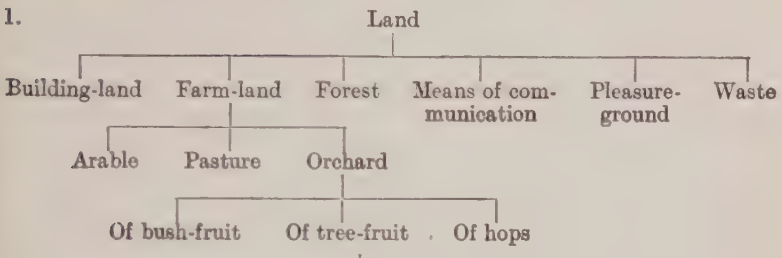
show that a division which is not dichotomous is nevertheless exhaustive, and the constituent species exclusive of each other.

The reason why dichotomy is out of place as the principle of a classificatory division is that we desire in a division to exhibit our various species as alternative developments of a common notion; at every stage the genus is further particularized by the differentiae which we introduce in constituting its species; thus the division of the genus soldier, according to mode of fighting, into artillery, infantry, cavalry, engineers, and flying corps, was carried further by particularizing the way in which the artillery may be constituted for different fighting purposes, or the cavalry armed, &c. But one side of a dichotomy is always characterized negatively, by the non-possession of the attribute which characterizes the other side; and there is therefore no positive notion, except the original genus, which we can develop in the subdivision of this side. Now it may be sometimes convenient to use negative differentiae in the course of a classification, when one species or subaltern genus is distinguished from the rest by lacking a character which they exhibit.[1] But this is not done upon any principle of dichotomy; for there might be several co-ordinate species or subaltern genera distinguished by different forms of that character which the one lacked; and then the division would not be dichotomous, but as manifold as the facts required. Thus albinism might be co-ordinated with several varieties of pigmentation. And the further differentiation of the subaltern genus differentiated negatively would be made by means of some fresh generic character; whereas when dichotomy is adopted as a principle, the negative differentia is introduced before exhausting the co-ordinate forms of the generic character first used as a basis; so that at each stage the remainder of these appear as variations of the lack of the last form taken as a positive differentia. Thus the land of a country may be divided, according to the use to which it is put, into building-land, farm-land, forest, means of communication, pleasure-ground, and waste: each of these 'subaltern genera' may be subdivided, farm-land for example into arable, pasture, and orchard: orchard again according as bush-fruit, tree-fruit, or hops are cultivated. But if we were to proceed by dichotomy, we should divide land into building-land and land not used for building: the latter into farm-land and non-farm-land: non-farm-land into forest and not forest, and so forth. Now such a division would not only be

[1] Cf. *supra*, p. 114, n. 2.

far more cumbrous than one unhampered by the method of dichotomy, as may be seen by setting both out in scheme as follows :—

1.



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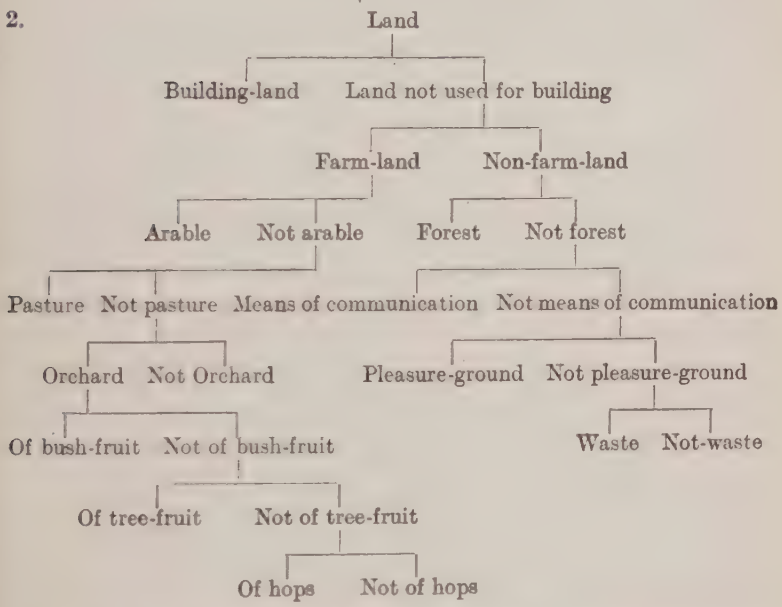


but it fails entirely to exhibit its species as alternative developments of a common notion, or (as it was put in the last chapter) variations on a common theme. To build on it, to farm it, to let it grow timber, &c., are so many ways of using land ; to plough, to graze, and to raise fruit from permanent stocks on it are three ways of farming, and therefore of using it ; to grow bush-fruit, tree-fruit, and hops on it are three ways of raising fruit on it from permanent stocks, and therefore of farming and therefore of using it.[1] But

[1] Perhaps orchards (if they may be held to include all ground used for raising fruit from permanent stocks) should be divided according as they

to farm land is not a way of not building on it; a forest is not a form of not being a farm; roads and railways, which occupy land that is used as a means of communication, are not modes of not being a forest; to use land as pleasure-ground is not a particular way of not making a road or a railway along it; to leave it waste is not a particular way of not using it as pleasure-ground. Neither again is grazing a particular way of not ploughing land, nor growing tree-fruit a particular way of not growing bush-fruit on it. The positive differentia of any subaltern genus negatively characterized is therefore really a differentia of the nearest positive genus: forest-land and farm-land, e.g., are species of land, not of non-farm-land and land-not-used-for-building. A negative concept affords no basis for further subdivision, and in a division which attempts to classify by dichotomy half the differentiae are useless for the development of the generic notion.

[This is the main objection to a classificatory division by dichotomy; which is strangely defended by Jevons, *Principles of Science*, 2nd ed., c. xxx, pp. 694–698, and *Elementary Lessons in Logic*, Lesson XII. Other objections, which it seemed unnecessary to add in the main text, since the first is fatal, may nevertheless be pointed out. The proper division co-ordinates concepts of the same degree of speciality; but the division by dichotomy subordinates them in several stages; so that waste-land is placed level with orchards of bush-fruit. The serial order in which the subaltern genera are placed (except where a positive concept is divided) is also quite arbitrary; building on land might as reasonably be called a way of not farming it, as farming it a way of not building on it. Lastly, it is claimed for division by dichotomy that it is the only method which secures us from possible oversight of a species: if *man* be divided into Aryan, Semitic, and Turanian, a race may turn up that is none of these; whereas if it be divided into Aryan and non-Aryan, non-Aryan into Semitic and non-Semitic, and non-Semitic into Turanian and non-Turanian, we have a class ready (non-Turanian) for any new race that may turn up. But it must be observed that to say that a race is non-Turanian does not characterize it: that the Aryan and Semitic races are also non-Turanian (so that the constituent species are not mutually exclusive); and that if the last objection is con-

grow bush-fruit, tree-fruit, or bines; and bine-orchards might be subdivided into hop-yards and vineyards. Even then it is not clear where strawberry-gardens would come. Such are the practical difficulties of making a perfect division. In the text something has been sacrificed to compendiousness, else nursery-grounds, brick-fields, and other varieties of land distinguished according to use would need to be included.

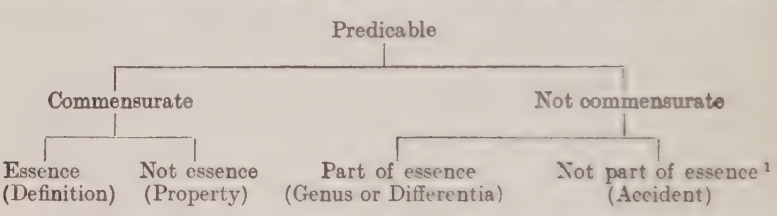
[sidered captious, because the non-Turanian is expressly made a branch of the non-Semitic, and that in turn of the non-Aryan, then it means what is neither Aryan, Semitic, nor Turanian; now if we are uncertain that our division is exhaustive, and wish to reserve a place for things that may fall within none of the species we set up, it is easy to do that without the pains of all this dichotomy; we may divide man into Aryan, Semitic, Turanian, and *anything that is none of these*; this last heading expresses what non-Turanian means in the dichotomy, and stands, as it should, *upon a level* with the rest.]

For this reason, a classificatory division should never use dichotomy as a principle; the numbers of species into which a summum or subaltern genus is to be divided can be determined not on any general logical grounds, but solely with reference to the nature of the genus in question. Even where, as in the case of the four elements, the basis of division is the combination of attributes, the number of possible species that can be formed by different combinations is determined, under the restriction that contraries cannot be combined together, not by logic but by mathematics. Of course, if a genus falls naturally into two species, it ought to be divided in two; as number is divided into odd and even, and line into straight and curved. But this is not mere dichotomy; for it is not the same to divide number into odd and even as to divide it into odd and not odd. The claim made for dichotomy is that its branches exhaust the genus and exclude each other in virtue of the mere *form* of the division[1]; since everything in a genus must either be or not be, and cannot at once be and not be, characterized by any differentia that can be taken. And this is true; and we need realize no more than this, in order to see that number is either odd or not odd; but in order to see that it is either odd or even we need to understand the

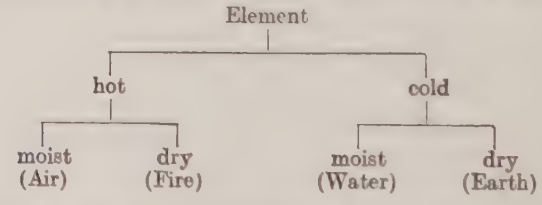
[1] Cf. S. H. Mellone, *Introductory Text-book of Logic*, c. v. § 10, who points out that although division by dichotomy 'has been adopted by the mediaeval and formal logicians because it appears to provide a theory of division which does not make the process depend entirely on the matter of our knowledge, as classification does', yet this appearance is illusory. I know on formal grounds that of any genus x the species either are or are not characterized by any attribute a; but I cannot therefore divide x into the two species a and *not-a*, since in fact a may be an attribute never found in the genus at all. Every circle must be either rectilinear or not; but there are not two species of circle, the rectilinear and the non-rectilinear. For this reason, in Symbolic Logic (cf. *supra*, p. 120, n. 1), XYZ, $X'YZ$, &c., represent not classes but class-compartments, which may be necessarily empty; and some writers, like Mr. Bertrand Russell, recognize by the name of null-class a class which has no members.

peculiar nature of number, and not merely the general 'laws of thought', as they are called, that hold of every subject. The completeness of the division of number into odd or even is not therefore vouched by logic, any more than the completeness of the division of rectilinear triangle into equilateral isosceles and scalene ; nor in the fact that it is twofold does the first possess any guarantee which the second lacks in being threefold. And if a genus is seen to fall into thirteen species instead of three, it should be divided into thirteen ; just as rectilinear triangle should be divided into three and not two. Unfortunately there are few subjects where we can see at once that a genus contains necessarily so many species and no more ; and that makes our divisions precarious, but there is no remedy in the use of dichotomy.

It may, however, occasionally be possible to show by dichotomy that a division which is not dichotomous is exhaustive or its species mutually exclusive. Aristotle thus supported his list of predicables.



But there is no particular logical interest attaching to this mode of establishing a division ; it is in principle the same as where our basis is the combination of certain attributes, and we show the division to be exhaustive by showing that no other possible combinations remain, as in the case of the four elements already given.



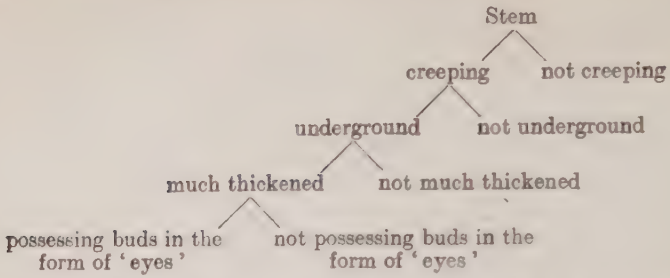
Dichotomy is really appropriate when we are seeking not to divide a genus but to define a species. There are two contrasting ways in

[1] But generic properties would have to be ranked in this division as accidents. Cf. p. 104, n. 1, *supra*.

which we may set about to seek a definition. We may take instances of that which is to be defined, and try to detect what they have in common, which makes them instances of one kind, and on the strength of which we call them by the same name. This is the 'inductive' method. We might thus define 'snob', comparing those of our acquaintance to whom we could apply the name, or those whom Thackeray has drawn for us; and if we thought that among all their differences they agreed in prizing rank or wealth above character, we might accept that as our definition. The other method is that of dichotomy, and in this we try to reach our definition rather by working downwards from a genus, than upwards from examples. Some genus is taken, to which the subject we wish to define belongs. This genus we divide into what possesses and what does not possess a certain differentia. The differentia taken must be something predicable of the subject to be defined; and if genus and differentia together are already commensurate with that subject, the definition is reached; if they form only a subaltern genus predicable of it, this subaltern genus must be again divided in the same way: until we reach a commensurate notion. At every stage of our division, the differentia taken must either be a modification of the differentia next before it, or at least be capable of combining with those that have preceded it in the construction of one concept in such a way that we are throughout specifying the general notion with which we started[1]; and there should be so many steps of division as there are stages which our thought recognizes as important in the specification of this concept. At every stage also we proceed by dichotomy because we are only interested in the line that leads to the subject we are defining; all else contained within the genus we thrust aside together, as what does not exhibit the differentia characterizing that subject. Had we further to consider and subdivide it, we could not be satisfied with characterizing it only negatively; for a negative notion furnishes, as we have seen, no basis for any further specification. But we may disregard, or cut it off: a step to which the technical name **abscissio infiniti** has been given, i. e. the cutting off of the indeterminate.

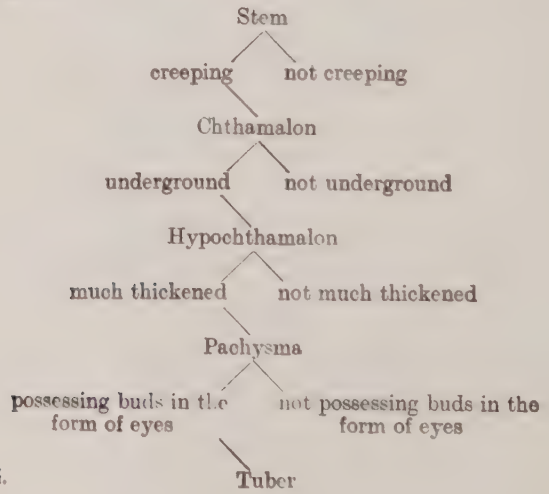
The following example of definition by dichotomy will illustrate what has been said. The term to be defined is *tuber*; the genus to which it is to be referred is *stem*.

[1] Cf. *infra*, pp. 130–131, 133–134.



In this process, we reach as our definition of a tuber 'a stem creeping underground, much thickened, and possessing buds in the form of eyes'. At every stage by an *abscissio infiniti* we rejected from further consideration a large part of the genus we had so far reached: first all stems not creeping, then all creeping stems not underground, then all underground creeping stems not much thickened, &c.; and at every stage we subdivided that part of the genus which we had retained by a differentia that specified further the form to which we had so far brought it.

It might have happened, that creeping stems had a name to denote them, say *Chthamala*[1]; and that underground Chthamala had a special name, say *Hypochthamala*; that these when much thickened had again a different name, say *Pachysmata*; and that tubers were pachysmata that possessed buds in the form of eyes. In this case, the matter would be set out in somewhat different form, as follows—



[1] *χθαμαλά.*

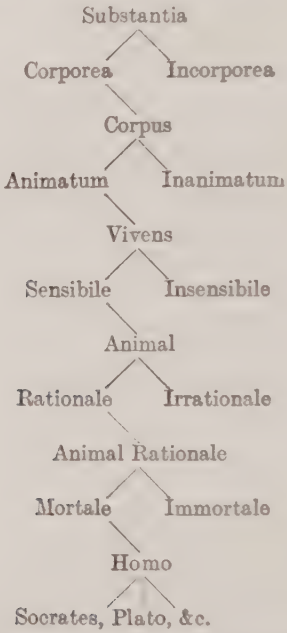
This mode of setting out the definition of anything implies a classification, in which names have been given to every wider and narrower genus, and the differentia which distinguishes each within its proximum genus has been settled. It may indeed be regarded as an extract from a classification, made for the purpose of exhibiting the nature of a single species. And this is more or less the character of all definition by dichotomy; though the classification may be only in the making, in the very process by which we seek for our definition. It is only after considerable study of the parts of flowering plants, enabling us to group them by their less superficial characters, that a tuber would be referred to the genus stem at all, instead of root; by that time, the distinction between creeping and other stems, between those that creep above and those that creep below the ground, would have been already made; so that the method of dichotomy does not so much help us to discover, as to set out and arrange what we know of, the definition of a tuber. There may, however, be cases where the method will guide us in the construction of a definition of that whose nature has not yet been carefully investigated; the genus to which a term is to be referred may be clear, but the appropriate differentiae unconsidered; snob, for example, belongs clearly to the genus man; but even here, the process of finding a differentia, by which to distinguish snobs from other men, is classification in the making. Let us take the prizing of rank or wealth; if that by itself does not constitute a snob, we need some further differentia, to distinguish snobs from other men who prize rank or wealth; say they are distinguished by prizing these beyond character; we then have a definition of a snob, but in getting it, we have taken note of a wider class of men within which they are included.

There are three things which Aristotle[1] says that we must look to, in reaching definitions by the division of a genus. All the terms (the summum genus and the successive differentiae) must be of the essence of the subject defined, they must be placed in their right order, and none must be omitted. These are requirements also of an ideal classification, though in the practice of classification, as of definition, many compromises are necessary; but just as a study of the general form of classification does not enable us to classify any particular set of things, so we are not enabled to define any

[1] *Anal. Post.* β. xiii. 97ª 23 *sq.*

particular subject, merely by familiarizing ourselves with the scheme of definition by dichotomy.

[A definition of *man*, displaying the series of subaltern genera to which he may be assigned below the summum genus substance, and the differentia by which each subaltern genus is successively distinguished within the genus next above it, was long known in logical textbooks by the name of *Arbor Porphyriana*. It may be transcribed here. That of *tuber* given above on p. 128 is in the same form.



The material for the scheme is to be found in Porphyry's *Isagoge*, c. iii ; where the writer points out that the same differentia which is divisive (διαιρετική) of one genus is constitutive (ποιοτική) of that immediately below it. The scheme has the advantage of exhibiting the series of differentiae by which the definition of the species is reached from the summum genus. Aristotle in *Met. Z.* xii. discusses how many differentiae there really are constitutive of the species ; and decides that if each differentia is itself a true differentia of the one before it, then the species has only one differentia, namely the last. For example, if *animal* is divided into *footed* and *footless* (ὑπόπου and ἄπουν) and if the *footed* are divided into *biped* and *quadruped*, the latter differentia *biped* is a differentia of *footed* as such ; for to be a biped is a particular way of having feet. In the species

[*animal bipes* therefore, the correct analysis is into *animal* and *biped*, and not into *footed animal* and *biped*, and though we may proceed through successive stages to *biped*, there is nothing in the thing corresponding to the serial order. If, on the other hand, at any stage we introduce a differentia which is not merely a further specification of that which we have used before (as e. g. if we were to divide *biped* into *feathered* and *featherless*, or *rational* and *irrational*), then the species is constituted by more differentiae than one; e. g. if we take *animal* again as the genus, the species *man*, defined as a *featherless* or *rational biped*, would really be constituted by two differentiae. We might endeavour to avoid this conclusion by calling *biped* the genus and *featherless* or *rational* the differentia; but that ignores the fact that *biped* is obviously not summum genus of *man*. And if we select a fresh basis of differentiation at more than one stage, we are each time adding to the number of differentiae that must be recognized in the species. In doing so we ignore the precept, to proceed throughout any division upon one basis; and Aristotle certainly speaks of the introduction of a differentia which is not continuous with that before it as dividing κατὰ τὸ συμβεβηκός and not κατὰ τὸ ὀρθόν. We may notice too, that where a differentia which is a continuation of that before it would be inapplicable to the other member of the preceding genus (e. g. *biped* is not applicable to *footless*, the other member along with *footed* of the genus *animal*), a differentia which is not of that nature might, for all that we can tell *a priori*, be applicable to both members (e. g. *feathered* and *featherless* might be applicable to *footless* no less than to *footed animals*); hence we shall characterize our species by the combinations presented in them of the various alternative modifications of several generic attributes.[1] The fullness and complexity of natural kinds constantly leads to the introduction of fundamentally new differentiae, especially where, as in the classificatory sciences often happens, our differentiae are intended as much to be *diagnostic*—i. e. features by which a species can be identified—as to declare the essential nature of the species. Cf. pp. 133–135.]

Before distinguishing Logical Division from the other processes to which the name Division is applied, it may be well to emphasize that it deals entirely (like the doctrine of Predicables) with concepts or universals. The genus which we divide is divided into *kinds*; itself a universal, the specification of it by various differentiae can only give rise to more determinate universals. The division of it

[1] Some of these may be attributes not of the *summum genus* but only of some subaltern genus; and in some combinations, a particular generic attribute may be altogether absent; hence the occurrence of negative differentiae in scientific classifications. Cf. *supra*, p. 114, n. 2.

stops therefore with infimae *species*, and never proceeds to the *enumeration of individuals*. For if the infima species could be logically divided into individuals, we must apply some *fundamentum divisionis*; and that means, that we should have to distinguish individuals according to the different modes in which the common character of the species appeared in them; and to do that would be to distinguish these modes themselves, which are not individual but universal, for many individuals might exhibit the same mode. But individuals of any species are in fact distinguished from each other by the coincidence of innumerable attributes; it is not any attribute singly, but the particular combination of them, that is unique in each instance; and whether or not they are sufficient to constitute individuality, unique combinations of innumerable attributes cannot be exhibited in a logical division as differentiae of one species.[1]

There are two processes which have been called division, besides the division of a genus into its species. They are known as *physical* and *metaphysical division*. In **Physical Division,** we distinguish the parts of which an individual thing or aggregate is composed: as in a man head, limbs and trunk: in a flower bract, sepal, petal, stamen and pistil. This process is also called *Partition*. It is still a process of thought that is meant—not the actual tearing of a flower to pieces, or quartering and beheading of a man; it may be applied to the distinction of the parts composing either a determinate individual, or any individual of a kind: as Great Britain on the one hand can be divided into England, Scotland, and Wales, a tree on the other into root, stem, branch, leaf, and flower, or a forest into its component trees.

In **Metaphysical Division,** we distinguish in a species its genus and differentia, in a substance its different attributes, in a quality its different 'variables' or 'dimensions'; thus we may distinguish in man animality and rationality, in sugar its colour, texture, solubility, taste and so forth, in a sound its pitch, timbre, and loudness. This is obviously a division that can be carried out in

[1] Thus in the *Arbor Porphyriana* the enumeration of the ἄτομα Socrates, Plato, &c., in the infima species *man* is no part of the logical division. Cf. Porph. *Isag.* c. ii ἄτομα δὲ λέγεται τὰ τοιαῦτα, ὅτι ἐξ ἰδιοτήτων συνέστηκεν ἕκαστον, ὧν τὸ ἄθροισμα οὐκ ἂν ἐπ' ἄλλου τινός ποτε τὸ αὐτὸ γένοιτο τῶν κατὰ μέρος· αἱ γὰρ Σωκράτους ἰδιότητες οὐκ ἂν ἐπ' ἄλλου τινὸς τῶν κατὰ μέρος γένοιντ' ἂν αἱ αὐταί. ('Such things are called individuals because each is constituted by peculiarities, the precise collection of which would never be the same in any other particular instance; for the peculiarities of Socrates would never occur identically in any other particular.')

thought alone. In Physical Division, the parts of an individual man or plant may be physically separated ; and in Logical Division, when the genus is concrete, individual specimens of the infimae species may be exhibited in different cases in a museum. But in Metaphysical Division, the ' parts ' cannot be exhibited separately ; though the colour of sugar may be exhibited without its taste in a thing of another kind—e.g. in a sample of salt—it can never be exhibited by itself.

It should be further observed, for the better distinguishing of these different kinds or senses of division, that in Logical Division the whole which is divided can be predicated of its parts—*animal*, e.g. of *man*, *ox*, &c.—and indeed unless it is so predicable of all its parts, the division is at fault ; in Metaphysical Division the parts can be predicated (paronymously, to use the Aristotelian expression,[1] or attributively) of the whole—e.g. whiteness, sweetness, &c., can each be predicated of sugar, in saying that sugar is white, is sweet, &c. ; in Physical Division, the parts can neither be predicated of the whole nor the whole of the parts—we cannot either say that a leaf or stem is a tree, or that a tree is a leaf or stem.

[A few words may be added on the relation of Logical Division, and its rules, to the practical work of Classification. Just as the *theory* of Definition, with its sharp distinction of essence and property, breaks down amidst the complexity and variety of concrete things, so it is with the theory of Division. Ideally when a genus is divided into species, whether once or through several stages, we ought at each stage to see that just such and so many species are possible in that genus ; we do see this in geometry, in the division for example of conic sections into hyperbola, parabola, ellipse, and circle ; but in other sciences for the most part we must wait upon experience. Now we do not in experience find that things fall into kinds which fit into any perfect scheme of logical division. Any actual division that can be made therefore of animals, or plants, or forms of government, would exhibit many logical defects ; every classification involves compromise ; the things, which it puts into the same class from one point of view, from another claim to be placed in different classes ; all that was said in the last chapter

[1] *παρώνυμα δὲ λέγεται ὅσα ἀπό τινος διαφέροντα τῇ πτώσει τὴν κατὰ τοὔνομα προσηγορίαν ἔχει, οἷον ἀπὸ τῆς γραμματικῆς ὁ γραμματικὸς καὶ ἀπὸ τῆς ἀνδρείας ὁ ἀνδρεῖος*, *Cat.* i. 1[a] 12. ('That is paronymous which receives its designation from something with a difference in inflexion, as a grammarian from grammar and a courageous man from courage.') The Latin for *παρώνυμον* is *denominatum* or *denominativum*, according as the subject or its attribute is meant.

[about the difficulty of defining concrete natural kinds might be repeated to show the difficulty of classifying them; and the same reasons which prevent our satisfactorily continuing a division down to a point at which it would find a separate specific concept for every individual prevent our *satisfactorily* classifying them at all. Classification is, as Jevons called it,[1] a tentative operation; its results are provisional; discovery may reveal new species, and show that characters which have been supposed always to go together may be separated, or those hitherto considered incompatible combined in the same individual: there are limits indeed to this, for there are 'laws of nature' with which all particulars must be consistent; but many so-called 'laws of nature' themselves rest on the same evidence on which our classifications are constructed.

Thus the ideal which Logical Division sets before us is very different from anything which Classification achieves. The first is or would be an *a priori* process; by which is meant that it would fain develop specific from generic concepts not indeed prior to any experience of that which belongs to the various species of the genus divided, but with a perception that the species revealed in experience are such as must necessarily have existed in that genus. Classification is an *a posteriori* process; it appeals for support to the facts which we are classifying, and argues that they reveal such connexions of attributes as we take to mark the classes proposed; it does not attempt to show that attributes could be connected in individuals of the genus in no other ways than these. Logical Division again would fain be exhaustive, and establish constituent species which do not overlap; but a classification may have to acknowledge that there are individuals or whole classes which might with equal right be referred to either of two co-ordinate genera, or seem to fall between them, or outside them all. For these reasons, Division, as treated in a textbook of Logic, is apt to seem unreal and fanciful to any one familiar with the work of scientific classification; its rules seem framed to suit not the world he has to deal with but a fictitious world of the logician's imagination; the consideration of a process which, outside geometry, can scarcely be illustrated by examples except by mutilating facts, is denounced as a barren pastime. And there is justice in the denunciation, when Division, or Definition, is studied without reference to the recalcitrant facts, and on its formal side alone. But if we realize with what great abatements the rules of Definition and Division can be fulfilled in the actual classification of concrete facts, we may yet profitably study these rules, as counsels and not precepts. That is the best classification which conforms to them most closely. The case of the logician may be compared with that of the

[1] *Principles of Science*, c. xxx. p. 689, 2nd ed.

[geometer. The geometer studies such figures as he conceives, and he believes that his conclusions are true of the squares or triangles that exist eternally in space, bounded by the distances between points therein ; but he does not imagine they would apply without qualification to a square table, or a triangular lawn. The figures of these concrete things are much more complex than a simple square or triangle. So (though the cases are not identical) the logician studies the problem of classification as it presents itself to thought ; but is prepared to expect that real things are cross-related to each other in far too complicated a manner for any single and simple scheme of classification to embrace them as they stand. We must consider aspects of them, and attempt to ascertain what various forms some particular property may assume, and under what conditions. In tracing a property through all the phases in which it appears in different instances, we are in a sense pursuing a genus into its species ; we are realizing its generic identity under divers forms, and this is part of the business of a logical division. The things themselves which we have to classify, if we take them in their completeness, cannot be caged in a neat logical arrangement ; yet even so, the ranking of them in genera and species at all, which is not the work of logic, but the natural bias of our thought (for the distinction of man and animal is older than that of species and genus), implies an effort at such arrangement ; the logician does no more than render explicit the aims which underlie all classification : except that the form of his theory takes too little account of the modifications which are imposed by the particular nature of the subject-matter with which we may have to deal.[1]]

[1] Some useful remarks on Classification, on the difference between so-called natural and artificial classifications, and on the relation of different classifications of the same set of facts to our different purposes, will be found in J. Venn's *Empirical Logic*, c. xiii.

CHAPTER VI

OF THE INTENSION AND EXTENSION OF TERMS AND OF THEIR DENOTATION AND CONNOTATION

We are now in a position to consider certain distinctions in regard to terms which, owing to the erroneous identification of them, have become involved in much confusion. These are the distinctions (1) between Extension and Intension, (2) between Denotation and Connotation. It was observed by Aristotle,[1] that in one sense the genus is in the species, in another sense the species is in the genus. 'Animal' is in 'man', in the sense that you cannot be a man without being an animal, so that being animal is included in being man. 'Man' is in 'animal', in the sense that among the forms of animal nature, man is included.

In the technical language of later Logic, this distinction may be expressed by saying that in intension the species includes the genus, in extension is included in it.

The **intension** of a term is what we *intend* by it, or what we mean by it when predicating it of any subject[2]: the **extension** is all that stands subordinated to it as to a genus, the variety of kinds over which the predication of the term may *extend*.[3] Or, if by term we mean purely the concept, we may say that the extension is the variety of species in which a common character is exhibited, the intension the common character exhibited in this variety. The distinction may be more readily apprehended, if it is noticed that we analyse the intension of a term in defining it, and its extension in dividing it.

It is clear that as between two terms subordinated one to the

[1] *Phys.* δ. iii. 210ᵃ 17–19. Cf. p. 133, *supra.*

[2] I do not wish to imply that we may not 'intend' the same by a term when it is subject of a proposition, as when it is predicate. But as in the subject the extension may be more prominent than the intension, while the predicate is always understood primarily in intension, the expression in the text is less ambiguous than if I said 'What we mean by it in a proposition'. Cf. *infra*, c. ix.

[3] For another use cf. p. 143 *sq.*, *infra.*

other in a classification, the higher, or superordinate, will normally[1] have the greater extension; *animal*, for example, is a term of wider extension than *man*, and *conic section* than *ellipse*; for the concept 'animal' extends or applies to much besides man, and that of 'conic section' to hyperbola and parabola, as well as to ellipse and circle.[2] Many hold also, that the superordinate term, as it is of greater extension, so is of less intension; less being meant by calling anything an animal than by calling it a man; or by the term 'conic section', than by the term 'ellipse'. Hence it has been said that the extension and intension of terms vary inversely: 'when the intent of meaning of a term is increased, the extent is decreased; and vice versa, when the extent is increased, the intent is decreased. In short as one is increased, the other is decreased.'[3]

This inverse relation of intension and extension in terms may be illustrated not only by reference to classification, but in another way. We may take any term, such as *Christian*, and qualify it by an adjective or adjectival phrase: as if we were to say 'Armenian Christian' or 'Christian of Caesar's household'; by the qualification we clearly make a term of narrower extension than 'Christian' simply, for we conceive that there may be Christians not Armenians, or not of Caesar's household; and at the same time we add to the intension, for it is no part of the concept of a Christian to be an Armenian, or of the household of Caesar.

Still, when we thus qualify a general or an abstract term, we are instituting a sort of classification; we make an Armenian species within the genus Christian, or a class, say, of bright colours within the genus colour. Therefore we may say generally that it is only to terms in a classification, and in one 'series of subordination' in

[1] Occasionally, as we have seen (*supra*, p. 73, n. 2), we find in a classification species whose members differ from their nearest kindred as widely as members assigned to different genera in it differ, so that they are referred to a distinct genus, although no other species is found belonging thereto; as in zoology men are placed in the species *Homo Sapiens*, which is the only species of the genus *Homo* and of the class *Hominidae*. But that means that we think there might be other genera of Hominidae, and species of Homo: and if there were, the relation stated in the text would hold.

[2] Porph. *Isag.* c. viii ἔτι τὰ μὲν γένη πλεονάζει τῇ τῶν ὑπ' αὐτὰ εἰδῶν περιοχῇ, τὰ δὲ εἴδη τῶν γενῶν πλεονάζει ταῖς οἰκείαις διαφοραῖς. ('Further, genera exceed species in the compass of the species under them, species genera in the differentiae belonging to them.')

[3] Jevons, *Principles of Science*, 2nd ed., c. ii. p. 26. Cf. Sir W. Hamilton, *Lectures on Logic*, viii. ¶ xxv; Thomson, *Laws of Thought*, § 28; Bain, *Logic, Deductive*, p. 51 ('the greater the one the less the other').

it, that the doctrine of the inverse relation of intension and extension applies. It would be ridiculous to compare in this respect such different concepts as *democracy* and *steam-engine*; it is even unmeaning to compare terms belonging to the same classification but to different lines, or 'series of subordination', in it; *bird* and *reptile*, for example, both belong to a classification of animals, but are not subordinate one to the other, and nobody can well tell which has the greater intension, nor if that were decided would he be able to infer from the decision, which had the greater extension, or comprised the larger number of subordinate species.

Applying only to terms subordinated one to another in a classification, the doctrine is an attempt to explain the nature of classification, as a series of terms so related that each is of wider extension and narrower intension than the next below it.

Now it may be questioned whether the doctrine is just. The generic term undoubtedly exceeds the specific in extension, but does it fall short in intension? This question may be put in another form: is the process of classification one of mere *abstraction*? do I reach a generic concept from specific concepts merely by leaving out part of the latter, and attending only to the remainder? If our concepts of species and genus were constituted by sets of attributes disconnected but coincident, then this would be the case. The generic concept would be formed by picking out from several sets those attributes, or marks, which occur in them all; it would contain fewer marks, or be of less intension, in the same sort of way as one man may have fewer decorations than another. On these principles the nature of a classification might be satisfactorily expressed by the following symbols :—

But we have seen [1] that the genus is not something which can be got by any process of subtraction from the species; it is not the same in all its species, and does not enter unchanged into them all as water into every pipe that leads from a common cistern. You

[1] Cf. p. 83, *supra*.

cannot form a concept of it apart from all the species, as *a* can be read and written apart from other letters with which it may be combined. Attributes that are really independent, such as blue, and sweet, and heavy, can be thus conceived apart; but they cannot stand to each other in the relation of genus and species.[1]

If we look at terms which are really in a relation of genus and species, it is not clear that the wider term has the less meaning. Take *animal* and *man*; if I say of anything that it is an animal, I certainly convey less information about it than if I say it is a man; but it does not follow that the concept *animal* is of less intension than *man*. For it must be noted, that I should not say of anything that it is *animal*, but *an* animal; which implies that I am aware of other animals, and that the concept *animal* includes alternatives, among which I cannot or do not at present choose. But if so, the generic concept would seem to exceed the specific in intension; 'animal' means 'man, or horse, or crab, or jellyfish, or some other form in which the general nature of an animal may manifest itself'. As we become familiar with the infinite variety of animal life, the term comes to mean not less to us, but more.

Or take another illustration. Say that a boy first makes acquaintance with the steam-engine in the form of railway locomotives. For a long time the term means that to him; but by and by he meets in his experience with traction-engines, ship's-engines, and the stationary engines of a factory. His earlier concept of a steam-engine—the earlier intension of the term for him—will alter; much which he included at first in it, because he found it in all railway locomotives, he will learn to be unessential—first running on rails, then the familiar shape, then the moving from place to place. And according to the doctrine before us, he will leave out from the concept one point after another, and at the end his notion of a steam-engine will be the unexcised residuum. But surely his notion of a steam-engine will have become richer and not poorer in the process; it is not that he finds that a steam-engine need not run on rails, so much as that it may run on the roads, nor

[1] And therefore the introduction of differentiae into a division which are not differentiae of those before them is not *κατὰ τὸ ὀρθόν* (cf. *supra*, p. 131), though they may still be such of which only the genus from which we started is susceptible; and the introduction of them may be justified as well by considerations of practical convenience as on the ground that species are distinguished by variously combining the variations of many generic characters, or characters not pervading the whole genus.

that its familiar shape is unessential, so much as that it may be built in quite a different manner; nor that it need not move from place to place, so much as that it may work as a stationary engine. It becomes a genus to him, because it becomes a thing of alternative possibilities; and the experience which leads him to extend the term to new kinds of subjects leads him to use it with a wider range of meaning. It is true that in becoming generic, the term comes to have a *less definite* meaning, when applied to any subject; but it does not therefore come to have *less* meaning.

The doctrine of the inverse relation of extension and intension in terms may seem therefore to misrepresent the nature of a classification. But a doctrine which has been accepted so widely,[1] and is at least at first sight so plausible, must have some degree of justification. Its justification, or excuse, seems fourfold.

1. The thought which general terms suggest to the mind is often vague, and the more so in proportion as they less suggest a definite sensible object. We do not realize all the alternative possibilities involved in animal nature each time that we use the term 'animal'. So, because in the term of wider, as compared with that of narrower, extension there is often little definite, we are apt to suppose instead that there is a definite little. This error is encouraged by mistaking for thought the imagery that accompanies thinking. The nature of this imagery differs with different people, and any illustration can be only arbitrary. But it might well be that when one thought of man or horse, he pictured to himself the look of either with fair completeness; but that with the notion of animal there went the kind of image which a child would draw of a quadruped—four lines sticking out of an elongated trapezium, with a few more for the head and tail. There is less detail in such an image than in that of a horse or a man; and it is not impossible that one might hence be led to suppose there was less intension in the term.

2. Our actual classifications, as we have seen, fall short of perfection in many respects; we often do not understand the interdependence of the various characteristics of an organic kind, or of the various properties of an elementary substance. In these circumstances, we are compelled at times to fix on certain characters as

[1] There are, however, eminent names on the other side, e.g. Mr. F. H. Bradley, Professor Bosanquet, and R. L. Nettleship. Cf. especially section xi of the 'Lectures on Logic' in *The Philosophical Remains of R. L. Nettleship.*

constituting a genus, and then distribute into species the subjects in which they are found by means of attributes whose connexion with these characters we cannot conceive. For example, there is a far-reaching division of angiosperms (already referred to) into monocotyledons and dicotyledons, based on the number of the seed-leaves; but in these two classes the sub-classes are distinguished by various characteristics of the calyx and corolla, of the mode in which the stamens are inserted, &c. Now we are ignorant why a plant with two seed-leaves should be capable of one series of flower-developments, and a plant with one seed-leaf of another series; the number of seed-leaves is, for all we can see, an irrelevant character, though it cannot really be so; and the concept of dicotyledon or monocotyledon is complete, without reference to the character of the flower. Here therefore the intension of the wider term is less than that of the narrower. To the botanist the term Dichlamydeae, whose extension is less than that of Dicotyledon, means plants which in the first place have two seed-leaves, and over and above that have both calyx and corolla; the term Dicotyledon means merely a plant with two seed-leaves. Such cases give colour to the doctrine, that where terms are subordinated one to the other, the intension varies inversely with the extension; but they do not embody the true spirit of a classification.

3. We have seen that a term may be qualified by an adjective which is really an accident of it: by which is meant that the adjectival concept is an addition to the original concept, rather than a further determination of it; as when we qualify the term Christian (which implies a certain religious belief) with the adjective Armenian (which implies a certain nationality)—there being no necessary connexion between creed and race, but any variety of one being capable of coinciding in individuals with any variety of the other. These cases (to which those considered in the last paragraph approximate) bear out the doctrine of inverse relation, so far as they go. But it may be observed that they only bear it out, because they have been as it were constructed to do so. We take a term, and qualify it by an adjective which in the first place is known not to be applicable to all instances (and therefore narrows the extension), and in the second place is not implied by the term in any way as a possible development of the genus: so that it is a sheer addition to whatever intension the original term possessed. Then we call attention to the fact that in the original term, and the term

composed of it and of an adjective, extension and intension vary inversely. Of course they do, because we have carefully arranged it, by so qualifying the original term that they must. But it is ridiculous to infer from this, that in all terms, where one is of wider extension than the other, its intension is less. Because this holds where the terms are not related as genus and species should be, it must not be concluded to hold where they are so related.

4. It may still be felt that there is more truth in the doctrine than has been conceded. Take the most unimpeachable examples of genus and species, such as rectilinear triangle, with its species equilateral, isosceles and scalene. Can we not and do we not conceive a rectilinear triangle with regard to those points in which equilateral, isosceles, and scalene agree, and without regard to those in which they differ? and may not this notion be perfectly precise and definite? and if such be the intension of the genus-term, is it not less than that of the species-term? We must admit that this is possible. In the words of R. L. Nettleship,[1] 'we may, for convenience' sake, mentally hold apart a certain fraction of the fact; for instance, *the minimum of meaning which justifies us in using the word "triangularity"*. We may call this the generic triangle, and distinguish it from particular forms of triangle.' But the true intension of the term is not the 'minimum of meaning' with which we can use it, but its 'full meaning'.

What has been so far said with regard to the relation of intension and extension in terms may perhaps be rendered clearer to some as follows. Wherever we have species of a genus, or distinguishable varieties of a common nature, we may contrast the unity which they present with the variety. To attend to the intension is to attend to the element of unity: to attend to the extension is to attend to the element of variety. Sometimes we are more interested in one, and sometimes in the other. When Socrates in the *Meno* asks what is virtue, and Meno begins describing the virtue of a man, the virtue of a woman, and so forth, Socrates explains that he wants to know what virtue is as one in all these, and not what the divers virtues are; in later language, he wished for the intension and not the extension of the term. Aristotle remarks[2] that an enumeration of these different virtues and a description of them severally are more valuable than a vague statement of their common nature:

[1] *Philosophical Remains*, i. p. 220. The italics are mine.
[2] Plat. *Men.* 71 D–72 D; Ar. *Pol. a.* xiii. 1260^a 20–28.

i. e. that here at any rate the element of variety is more worth consideration than the element of unity, if either is to be neglected. But if the two are realized together, the unity of the superordinate whole must be seen as the more comprehensive unity, not as the more jejune extract. So far however as we cannot realize them together, and see their necessary connexion, it will have the character of the jejune extract and be a whole of less meaning, even although we know that the variety of species into which it enters is great; and in these conditions, it may be said to be of less intension.

It follows that the *infima species* (or the term denoting it), in the unity of whose being we recognize no variety, has properly speaking no extension. Equilateral triangles may differ in the length of their sides, and we may if we like regard this difference as constituting a variety in their common nature. But if we do not—if we conceive the particular length of the sides to constitute no difference in equilateral triangularity—then we recognize no such variety in the unity as makes it possible to distinguish from the intension the extension through which it ranges. The term *equilateral triangularity* will denote to us a certain unitary nature, but no varieties of such.

Logicians have been withheld from acknowledging these terms to have no extension by two reasons, by one justifiably, by the other through a confusion. Justifiably by this, that the point at which logical division stops is generally arbitrary, and what are treated as *infimae species* are capable of subdivision into lower species, which would be their extension; ellipses may vary in their ellipticity according to focal length, Christians in their Christianity according to faith as well as practice. The consciousness of the variability of the specific nature which forms the intension of the term makes us regard it as still having extension, though less than its superordinate terms. Terms within whose intension there is no variety, like *point*, or none recognized, like *equilateral triangle*, are rare.

The other reason is this, that even where there is no variety within the intension of a term, there is multiplicity of instances. Though no species of equilateral triangle are distinguished, innumerable equilateral triangles are. Two such triangles interlaced are a favourite symbol in the decoration of churches; and the number of them delineated on church-walls and windows must be past counting. If the individual instances make the extension,

the *infima species* will have plenty, though still less than its superordinate terms, because there are more instances of the genus than of any one species [1]—more triangles, for example, than equilateral triangles.

It is plain that this reason involves a confusion between two different things, between the variety of *kinds* over which the predication of a term may extend—the variety of which we *conceive* an unity to be susceptible, and the various *individual instances* in which a common nature is manifested. On the former view, the extension of *man* is Aryan and Semitic, Negro and Berber, &c.: of *triangle* equilateral, isosceles and scalene; on the latter, that of *man* is Socrates and Plato, Alexander and Caesar, you and I, &c., that of *triangle* every triangle on a church wall or on a page of a copy of Euclid's *Elements*. But the relation of genus to species is not the same as that of universal to individual, of a kind to its instances, and the antithesis of intension and extension ought therefore not to be used indifferently in respect of both. We might perfectly well understand by the extension of a term either the various forms or the various instances in which the common nature that is its intension is manifested; but we ought not to understand both indifferently.

It is easy to see how the confusion arises. Though the antithesis between the intension and the extension of terms is based on that between the unity in *what* different individuals are, and the variety in which that unity is displayed, most of the terms in which this antithesis is illustrated are general terms predicated of individuals, like *man* or *ox* and *animal*, *gold* or *silver* and *metal*, *axe* or *hammer* and *tool*, *musician* or *painter* and *artist*, *triangle* or *square* and *figure*.[2] They are predicable of individuals, but in respect of their common nature. The superordinate term—*animal*, *metal*, &c.—is predicable of more individuals, the subordinate—*man* or *ox*, *gold* or *silver*, &c.—of fewer. Sometimes there are also proper names predicable of the individuals singly, but all alike are names of individuals. The distinction in the meaning of a general name between the individuals whereof it is predicable and the common nature in respect of which it is predicable of them is important and obvious. Language allows us to say that Caesar is a man, and that a man is an animal,

[1] Except in a species which is *sui generis*: cf. p. 137, n. 1, *supra*.

[2] In the last two instances the terms though substantival are attributive in meaning: cf. *supra*, p. 37, n. 1.

that Beethoven is a musician and that a musician is an artist, that this is gold, an axe, a triangle, and that gold is a metal, an axe a tool, a triangle a figure. Hence it is supposed that the relation of man to Caesar, or musician to Beethoven is the same as that of animal to man, or artist to musician; the relation of axe or triangle to 'this' the same as that of tool to axe, or figure to triangle. For we are misled by the common form of the proposition, *A is B*, and do not reflect sufficiently on the different senses in which one thing is said to be another.[1] When I say that a man is an animal or a triangle a figure, I mean that being a man is a way of being an animal, to be a triangle is to be a figure; and I could say instead that humanity is animality or triangularity figurateness. But when I say that Caesar is a man, or this a triangle, I do not mean that Caesarness is a way of being a man, or that thisness is triangularity; the concrete individual is something more than can be comprised in any concept.

With abstract terms and names of universals we are not tempted to make this confusion. We should not feel the same hesitation in allowing that 'equilateral triangularity' as that 'equilateral triangle' has no extension; and if we hesitated to deny extension to humanity or democracy, it would be only because we are conscious that these concepts are capable of further specification, that humanity is something different in different men, democracy in France and in the United States. No doubt attributes and relations have their instances, and abstract terms are names of attributes and relations; they are predicable of the several instances, and as such are general. But the instances can only be distinguished by referring to the particular subjects[2] in which the attributes inhere or between which the relations hold; and in abstraction we commonly ignore these, and consider the attribute or relation by itself; we may be interested in the divers forms that it may take, and have separate names for these, for the diversities of colour or constitution, consanguinity or proportion; but to be interested in the instances would be to be interested in the concrete individuals that display them, and from these we are abstracting. Hence it is that the abstract term becomes the name of the attribute or relation, of whose instances it is predicable as a general

[1] Cf. *supra*, pp. 23–24.

[2] Generally concrete individuals, but not always; I might e.g. direct attention to instances of degree by mentioning colour and heat, without reference to particular coloured or hot things.

term, and that even when we use it as a general term, e. g. when we speak of so many deaths, in the plural, we are still apt to think of the attribute or relation as identical in all its instances ; indeed, as we saw, it has been denied that there are instances of relations.[1]

It is plain then that by the extension of a term we should not mean indifferently species and individuals ; to be specified in divers ways is not the same as to be found in many instances. And there is the less necessity for using the word *extension* thus confusingly, that another word, *denotation*, will serve where the instances are meant. A word denotes anything of which it can be predicated as a name ; *man* denotes Socrates and Caesar, *artist* Beethoven and Giotto, *triangle* this and that triangular figure. It is true that universals are denoted also by their names; *animality*, *triangularity*, *proportion*, each denote something; and abstract terms denote not only instances of attributes and relations, but the attributes or relations considered each as one in its several instances.[1] But this fact need not disturb us. We use *denote* in the same sense, in each case.

It will be observed also that the inverse relation of extension and intension does not hold equally when by the extension of a term we mean the forms in which the intension is displayed, and when we mean the instances. We saw how the intension of the term *animal* might from one point of view be said to increase, as one becomes acquainted with fresh forms of animal life ; and how from another point of view, because what at first one might have regarded as essential to an animal turns out not to be indispensable, it might be said to diminish, shrinking to a jejune residuum. But whichever way we look at it, it is only acquaintance with fresh *forms* of animal that produces this result; a mere increase in the *number* within one's acquaintance would not produce it. It is said that you cannot widen or narrow the extension of a term without restricting or enlarging its intension, and vice versa. But change in the meaning of a term comes by extending its application to new *kinds* of subject, or confining it to some kinds only of those to which it was before applied. The intension of the term *baby* does not increase and decrease with the fluctuations of the birth-rate.[2] A change in the intension of a term will indeed commonly affect its denotation as well as its extension, just as the superordinate term in a classification commonly denotes more individuals than the subordinate, besides having a wider extension; but only a change in the extension, that is,

[1] Cf. *supra*, pp. 27, n. 3, 33–35. [2] F. H. Bradley, *Principles of Logic*, p. 158.

in the kinds of individual denoted, not in the mere denotation, will affect the intension.[1]

In place of the terms Extension and Intension, various writers have used others to mark either what is, or what they wrongly thought to be, the same distinction; and in particular, since the publication of Mill's *System of Logic*,[2] the antithesis of Denotation and Connotation has come into favour. Mill regarded this antithesis as identical with that of Extension and Intension; but he claimed for his expressions that they possess an advantage lacking to others, in the existence of the corresponding verbs, to *denote* and to *connote*; we may speak of a term denoting or connoting this or that, but with other expressions we must use a periphrasis and say, e.g., that so and so is included in the extension, or constitutes the intension, of a term. This advantage and the jingle of the antithesis have combined with Mill's authority to bring the word *connote* into common use; for we do require at times, as the passage above referred to in the *Meno* shows, a word that will distinguish a term's meaning in intension from its meaning in extension. In other respects Mill's expressions are less appropriate; for *extension* suggests, and *denotation* does not, the range through which the intension is manifested; *intension* suggests, and *connotation* does not, what we intend by a term; and *con*notation contains a suggestion, inappropriate in many cases, of *additional* meaning. But the trouble is that the two antitheses are not really equivalent. A term may denote, which has no extension; and may have intension, which, in the prevalent meaning of the word, has no connotation. Mill drew his distinction with his eye mainly on two classes of terms, attributives and general concrete names. The functions of denoting and connoting which he found in these he thought to be the only functions of any term. Then, because certain terms do not connote like them, viz. proper names and the names of *infimae species* of attributes or relations[3] (like *length* and *whiteness*), he thought they only denoted; and he made a division of 'names' into *connotative* and *non-connotative* (by which he understood *unmeaning*), which he described as 'one of the most important distinctions which we shall have occasion to point out, and one of those which go deepest into the nature of language'.

[1] Of course, when the term denotes kinds, its intension will be affected by a change in the denotation.

[2] v. Bk. I. ii. § 5.

[3] Mill does not mention relations, but the argument applies equally in their case; and if they are not always mentioned in the following discussion, that is only for brevity's sake.

As he expounded it, however, it has been a source of little but error and confusion. He confounded different distinctions, and raised a controversy about the connotation of proper names, to which there has been no satisfactory issue, because he never clearly realized to himself what he meant by connotation, nor that it was something different from intension; and so the word has been used in the controversy in different senses.

In order to clear up the ambiguities of the word, we must examine the passage in which Mill expounds his doctrine. It runs as follows. 'A non-connotative term is one which signifies a subject only, or an attribute only. A connotative term is one which denotes a subject, and implies an attribute. By a subject is here meant anything that possesses attributes. Thus John, or London, or England, are names which signify a subject only. Whiteness, length, virtue, signify an attribute only. None of these names, therefore, are connotative. But *white*, *long*, *virtuous*, are connotative. The word white, denotes all white things, as snow, paper, the foam of the sea, &c., and implies, or in the language of the schoolmen,[1] *connotes*, the attribute *whiteness*. The word white is not predicated of the attribute, but of the subjects, snow, &c.; but when we predicate it of them, we convey the meaning that the attribute whiteness belongs to them . . . All concrete general names are connotative. The word *man*, for example, denotes Peter, Jane, John, and an indefinite number of other individuals, of whom, taken as a class, it is the name. But it is applied to them, because they possess, and to signify that they possess, certain attributes. . . . The word *man*, therefore, signifies all these attributes, and all subjects which possess these attributes. . . . Even abstract names, though the names only of attributes, may in some instances be justly considered as connotative; for attributes themselves may have attributes ascribed to them; and a word which denotes attributes may connote an attribute of those attributes. Of this description, for example, is such a word as *fault*; equivalent to *bad* or hurtful quality. This word is a name common to many attributes, and connotes hurtfulness, an attribute of those various attributes.[2] . . . Proper names are

[1] Mill means that in the case of such terms as these, the schoolmen spoke of attributes being connoted; but not that his use of the word *connote* conforms generally with that of the schoolmen: cf. *infra*, pp. 156–158.

[2] Mill instances 'slowness in a horse' as an attribute denoted by the word 'fault'. It is clear that if 'fault' is connotative, 'virtue' should not have been given as an example of a non-connotative name. The italics in this quotation are his.

not connotative: they denote the individuals who are called by them; but they do not indicate or imply any attributes as belonging to those individuals.'

Thus Mill considers three classes of terms to be connotative—

(*a*) attributive terms, like *white, long, virtuous*, &c.;

(*b*) general concrete names, like *man, snow*, &c.;

(*c*) abstract terms, if they are names of a genus of attributes, like *fault*;

and two classes to be non-connotative—

(*a*) proper names;

(*b*) abstract terms, if they are names of *infimae species* of attributes, like *length, whiteness*, &c. Designations, i. e. phrases indicating an individual that contain connotative terms, he regards as connotative.

Now it is true of all his 'connotative' terms, that they signify more or less of *what* that is, whereof they are predicated; and they are therefore said to *denote* the subjects of which they are predicable, and to *connote* whatever character[1] they indicate these subjects to possess. But further, they are used of these subjects *because* of their possessing such character. Mill means then by the *denotation* of a term the subjects of which it can be predicated, by the *connotation* that character, to indicate the possession of which we use the term of any subject.

It might seem that we could say simply, that the connotation of a term is its meaning. But there are two reasons why this is not so. In the first place, terms have two functions, both of which may be called meaning. They direct our thought to some subject, and they suggest what that subject is, to which our thought is directed. I may be said e.g. to mean by *tools* either spades, hammers, axes, &c., or 'things made in order that we may by their means in handling them do what we could not do, or do so well, with our unaided hands'. Mill would say that the former is what the word denotes, the latter what it connotes. In the second place, a term may discharge the function of signifying what the subject is, to which it directs our thought, in two ways. It may signify the subject in its entirety, or some character in the subject, with which the subject is not identical. It is only the latter function which Mill calls connoting, as in the example *tool* just given. Terms which mean what the subject is in its entirety he calls non-connotative, and he

[1] Mill says *attributes*, because he regards e.g. being gold as an attribute or aggregate of attributes in any piece of gold.

does not think that they signify what the subject is at all. And there are further differences within both kinds of terms, in their way of discharging the function of signifying *what* the subject denoted by them is, which Mill ignores.

The most important class of his 'non-connotative' terms is proper names. If connotation in a term is signifying some character in a subject, to indicate its possession of which we use the term of any subject, proper names certainly do not connote. But besides this signification in a term Mill recognized no other function, except denoting. Hence he thought that proper names only denoted, and were 'unmeaning marks'. 'A proper name', he says, 'is but an unmeaning mark which we connect in our mind with the idea of the object, in order that whenever this mark meets our eyes or occurs to our thoughts, we may think of that individual object'[1]; and he contrasts connotative names as 'not mere marks, but more, that is to say significant[2] marks.' Now in thinking that a proper name merely denotes, and signifies nothing, Mill was wholly wrong. It is the sense of this error which has led critics to say that proper names have connotation[3]; and if we had to make the antithesis of denoting and connoting cover the ground in regard to the functions of every kind of term, that would certainly be the less misleading doctrine. But Mill was calling attention to a real difference distinguishing his 'connotative' from proper names, which may be well expressed by saying that proper names have no connotation, if we accept the sense of 'connotation' which may be extracted by considering the classes of term to which he ascribes it, and reject his identification of it with 'signification' generally. We may the more

[1] This account of a proper name closely resembles Hobbes's definition of a name generally (quoted p. 20, *supra*), which in the first section of the same chapter Mill approved. Hobbes says that a name is 'a word taken at pleasure to serve for a mark which may raise in our mind a thought like to some thought we had before'. To say that it is taken at pleasure means that it is not taken on account of any pre-existing signification. This is true at the outset of all names, proper and general alike, except derivatives. A general name was unmeaning *before it was given to anything*: so also is a proper name. But a proper name, like a general name, has a meaning *after it is given*.

[2] Mill obviously means by *signifying* being the sign of *what* a thing is; else he could not distinguish 'mere marks' from 'significant marks'; for a mere mark denotes. It is possible to use the word 'signify' in the sense of 'denote'. But throughout the following discussion it will be used as Mill uses it.

[3] e.g. Jevons, *Elementary Lessons in Logic*, Lesson V: Bosanquet, *Essentials of Logic*, Lect. V. § 6; also the first edition of this book.

conveniently do this, because the signification which proper names do possess is perfectly well indicated by the word ' intension '.

A proper name certainly has intension as well as denotation. It is a mark directing our thought to an individual; but that which is to be a mark must have meaning. A scratch may be a mark *on* a coin which I am looking at; it is not a mark *of* the coin that I am looking at, but of its being the same coin which I had put in the way of a suspected thief. I may of course be ignorant of the meaning of a mark. The broad arrow ↑ which is occasionally seen on gate-posts, milestones, &c., is a mark; a traveller might not know what it meant; but he would not call it a mark, unless he guessed that it meant something. By enquiry he might learn that it meant that the spot where it was placed was the precise spot whose height was recorded in that portion of the ordnance survey. Here the mark is general. But the mark by which his nurse recognized Odysseus was equally significant. In its own nature it was a scar, the consequence of a wound, and not (like a brand) intended as a mark. Yet this scar (its precise form and position being taken into account) to those who had observed it in Odysseus became a mark by which to know him. He had been absent twenty years, and was changed otherwise beyond recognition; he was supposed to be dead; but his nurse, seeing the mark, knew the man before her to be him—knew that about the man before her which otherwise she would not have known. How can it be said that it was an unmeaning mark for her? And suppose that instead he had at once told her that he was Odysseus; the name would have given her precisely the same information; how then could the name be unmeaning? The doctrine that proper names have no intension is refuted by every criminal who assumes an alias.

And not only, to any one who knows of what individual it is the name, has a proper name meaning, but it has more meaning than a general term. The cry ' man overboard ' would have conveyed to Aeneas and his companions not more but less information than the cry ' Palinurus overboard '. It cannot indeed convey to any one, for he cannot know, the whole character of the individual denoted; but it excludes from its meaning designedly nothing of that character; whereas another term, if it is not the name of an *infima species* of attributes,[1] is designedly confined to signifying only some deter-

[1] This is intended to signify the whole character of what it denotes: cf. *infra*, p. 154.

minate character in what it denotes. Mill speaks as if, were I to point to some individual person or thing, and to ask who or what is that, and were another to reply by a proper name, I should only learn what it *is called*, not what it *is*. And if I now heard the name for the first time, that is true. But it is equally true of a general name, when I hear it for the first time. If I point in a foreign country to an unfamiliar object, and ask what is that, though I am answered with a general name, I shall only learn what in that language it *is called*. On the other hand, if a proper name is, and I know it to be, the name of something with which I am already familiar, either personally or by report, it may be very instructive. What would not a man have given to be once truly told, in reply to the question 'Who is that?' 'Napoleon'? Or if I cross a country road, and am told 'That is Watling Street', do I not learn much more about it than what it is called, and more than the word 'road' conveys?

What then is the important difference between a proper name and other classes of term, which Mill wishes to indicate by saying that proper names have no connotation? It is that they cannot be used to convey information about an otherwise unknown individual. A general term, used of any subject, is instructive to those not acquainted with the subject. If I ask 'What startled you?' and you tell me a karait, I shall know that it was a very venomous snake. That is because the term 'karait' is used of a subject merely to indicate that it has a certain character, of any subject possessing which it might be used equally. It has a signification on the ground of which it may be predicated of one fresh individual after another. But a proper name is not used of any individual for the first time on the ground of a signification which it already possesses; the son of James I and VI was not called Charles on account of his Carolinity.[1] It acquires its signification from the individual to which it is given. Hence it is uninstructive to any one ignorant of the individual denoted. If I ask 'What startled you?' and you answer 'Glamby', I shall not know in the least what it was, unless I know already what that word denotes. On the other hand if I do already know that—if I know that Glamby is the name of your dog or your baby or the ghost that haunts your house—I shall learn not only what

[1] Hence, as was pointed out p. 47, *supra*, the name Charles is used equivocally of him, of his son, of the son of Pepin, &c. But an equivocal term is not a term with no meaning; it is a term with more than one meaning.

kind of individual it was that startled you, but what individual of that kind. For this is a further peculiarity distinguishing a proper name from terms of any class that Mill calls connotative : it is part of the meaning of a proper name that the subject denoted is precisely this or that individual. That is why a proper name can be the predicate of a proposition ; we make it a predicate when we wish to say not of what kind something is, about which information is offered, but what individual of some kind. If I were wandering for the first time in a country known to me by history, and, coming to a village, asked its name, the answer *Quatre Bras* would not tell me that it was a village, but which village it was. And since I may point to this village without knowing which it is, I can distinguish in a proper name the function of designating or denoting an individual from that of signifying which individual, with all its being and history, is denoted ; and so I must say that, besides denoting, it has intension ; only, part of its intension, concerning what it denotes, is that this is such precise individual.[1] Did it signify nothing concerning that which it denoted, it would not even have denotation. If you say that you have been reading about Quatre Bras, and I do not know whether that is a village or a general or a poem or a star, it denotes nothing to me. A name could only denote and have no signification, if that could be discriminated which had no character.[2]

All this indeed only amounts to saying that a proper name has not general meaning. Mill really intended by connotation general meaning, but thought that to lack it was to lack meaning altogether. ' General meaning ' is not however a complete account of what he intended by the word. Connotation is signifying some character in a subject, which can be distinguished from that subject. ' A connotative term is one which denotes a subject and implies an attribute.' When a term signifies a subject in its entirety, what it signifies is not an attribute of what it denotes. Hence Mill denied connotation to another class of terms, the names of *infimae species* of attributes. He was assisted to do so by confusing the relation of species to genus with that of individuals to their kind. *Man* denotes ' Peter, Jane,

[1] If we had to make the words *denotation* and *connotation* express all the functions of all kinds of terms, we might say, though a trifle loosely, that the denotation of a proper name is part of its connotation.

[2] Mill's confusion between an individual's substantial nature and an attribute perhaps helped to mislead him here. Because, if I think away its being white, snow still remains something that I can discriminate, therefore he seems to suppose that, if I think away his being man, John still remains something which I can discriminate.

John,' and connotes their common character; *fault* denotes slowness, stupidity, &c., and connotes their common character. *Man* is connotative, *Peter* or *Jane* or *John* is not; and similarly, he thinks, *fault* is connotative, *slowness* or *stupidity* not. But this result, unlike his similar view about proper names, is both devoid of plausibility and in contradiction with his teaching elsewhere. So unplausible is it, that some, being unable to bring these terms under the formula just quoted, have preferred to deny to them denotation. And it flatly contradicts Mill's doctrine, that definition declares the connotation of a name. For the name of a species of attribute may be definable [1] by giving its genus and differentia, even where that of the genus is not; and yet according to Mill the latter has connotation, and the former none.

That these terms are unmeaning is clearly absurd. When Wolsey says to Thomas Cromwell, 'Cromwell, I charge thee, fling away ambition; By that sin fell the angels', the word *ambition* does not denote an object of thought, without signifying what that is which it denotes, but signifies the nature of the sin which Cromwell is warned to avoid. Yet it also denotes it. Such terms are names of universals, of the common character in many instances of an attribute or relation.[2] But of what they denote they signify the entire being; and what they denote is general. Hence, if to be connotative is to have general meaning—and that is why Mill denies connotation to proper names—they are connotative. If it is to signify something general in a subject of which that is not the entire being, they are not. This is the ambiguity which is the source of Mill's vacillating language.

Proper names, then, and the names of *infimae species* of attributes and relations both signify the entire being of what they denote, but with a difference, because what the latter signify is general, and may be definable. Those are the classes of term which Mill calls non-connotative. But the classes which he calls connotative, though he offers a single account of them all, are not really alike. An attributive term, like *long* or *white*,[3] denotes that which, *being constitutively something else*, is long or white also [4]; and it is connotative because

[1] It need not be; e.g. blue is a species of colour, but can no more be defined than colour, because to know the specific difference presupposes that I know the species; I could only say that it is a blue colour.

[2] Or names of the instances considered merely in their common character.

[3] These terms were called connotative by the schoolmen: v. *infra*, p 157.

[4] Hence they are commonly combined with a general term, and we speak

it notes a character found *along with* the constitutive being of the subject which it denotes.[1] Its 'connotation' is not the constitutive or general being of that subject, but some detail in its being, which would be denoted by the corresponding abstract term *length* or *whiteness*.[2] But a general concrete name, like *man* or *snow*, does not denote that which, being something else, or having some other constitutive being, is man or snow also; and it is *con*notative because, *besides denoting* a subject, it signifies not some detail in its being,[1] but its constitutive or general being; the prefix, if it has any force, has not the same force in this case. And those abstract terms which Mill calls connotative he calls so because, *besides denoting* species of relation or attribute, they 'connote' their generic nature.[2]

We may now sum up the results of our investigation into the antitheses Intension and Extension, Connotation and Denotation. All terms may be said to denote the subjects of which they can be predicated, but those most directly which are names of, or can stand for, those subjects; hence adjectives, when used to denote the subject of a proposition, are often combined with a demonstrative word, such as an article. All terms have intension, or meaning; that is, they signify all or something of what that is which they denote. When the intension, or what is thus 'intended', is something displayed in divers forms or species, these are said to be the extension of the term; and sometimes the individuals in which the common nature, which is the intension of a concrete general term, is found are called its extension; the latter usage is not extended to general abstract terms, for in abstraction the instances are not discriminated. Either way, proper names have no extension; names of *infimae species* of substances can only be said to have extension, if the individuals are taken as the extension; names of *infimae species* of attributes can only be said to have it on the same condition, that we consider their individual instances. What is commonly

of long days or long shadows, white sails or white complexions. The fact that they may also be combined with proper names, so that we can say 'the envious Casca' or 'the melancholy Jacques', shows that proper names have intension. No one would say 'the envious *X*' if he did not know in the least *what* that was, which *X* denoted.

[1] A word like *traitor* or *artist* might be said to do this; but we have seen (p. 37, n. 1, *supra*) that these, though substantives grammatically, are attributive in function. They also were called connotative by the schoolmen.

[2] Attributive terms also may be predicated of attributes and relations, as when we say that impartiality is rare; here *rare* connotes an 'accident' in the attribute which it denotes, and not, as *virtue* would, its generic nature. Mill takes no account of this difference.

said about the inverse relation of intension and extension in terms refers only to terms subordinated one to another in a classification, and does not regard individuals as the extension. Lastly, terms have connotation which have general meaning; the connotation of a term is that character through signifying which it denotes the subjects of which it can be predicated.

[It may be added that the instance which Mill takes, on which to argue that proper names have no connotation, viz. *Dartmouth*, confuses the issue. He urges that the town would still have the same name if the river changed its course, though the name would not then connote the town's position; therefore it connotes nothing now. The argument is not good. If a town is called Dartmouth because it stands at the mouth of the Dart, so far the name is a designation. But meaning or intension in a proper name is not dependent on connotation belonging to a connotative word in it. Mill should have taken the river's name Dart, or Dartmouth in New Hampshire. The latter illustrates yet another point. Most proper names are chosen for a reason. A mountain may be named after its discoverer or first climber, a town or college after its founder, a child after its grandparent or godparent, a society after some one of whom its members wish to be considered the disciples. But this does not become part of the meaning of the name, which is derived from that to which the name is given. A similar remark applies to those names in which, as often happens, something may give a clue to the nature or nationality or sex of the subject denoted; the guess may be wrong; but even if it is right, the feature which gave the clue functioned as having general meaning; and the meaning which it is important to vindicate for proper names is not general meaning. It is however relevant to this vindication, that proper names often come to *acquire* general meaning; *Caesar* is a familiar instance, and we have all heard of a Daniel come to judgement, and that *Capuam Hannibali Cannas fuisse*. For this acquisition comes about through extending to another subject some part of the signification which the name derived from the subject to which it was originally 'proper'.]

[For the sake of the curious, a few words may be added on the history of the term 'connotative'. In William of Occam a distinction is found between *absolute* and *connotative* terms. Absolute terms have not different primary and secondary significations; 'nomen autem connotativum est illud, quod significat aliquid primario et aliquid secundario.' He gives as instances relative names (for *father* signifies a man, and a certain relation between him and another): names expressing quantity (since there must be something which has the quantity): and certain other words: *v.* Prantl,

[*Geschichte der Logik im Abendlande*, Abs. xix. Anm. 831, vol. iii. p. 364. Johannes Buridanus said that some terms connote nothing beyond what they stand for ('nihil connotantes ultra ea, pro quibus supponunt'); but 'omnis terminus connotans aliud ab eo, pro quo supponit, dicitur appellativus et appellat illud quod connotat per modum adiacentis ei, pro quo supponit'.[1] Thus *meus* and *tuus* stand for something which is mine or yours; but they connote or signify further and 'appellant me et te tanquam adiacentes' (id. ib. xx. 111, vol. iv. p. 30). Elsewhere we are told that 'rationale' 'connotat formam substantialem hominis' (xx. 232, vol. iv. p. 63: cf. Anm. 459, p. 109). Elsewhere again *album* and *agens* are given by Occam (ib. xix. 917, vol. iii. p. 386) as examples respectively of connotative and relative terms; and it is explained (ib. Anm. 918) that a connotative or a relative term is one which cannot be defined without reference to one thing primarily and secondarily another; thus the meaning of *album* is expressed by 'aliquid habens albedinem'; and when by any term anything 'connotatur vel consignificatur, pro quo tamen talis terminus supponere non potest, quia de tali non verificatur'[2], such a term is connotative or relative. Thus a term was called connotative if it stood for ('supponit pro') one thing, but signified as well ('connotat') something else about it; as Archbishop Whately says (*Logic*, II. c. v. § 1, ed. 9, p. 122), 'it "connotes", i.e. "notes along with" the object [or *implies*], something considered as inherent therein.' The Archbishop suggests the term *attributive* as its equivalent; and though connotative terms were not all of them adjectives, since relative terms also connote, and so do terms like 'mischief-maker' or 'pedant', which though adjectival in meaning are substantival in form, yet adjectives are the principal class of connotative terms, in the original sense of that word.

Connotation and *denotation* were thus originally by no means equivalent (as they have come to be treated as being) to *intension* and *extension*. Connotative terms were contrasted with absolute, and their function of connoting distinguished from that of standing for something. James Mill, who probably by his remarks upon the word *connote* had some influence in directing his son's attention to it, says that '*white*, in the phrase *white horse*, denotes two things, the colour, and the horse; but it denotes the colour *primarily*, the horse *secondarily*. We shall find it very convenient to say, therefore, that it *notes* the primary, *connotes* the secondary, signification' (*Analysis of the Phenomena of the Human Mind*, vol. i. p. 34, ed.

[1] i.e. to use J. S. Mill's terms, it denotes 'id pro quo supponit', and connotes 'id quod appellat'. For *appellatio* cf. Prantl, vol. III. xvii. 59 ('proprietas secundum quam significatum termini potest dici de aliquo mediante hoc verbo "est"'). Cf. also ib. xix. 875.

[2] Occam means that, e.g., snow can be referred to as *album*, but *albedo* not.

[1869). By the schoolmen it would commonly have been said to connote the colour, and the primary signification was that 'pro quo supponit'. J. S. Mill, in a note to p. 299 of the same volume, objects to his father's inversion of the usage. But he himself, by extending the term *connotative* to cover what the schoolmen called absolute, and opposed to connotative, names, introduced a complete alteration into its meaning.

John and *man* are both absolute names in Occam's sense. *Man*, no doubt, according to some (though not according to a nominalist like Occam) signifies in John, or anything else 'pro quo supponit', an universal nature; but John and this are not two things, of which it denotes one primarily and the other secondarily, or for one of which it 'supponit', and 'appellat' the other; for John *is* a man, and without what the word *man* signifies would be nothing for which that word could stand or by which it could 'call' him. With *white* it is different; I have a notion of paper, and a notion of whiteness, and whiteness is no necessary part of my notion of paper; and so with any other subject of which whiteness is only an attribute and not the essence. Hence the name *white* may be said to signify (or in James Mill's usage to denote) two things, the colour, and that which is so coloured; for these can be conceived each without the other, as John and man cannot; or, if we prefer, it may be said to denote or stand for one, and to connote the other.

(Cf. also on the history of the word *Connotative* a note in Minto's *Logic, Inductive and Deductive*, p. 46.)]

CHAPTER VII

OF THE PROPOSITION OR JUDGEMENT

A GENERAL acquaintance with the nature of the judgement or proposition has been hitherto assumed. It would be impossible for Logic to be written, or if written to be understood, unless the acts of thought which it investigates were already in a way familiar; for Logic arises by reflection upon an already existent thought of things. Now judgement is the form in which our thought of things is realized, and it is primarily in judgement that we use terms. Their use in question, command, exclamation or wish presupposes earlier judgement. The varieties of terms, the different relations of one to another which form the basis of the distinction of predicables, would be unintelligible, unless it were realized that, in the first instance, terms come before us only as elements in a judgement. They live, as it were, in a medium of continuous judging and thinking; it is by an effort that we isolate them, and considering subject and predicate severally by themselves ask in what relation one stands to the other, whether they are positive or negative, abstract or concrete, singular or general, and so forth. Without presuming some knowledge of this medium in which they live it would be of as little use to discuss terms, as to discuss the styles of Gothic architecture without presuming some knowledge of the nature of space.

We must now consider more closely what judgement is, and what varieties of judgement there are that concern Logic.

A discussion of judgement raises many metaphysical problems, into which such a work as this cannot enter fully. But a few things may be pointed out about it.

To judge, in the logical sense of the word, is not to acquit or condemn, but to affirm or deny a predicate of a subject. There is however a connexion between the logical and judicial uses of the word. Judgement, in the logical sense, is often preceded by what must indeed be called thinking, but is not judging, viz. questioning

or 'wondering'; but this process, if we do not give it up, is ended or decided by a judgement, as the judge by his judgement after considering decides the case. It is true that as the judge may be mistaken in the opinion which he reaches on the facts, so we commonly in our judgements form fallible opinions only; and Logic can render no greater service than to make us more alive to the distinction, which the grammatical form of the proposition fails to reflect, between opinion and knowledge. We shall meet it in discussing what is called the modality of judgements. So important is it, that some would hesitate to bring knowledge and opinion under one genus, judgement. But there is much which may be said about them in common.

Every judgement makes an assertion, which must be either true or false. Its propositional form claims truth: i. e. I ought not to make a statement, such as that the earth is round, unless I think that it is so, and mean that it is so; although in fact we often express in this form opinions which we hold doubtfully. This capacity of truth or falsehood is the peculiar distinction of judgement, expressed grammatically in a proposition by the indicative mood. Imperatives, optatives, exclamations, and interrogations are not propositions as they stand, though they imply the power of judging. 'I say unto this man "Come", and he cometh.' Here the indicative sentence 'I say unto this man "Come"' may be true or false, the indicative sentence 'He cometh' may be true or false, and both these are propositions, and express judgements; but we cannot ask of the imperative 'Come', is it false or true?—it is not a proposition. Again the question 'Art thou he that troubleth Israel?' is not a proposition; it is not itself true or false, but enquires whether the judgement implied is true or false. An optative, as in the line 'Mine be a cot beside the rill', is not as it stands a proposition; it could hardly be met with the rejoinder 'That 's true', or 'That 's a lie'; if it were, and we were to ask 'What is true?' or 'What is a lie?' the answer would be 'That you really wish to live in a cot beside the rill'; so that, although an assertion is implied about the wishes of the person speaking, it is not so expressed in the optative. Exclamations may in like manner imply an assertion which they do not express, as when we say 'Strange!' or 'Incredible!' They may also be mere modes of expressing feeling, like an action and a gesture; and in such cases, though something doubtless 'passes in the mind', the exclamation can hardly be regarded as an attempt

at asserting [1] anything. It is not, however, necessary to go into any subtleties; the same grammatical form may indicate different acts of mind, and the same act of mind be indicated by different grammatical forms; 'Let the king live for ever' may be called imperative or optative: 'Angels and ministers of grace, defend us,' imperative, optative, or exclamatory: 'I would that I were dead,' optative or indicative. It is enough for us to realize that a judgement being an assertion, capable of truth and falsehood, the full and proper expression of it is in the indicative mood.

In judging, I affirm or I deny; in either case, I assert. I can express doubt—'matter may be eternal'; and herein I neither assert that it is nor that it is not eternal; still, I assert something, though it is not so easy to say what.[2] Propositions of the simple form '*S* is *P*', or '*S* is not *P*', are called *categorical*, but in all there is a categorical element. We can best elucidate the general character of judgement by considering examples of this form in the first place.

A proposition makes *one* assertion [3]; an assertion is one, when there is one thing said of one thing—ἓν καθ' ἑνός, i. e. when the subject is one, and the predicate one; though the subject and predicate may be complex to any degree. Thus it is one proposition that 'The last rose of summer is over and fled'; but two that 'Jack and Jill are male and female'; for the latter is equivalent to 'Jack is male and Jill is female'; one thing is asserted of Jack and another of Jill; one grammatical sentence expresses two judgements.

Subject and predicate are terms which have already been explained, as that about which something is asserted, and that which is asserted about it. A proposition—at least a categorical proposition—is often said to be composed of three parts, subject, predicate, and copula; the copula being the verb substantive, *is*, ἐστίν, *est*, *ist* (or *is not*, οὐκ ἐστιν, *non est*, *ist nicht*), sometimes, though mischievously, represented in Logic books by the mathematical sign of equation, = (or *not* =). We may consider at this point the nature and function of the copula, and the propriety of thus reckoning it as a third member of a proposition.

Common speech does not always employ the copula. Take the

[1] The reasoning which would make all exclamations imply a judgement was extended to actions by Wollaston, when in his *Religion of Nature Delineated* (first published 1724) he regarded all wrongdoing as a particular mode of telling a lie.

[2] Cf. *infra*, pp. 197 sq.

[3] Some difficulties about the singleness of judgement are discussed in Mr. F. H. Bradley's *Essays on Truth and Reality*, c. xiii. pp. 393 sq.

line 'It comes, it comes; oh, rest is sweet'. Here in the proposition 'Rest is sweet', we have subject (*rest*), predicate (*sweet*) and copula all severally present; whereas in the proposition 'It comes', we have the subject (*it*, referring to the omnibus), and for copula and predicate together the one word, *comes*. But that word contains what is said about the omnibus (for it is said to be coming, as rest is said to be sweet); and it also contains, in the inflexion, a sign that this is said about a subject; and the judgement may, if we like, be put in a form that exhibits predicate and copula separately, viz. 'it is coming'. It is true that this change of verbal expression may sometimes change the sense; it is not the same to say 'he plays the violin', and to say 'he is playing the violin'; we must say, 'he is one who plays the violin', or 'he is a violinist'. But it is clear that what the copula expresses is present as much in the proposition 'he plays the violin' as in the proposition 'he is a violinist'; just as it is present alike, whether I say *Beati immaculati in via* or *Beati sunt immaculati in via*. The inflexion of the predicate verb, or the inflexion of the predicate adjective together with the form and balance of the sentence, replaces or renders superfluous its more precise exhibition by the copula; which is, however, always understood, and if we set down the subject and predicate in symbols whose meaning is helped out by no inflexion, we naturally insert it. We symbolize the judgement generally by the form 'A is B'[2]; we may write it '$A\ B$', but that is an abbreviation; to write it '$A = B$' is an error.

If the copula thus expresses something present or implied in every judgement, what is its function, and can it be regarded as expressing one of three parts composing a proposition? Its function is to express that the subject and predicate are brought into the unity of a judgement: that the predicate is asserted of the subject, and that the subject is qualified by the predicate. I may think of rhetoric and I may think of trickery, but they may remain apart in my thought—subjects successively contemplated, like breakfast and a morning's work; if I say that 'rhetoric is trickery', I show that they are not unconnected, to my thinking, but that one qualifies the other.

Is the copula then a third member in the judgement, distinct from subject and predicate? Strictly speaking, no. For two terms are not subject and predicate, except in the judgement; and the act

[1] C. S. Calverley, *Lines on the St. John's Wood Omnibus*.

[2] Or 'A is not B', if the judgement is negative; and so elsewhere, *mutatis mutandis*.

of judging, whereby they become subject and predicate, is already taken into account in calling them subject and predicate ; it ought not therefore to be reckoned over again in the copula. In the verbal expression of judgement, which we call a proposition, we may distinguish as a third member a word showing that other words are subject and predicate ; but the whole proposition '*A* is *B*' expresses a single act, in which though we may distinguish subject and predicate from the predicating, we cannot distinguish them from it as we can from one another. To think the copula *is* the synthesis (or linking) of judgement : it is the form of the act, as distinguished from thinking the subject and predicate ; this is the matter, for judgement varies materially with variation of the subject and predicate. The copula is a word used to express the performance of that act.

Is it of any consequence how that act is expressed—(1) whether by an inflexion or by an independent word ; (2) if the latter, whether by the verb substantive or some different word or sign (such as the mathematical sign of equality) ?

(1) Every categorical judgement is analysable into subject and predicate ; in the act of judgement we affirm or deny their unity ; but, whether in affirming or denying it, they are distinguished ; and the predicate may in its turn become a subject of thought. The separation of the sign of predication from the predicate (as in the proposition ' He is a violinist ', compared with ' He plays the violin ') frees the predicate, as it were, from its immersion in the present judgement. If therefore we wish to set out a judgement in a form that shows clearly what is the subject, and what the predicate, each separately considered, an independent word is better, as a sign of predication, than an inflexion. For the purposes of a logical example, we should prefer to express a judgement in a form that shows this ; but it would be pedantry to do it, where, owing to the idiom of the language, it perverts the sense ; and we do not need to do it at all when we have no such need to extricate the predicate.

(2) Different languages agree to use the verb substantive, or verb of existence, as the sign of predication : *Homo sum*, I am a man : *Cogito, ergo sum*, I think, therefore I am.[1] The use of the verb of existence as copula suggests that every judgement *predicates*

[1] Propositions in which the verb of existence was predicate used to be called propositions *secundi adiacentis* ; and those which had some other predicate, where the verb *to be* was present or implied as copula only, were called propositions *tertii adiacentis*.

existence, that if I say 'government is a science', I declare not only that it is a science, but that it is or exists; on the other hand, the content of many judgements seems to negative this; for in saying 'a griffin is a fabulous monster', or 'Queen Anne is dead', I do not assert that a griffin or that Queen Anne exists. Hence some have boldly said that the verb 'to be' is a mere equivocal term employed sometimes to signify existence, and sometimes to signify predication: with no more identity of meaning in these two uses than there is between *est* = 'is' and *est* = 'eats'.[1] From this it would follow, that there is no special appropriateness in using the verb *to be* as sign of predication, rather than any other sign.

Yet if there were no special appropriateness in the verb *to be*, as the sign of predication, it is strange that so many languages should have agreed to use it. The case seems to be thus: that every judgement does imply existence, but not necessarily the existence of the subject of the sentence. The distinguishing characteristic of a judgement is, as we have seen, that it is true or false. With the false we need not here concern ourselves; for the man who makes a judgement, unless he says what he does not really think, says what he thinks to be true, and therefore intends to declare the truth. All judgements therefore, besides affirming or denying a predicate of a subject, implicitly affirm themselves as true.[2] But a judgement which affirms itself as true claims to express, so far as it goes, the nature of things, the facts, or the reality of the universe. In doing this it may be said to imply existence, not of its grammatical subject, but of the whole matter of fact asserted in it.

When I say that a griffin is a fabulous monster, I do not affirm that griffins exist like pigs and cows. But my judgement implies the existence of a mass of fable, in which griffins have their place as fables too. If there were no fables, I could not say that griffins were fabulous; but fables are an element in reality—i. e. in the totality of what is real—no less than pigs and cows. Again, when I say that Queen Anne is dead, I do not affirm the present existence of Queen Anne; I do imply her existence in the past; and the

[1] Cf. James Mill, *Analysis of the Phenomena of the Human Mind*, vol. i. p. 174 (ed. 1869); J. S. Mill, *System of Logic*, I. iv. 1.

[2] Cf. F. H. Bradley, *Essays in Truth and Reality*, p. 382: 'We cannot, while making a judgement, entertain the possibility of its error.' It may be noted that a lie is not a judgement, but rather an action intended, through the use of words that commonly express a judgement, to influence the action or opinion of others.

copula therefore still has the meaning of existence. It may be asked why it should be in the present tense, when the existence meant is past. The answer is, first, that the predicate corrects this so far as is necessary; but secondly, that the past (like fable) has a kind of existence. If I am the same to-day as I was yesterday, then I do somehow unite in me at once the present and the past; the past has ceased to be present, but it still somehow belongs to me. What is true of me is true of others, and of reality as a whole. Its history is in time; but it is one through that history; and the past belongs to it now, as well as the present. Queen Anne, it may be, does not exist now; but that exists now in whose past the life and death of Queen Anne have their place. They belong to the whole system of things which we call the universe; therein they exist, and only in belonging to it can they or anything else exist. The moon, if it had no place there, would not be; neither would justice, nor triangularity; though these different things play different parts in the whole.[1] When I say what triangularity is, the present tense is not used because it is contemporary with the time of the utterance; for it is not temporal at all. Not everything real belongs to the succession of events in time.

Every judgement then that I make claims to declare some portion of the whole truth that is to be known about the universe: in what form (so far as its purview goes) the universe exists. Hence it is no accident that the verb of existence is employed to express the act of judgement. There is a kind of thinking called questioning or wondering, in which we think of various things, and imagine them connected in various ways, without deciding in our minds whether they are so connected or not. Thus I may think of Public Schools, and ask myself whether they are liable to stifle originality in their pupils; and I shall be thinking also of that liability, and of the relation of subject and attribute, and imagining that relation to be exemplified between these terms. But if I judge one way or the

[1] Some writers have used the notion of a 'universe of discourse' or 'limited universe' to express the foregoing contention. In the whole universe fact and fable, savages and Rousseau's conception of savages alike have their place; but I can make statements which are true about Rousseau's conception which would be false about savages themselves. It is said that these are different 'limited universes'; and that propositions which do not assert the existence of anything in the material universe may assert it in some other. 'The royal dragon of China has five claws'—I do not affirm its existence in the universe of zoology, but in that of Chinese heraldic design. Cf. p. 44, n. 2, *supra*.

other, that public schools are or are not liable to stifle originality in their pupils, then I believe that this relation really holds, or does not hold, between these terms, and that what I think of exists independently of my thinking. And to express that a combination of which I think is real, I use the verb *to be*. 'Public schools *are* liable (or not liable) to stifle originality in their pupils'; i.e., the liability of public schools to do so, or their freedom from such liability, *exists*.

[It will be observed that on p. 164 the copula was said to *imply*, not to *predicate*, existence. For existence by itself is not a significant predicate, as we have already seen,[1] and therefore cannot strictly speaking be predicated. We may ask, for example, whether griffins exist, as we may ask whether ostriches fly; but whereas in the latter case the subject is assumed to exist, and the question is whether it possesses a certain predicate, in the former case we do not assume that there are griffins, and enquire whether they possess the predicate of existence. Their existence would consist in being griffins, and not merely in being; and to ask whether griffins exist is to ask whether anything existing has the character intended by the term griffin. The existent is thus assumed as the subject of our judgement, and the judgement claims to declare its nature; we do not assume its nature as a subject of which to predicate existence. Hence it has been said that reality is the ultimate subject of every judgement; that, as the distinction of its terms is not a distinction of two independent things, but of two factors in the being of one, this whole being, conceived by us in subject and predicate together, is really one 'content', and though judgements differ in their content, these contents are all predicated of the one reality; and the contents of all true judgements are factors co-existing in the being of that reality. To ask 'Is such and such a proposition true?' is to ask whether in its subject and predicate together I apprehend in part the nature of reality; and it is because of this 'reference to reality' in every judgement that we use in expressing it the verb *to be*.

This view that reality is the *ultimate* subject of every judgement is wrong if it be understood to mean that it is the *logical* subject, or be taken as destroying the force of the logical distinction between subject and predicate. We may distinguish in fact three subjects, the logical, the grammatical, and the ultimate or metaphysical. That the logical subject is not the same as the grammatical subject of the sentence is readily apprehended. The proposition 'Belladonna dilates the pupil' may be an answer either to the question 'What dilates the pupil?' or 'What do you know of belladonna?'

[1] Cf. *supra*, p. 65.

[In either case the grammatical subject is belladonna; but the logical subject is in the former case 'dilating the pupil'; that is what we are thinking about, and about that the judgement informs us that belladonna will effect it; in the latter case, the logical subject is belladonna, and about that the judgement informs us that it produces this effect. This distinction of logical subject and predicate is always present in thought when we judge, though sometimes the logical subject may be very vague, as when we say 'it rains' or 'it is hot'. But subject and predicate together may qualify something further. This is easily seen when the subject is an abstract term. 'Jealousy is a violent emotion': jealousy may be the logical subject here, but it only exists in those who are jealous. It is not then the ultimate subject, for it inheres in something else. Where then do we reach the ultimate subject? According to our ordinary way of thinking, in concrete individuals; and this is the view also of many philosophers, who have thought (and Aristotle seems to have been among them) that there was no single metaphysical subject, but as many as there are concrete individuals. In the *Categories*[1] the concrete individual is defined as that which can neither be predicated of nor inhere in anything further.[2]

But the doctrine which makes Reality the ultimate subject of every judgement holds that in a sense the metaphysical subject is always one and the same: i. e. that there can be only one real system, to which all judgements refer, and which they all contribute to determine and qualify. That a particular thing should exist or be real means that it has its place in this system; and what is called the existential judgement—the judgement whose predicate is the verb *to be*, in the sense of to exist—as in 'Sunt qui non habeant, est qui non curat habere', or 'Before Abraham was, I am'—declares a part of the nature of the one system of reality. The content of an existential judgement cannot indeed be predicated of reality as a quality or attribute. When I say that jealousy is a violent emotion, I think of it as an attribute of jealous

[1] ii. 1[b] 3–9, v. 2[a] 11–14. Cf. *supra*, pp. 50 sq.

[2] It is true that a singular term may appear as predicate of a judgement, as, for example, if we say 'The greatest epic poet is Homer' or 'The first man was Adam'. But in such a case Aristotle regards the predicate as only accidentally predicate, or *κατὰ συμβεβηκός* (cf. *Met.* Δ. vii): by which he means that the concrete individual does not really qualify or belong to what figures as its subject, but that because these two come together, or because it befalls Homer to be the greatest epic poet, and Adam to have been the first man, therefore you can say that one is the other, as you can also say that a grammarian is a musician when the two characters coincide in one individual, though 'musician' is not what 'being a grammarian' is, any more than Homer is what being the greatest epic poet is, or Adam what being the first man is. In fact, in making a judgement whose predicate is a singular term, we cannot help at the same time thinking of the predicate as qualified by what figures as subject. But cf. *supra*, p. 153.

[men ; when I say ' Est qui non curat habere ', I do not think of Horace as an attribute of reality. Nevertheless, his existence is bound up with the existence of the whole universe ; the universe of reality is found (when we think the matter out) to be presupposed by the existential judgement as much as by any other ; and though in it existence appears to be first affirmed in the predicate, and therefore not assumed in the subject, yet this cannot represent the true course of our thought. We could make no judgement at all, if we did not presume a reality about which it was made. Even the negative existential—' Joseph is not, and Simeon is not '—implies this ; for not to be means to have no place in that which is.

We are indeed accustomed to think of things and persons as if each were complete and independently real ; and in that case, the metaphysical subject of any judgement would be some concrete individual or other. The doctrine we are considering carries the question further, and holds that, since what is predicated of the concrete individual is not true of him in complete isolation from all else, therefore he is not, metaphysically speaking, or in the last resort, the subject of which it is true. There is no desire to deny to individuals a relative independence, or to pretend that the relation of attributes or universals to the concrete individual is the same relation as that of an individual to the system of reality which includes him. The judgement ' Jealousy is a violent emotion ' can be so restated as to make the concrete subject man the logical subject of the judgement ; I may express it, for example, by saying that jealous men are violent in their jealousy. I cannot so restate the existential judgement, or any other in which the logical subject is already a concrete term, as to make Reality the logical subject instead. But it is the metaphysical subject in the sense that it is presupposed and referred to even in those judgements. We cannot maintain the view that the metaphysical subject of every judgement is always in the last resort a particular individual. ' Civilization is progressive.' Doubtless civilization is only seen in the lives of men ; but it is seen in the lives not of this and that man singly but of the communities to which they belong. We have to think of men as forming a system and an unity, if we are to give meaning to a judgement like this. We saw too that the process of biological evolution, which seems in some way single, yet cannot be exhibited in any single organism ; nor is it easy to know what is a single organism. What is contended is, that all judgements involve us in the thought of one all-embracing system of reality, whose nature and constitution none can express completely, though each true judgement declares a part of it. Logic, as has been said before, cannot be rigidly separated from metaphysics ; indeed, it derives its chief importance from its connexion therewith. If it had merely

[to work out the scheme of syllogistic inference, and such-like matters, the problem which the present note has raised would be superfluous; but it investigates what is involved in thinking; and whether we must think of the universe as a sum of independent reals or as a system is a fundamental problem.[1]]

In the act of judgement, the subject[2] with which we start is thought of as modified or enlarged by the predicate, and in that form declared to be real. We end with the subject with which we began, differently conceived.[3] The thought of a combination of elements, and the affirmation of its reality,[4] are common features of every judgement, and the copula expresses them always, and so far has always the same meaning. Whatever sign be *used*, whether an inflexion, or the verb substantive, or the mathematical symbol for equality, or anything else, this combination, and the affirmation of its reality, must be *meant*. The verb *to be* naturally lends itself to this meaning. The mathematical symbol of equality has a different meaning; it is not a sign of predication, but an incomplete predicate; it expresses, of one thing, quantitative identity with some other. If I say $A = B$, the predicate is not B but 'equal to B': the special force of the sign ' = ' is 'equal to'; I must still perform in thought the act of predication, whether I say 'A is equal to B', or 'A is the first letter of the alphabet'; and if = were adopted as the sign of predication, the equation '$A = B$' (which means 'A *is* equal to B') must be written '$A = = B$'.

A judgement then contains subject and predicate; subject and predicate in their combination are declared real. To the words which signify the subject and the predicate separately is added

[1] The view that Reality is the ultimate subject of judgement is of course familiar to all readers of Mr. F. H. Bradley's or Professor Bosanquet's logical work. Cf. Bradley, *Principles of Logic*, c. i. pp. 12–14, and *Essays on Truth and Reality*, c. ix. pp. 253–254. Mr. Bradley does not distinguish between logical and metaphysical subject.

[2] i.e. the logical subject.

[3] Sigwart has pointed out that the movement of thought in a judgement is different for a speaker communicating information and for his hearer. The speaker knows the whole fact, when he starts putting forward one aspect of it in enunciating the subject, and supplements it with the other by adding the predicate: if I say 'This book took a long time to write', the whole fact is present to my mind in its unity before I begin speaking. To the hearer I present a subject of thought, 'this book', which awaits supplementation: to him the predicate comes as new information, which he has now to combine with the concept of the subject hitherto formed by him. v. *Logic*, § 5. 1.

[4] Even in a negative judgement, subject and predicate are elements thought of together, as standing in a relation of mutual exclusion.

a word which signifies that these are thought to be combined in the real. This word is called the copula ; it may be omitted in speech or writing, or be replaced by an inflexion ; but the act of thought which it indicates cannot be omitted, if there is to be a judgement. This act, however, is not a part of the judgement in the same way that subject and predicate are. It is the act or form of judging, and they determine the matter. Hence it is, at least generically, the same, while subject and predicate change ; and for this reason the scheme of a proposition '*A* is *B*' represents subject and predicate by symbols, but retains the 'copula' itself. We write *A* and *B* for subject and predicate,[1] because they represent indifferently any subject and predicate, being themselves none ; we write 'is', and not another symbol in its place, because whatever be the subject and predicate, the act of judgement is, generically, the same.

But judgements are not all so much alike that they can all be equally well expressed in propositions of the form '*A* is *B*' ; they do not differ merely as the places of these symbols are taken by different terms. For some propositions are of the form '*A* is not *B*'; and *A* may be replaced by a singular or by a general term ; and if by a general, we may judge either that all or some *A* is (or is not *B*), and this difference is one of form, in the sense that it is not a difference in the terms that replace our general symbols *A* and *B*. And there are other differences in propositions which are not differences in their terms. Having got some notion of what judgement is in general, we must now turn to the differences which are expressed in these differences of propositional form. With differences merely of the terms, as between 'men are animals' and 'roses are plants', we are not in Logic concerned.

[1] Of course any other indifferent symbols will serve, such as *X* and *Y* or *S* and *P*.

CHAPTER VIII

OF THE VARIOUS FORMS OF THE JUDGEMENT

JUDGEMENTS, or the propositions in which they are expressed, have for long been commonly distinguished according to Quality, Quantity, Relation, and Modality :—according to Quality, into affirmative and negative : according to Quantity, into singular, universal, and particular : according to Relation, into categorical, hypothetical, and disjunctive : according to Modality, into assertoric, problematic, and apodeictic. The distinctions in Quality and Quantity, as the simplest and most familiar, will be discussed first ; they can only be fully illustrated in categorical judgements or propositions.

In respect then of **quality**, categorical judgements are distinguished as *affirmative* or *negative*. An **affirmative** categorical judgement assigns a predicate to a subject ; a **negative** puts it from it. But the distinction between affirming and denying is too familiar to need and too simple to admit of being expressed in any other way, in order to indicate what is meant.

There are certain difficulties connected with negative judgements, which have already met us in dealing with negative terms. Judgement, as we have seen, refers to the existent, whose manner of being (so the judgement declares) is as we conceive. But the real is positive ; it only exists by being something, not by being nothing. A negative judgement declares what it is not, and how can this express it as it is ? *Dead-nettles don't sting.* How does that tell me anything real in dead-nettles ? You may say that I formed an idea of a stinging dead-nettle, and in the negative judgement declare it false, an idea of nothing real. But that only means that I had thought that, or asked myself whether, dead-nettles sting, and in correction or reply now judge that they do not. My 'idea' means my opinion, or a supposed opinion ; I may reflect on that, and say that the opinion is false ; but in the example I am judging about dead-nettles, not about any past opinion about them. And when I say that they do not sting, what am I saying about them ?

in them, what is this property of not stinging? surely, it may be urged, just nothing: so that in the negative judgement I assert nothing real.

These misgivings are sometimes, though unfairly, met by ridicule. Still, in face of them, we must assert, that everything finite is what it is, by not being something different: and at the same time, that it is not something different, in virtue of what it positively is. Hence we must accept the negative judgement as expressing the real limitation of things; but we must allow that it rests upon and presupposes the affirmative. If dead-nettles do not sting, there must be some characteristic which they do possess, incompatible with stinging.[1] There is always a positive character as the ground of a negation. Snow is not hot, because it is cold; this is not indeed an explanation of the temperature of snow; but it means that a material body (which must have some temperature) can only not have one degree of temperature through having another. If snow had no other degree of temperature, it would have 212° Fahr.; if it had none but 32° Fahr., it must have that. And it may be noticed how often in the building up of knowledge we use negative judgements to reach affirmative: to know what anything is not is frequently a help to discovering what it is. In the inductive sciences this procedure is constant, and we shall find it a fundamental feature of the induction in them.

To say that negative judgements presuppose affirmative does not however get rid of the difficulties to which we have referred. If snow is not hot because it is cold, then the cold is not hot. No one will deny that; some people will think it a mere tautological proposition. But it is not tautological, though it is superfluous. It is tautological to say that the cold is cold; to say that it is not hot because it is cold informs us that hot and cold are mutually exclusive attributes. Cold is no more identical with not-hot, than odd with not-even; though the numbers which are odd are the same numbers as are not even. The reciprocal exclusiveness of certain attributes and modes of being is the real truth underlying negation. But for

[1] A critic (Miss Augusta Klein) has objected that this is only a *negative* character, viz. the absence of glandular stinging hairs. But the tissues forming any part of a leaf can only *not* be glandular stinging hairs if they *are* something else. A body can only not be here if it is elsewhere. However, a difficulty arises with empty space; by being what, is it not occupied by some body? is emptiness purely negative? Democritus, and Plato, called space μὴ ὄν, not-being. Some have denied that a vacuum can exist.

that, everything would be everything else; that is as positive, as these several modes of being themselves.

Negation, as Plato saw,[1] is as necessary as affirmation, if there are to be any differences or discriminations within reality; that *A* is not *B* means that it is different from *B*, and not that it is non-existent.

[The further pursuit of this subject would take us too far into metaphysics. It may be pointed out in passing that the notion of an infinite (or, as philosophers sometimes say, an absolute) being is of a being who is everything that there is to be; of whom it cannot be said that he has one attribute by lacking another; whereas finiteness comes by limitation and exclusion: whence Spinoza's *Determinatio est negatio*. Whether this is a tenable conception is another matter. In particular it raises the problem of the meaning, and reality, of evil. For if an infinite being is all things, and evil is something real, he ought *inter alia* to be evil. It has been contended therefore that evil is in reality just nothing, a view against which there are obvious objections on the surface: or at least that it is a mere appearance incident to limitation, but in itself no more than limitation; what is absolute and all-inclusive, having nothing outside it to limit it, would not be evil, though it would include what, taken in improper isolation, appears evil.]

It has sometimes been proposed to treat the negative judgement, *A is not B*, as an affirmative judgement, *A is not-B*,[2] by combining the negative with the predicate. But inasmuch as the reciprocal exclusiveness of certain attributes and modes of being is a positive fact, it is no use trying to ignore it by a verbal manipulation. Nothing will make *A is not-B* an affirmative judgement, unless *not B* is something positive; and if *not-B* is something positive, say *C*, the judgement is true because *B* and *C* are counter-alternatives; e. g. the fact that the path of a bullet is not straight may be expressed by saying that it is a curve, but only because straight and curved are mutually exclusive and sole alternative determinations of a line. It follows that *C* is not *B*, and *B* is not *C*; and these negative judgements cannot be evaded by writing '*C* is not-*B*', '*B* is not-*C*'. For if *C* means the very same as *not-B* (e. g. *curved* as *not-straight*), then *not-C* means the very same as *not-not-B*, and the proposition

[1] *Soph.* 256 E περὶ ἕκαστον ἆρα τῶν εἰδῶν πολὺ μέν ἐστι τὸ ὄν, ἄπειρον δὲ πλῆθει τὸ μὴ ὄν. 257 B ὁπόταν τὸ μὴ ὂν λέγωμεν, ὡς ἔοικεν, οὐκ ἐναντίον τι λέγομεν τοῦ ὄντος, ἀλλ' ἕτερον μόνον. ('About each Form then there is much that it is, but an infinite amount that it is not. . . . When we speak of not being, we speak, it seems, not of what is contrary to being but only of what is different.')

[2] Such judgements, with an infinite term (cf. p. 42, n. 2, *supra*) for predicate, have been called infinite judgements.

B is not-C means no more than *B is not-not-B* ('straight is not-not-straight'). That however is absurd; for *C* is positive, and the consciousness of the distinction between it and *B* and of their reciprocal exclusiveness cannot be reduced to the consciousness that *B* cannot be denied of itself. The above argument could equally be illustrated if we took for *B* not one of two counter-alternatives, but a term like *dog*; only then *not-B* would leave us to select in the dark among a large number of still remaining alternatives.

In respect of **quantity**, categorical judgements are said to be either *singular*, or *universal*, or *particular*. But the differences at the bottom of this distinction are not in reality purely quantitative, though they have sometimes been represented as being so.

The subject of a proposition may be either a singular term like 'Socrates' or 'Caesar' or 'the present Cabinet', or a common term like 'man' or 'triangle'. In the former case, the proposition too is called **singular**. In the latter, the proposition may affirm or deny the predicate of the subject either universally, i. e. in every instance of it, e. g. 'All equilateral triangles are equiangular', 'Nemo omnibus horis sapit': in which case it is called **universal**; or partially, i. e. in particular instances, or of a part of the subject, only, e. g. 'Some larkspurs are perennial', 'Some animals cannot swim': in which case it is called **particular**. The judgements which these propositions express[1] are correspondingly distinguished as **singular, universal, or particular.**

Now these three kinds of judgement may clearly be represented as concerned respectively with one individual, with all individuals of a certain kind or description, or with some part of such aggregate or class. For though when I say that all acids contain hydrogen, or that some larkspurs are perennial, I may be thinking primarily of the kinds or species of acid, or of certain species of larkspur, yet the statements, if true, are true in every instance of those species.[2]

[1] We judge, commonly, not about words but about what they stand for, but we express our judgements in words. A common term stands for and is predicable of not a common nature in things, but things in respect of their common nature. These things are the subject of the judgement, when a common term is the subject of a proposition.

[2] i. e. of the whole or part of the *denotation*, as well as of the whole or part of the *extension* of the subject-term, if the distinction made on p. 146, *supra*, be adopted. It should be remembered that the singular term has no extension; and that an individual cannot be called the *whole* denotation of a singular term in the same sense in which the divers individuals of a class can be called the whole denotation of a general or class-term.

And so they may be represented as concerned with all or part of what their subject terms denote. And as a singular term denotes only one individual, the singular proposition is also concerned with all that its subject-term denotes. Hence it has sometimes been said that propositions are of *two* kinds in respect of quantity, universal when they refer to the whole denotation of the subject-term, particular when they refer to part of it. We shall see later, when dealing with syllogism, that in some connexions it is unnecessary to distinguish between singular and universal judgements or propositions, because they both equally make certain inferences possible. But at present it is important to realize that what are called differences of quantity in judgements or propositions, are not primarily differences in respect of *how much* of the denotation of the subject term is the subject of our thought.

The subject of a singular judgement is individual (though it may be an individual collection); that of an universal judgement may be an universal, or concept, e.g. 'Fear is contagious'; or, though not a concept, it may be determined by a concept,[1] e.g. 'Letters in transit are the property of the Postmaster-General.' The latter statement, though it concerns individual letters, applies to them not as this or that individual, but as possessing the character signified by the words 'letter in transit'. The difference therefore between it or the former and a singular judgement lies not in the quantity of the individuals to which they refer (i.e. in the singular referring to one individual and the universal to all individuals of a certain collection), but in the logical character of the subject, which in the singular judgement is a determinate individual, in the universal judgement a concept or anything characterized and determined by a certain concept. We may include both these in the expression 'a conceptual subject'. No doubt an universal judgement has a quantitative aspect, for it does concern all individuals that share the subject-concept; but this aspect is secondary. Primarily, in making it, we have before us a relation between one character and another in individuals, not between individuals and a certain character. Neither therefore is the difference between an universal and a particular judgement primarily quantitative. A particular judgement refers to part only of the denotation of some conceptual subject, an universal to all; but this is because in the latter the

[1] The totality of things exhibiting a certain character is called a class, and the character which determines membership of the class a class-concept.

relation of concepts is taken to be necessary, and therefore the subject-concept sufficiently determines the application of the judgement; in the former it is not, and we indicate by the word *some* that the application of the judgement is not completely determined.[1] A criticism of the forms in which language expresses judgements of these different types will throw further light on what has just been said.

It is common to indicate an universal judgement by the words *all* or *no* (*none*) prefixed to the subject, according as the judgement is affirmative or negative; a particular judgement by the word *some*, similarly prefixed; these are called signs or marks of quantity. The idiom of language will indeed often express a universal judgement in other ways; we can say *Man is mortal*, as well as *All men are mortal*: *A barometer will not work in a vacuum*, as well as *No barometer will work in a vacuum*. But in the absence of a mark of quantity, it is not always clear whether a proposition is meant to be universal or particular; if I say *Women are jealous*, *A flower is a beautiful object*, I need not mean all flowers, or all women. Precision requires the quantity of a judgement to be expressly indicated: particularly where (as in logical examples) the proposition is taken out of context and we lack the help which context often affords us in divining the writer's intention: and at least where the subject is in the plural,[2] the words *all*, *none*, *some* are appropriated to that service. A proposition without any mark of quantity is technically known as an **indefinite** proposition; because it is not clear whether the whole, or only a part, of the extension or denotation of the subject is referred to, and so the scope of the proposition is undetermined: the examples just given,

[1] The Aristotelian division of political constitutions (or rather Platonic—for it occurs in Plato's *Politicus*) is another example in which differences not really quantitative have been presented under a quantitative form. A monarchy, an aristocracy, and a democracy, though said to differ according as power is in the hands of one man, of the few, or of the many, really differ, as Aristotle himself pointed out, in quality or kind. It must be added that Aristotle does not put forward a purely quantitative division of judgements (cf. *de Interpr.* vii. 17^a 38 ἐπεὶ δ' ἐστὶ τὰ μὲν καθόλου τῶν πραγμάτων τὰ δὲ καθ' ἕκαστον—'since of things some are universal and some several'), though in expounding the syllogism in the *Prior Analytics* he often lays stress on the quantitative implications of the contrast between universal and particular judgements.

[2] 'Man is mortal' is clearly universal; but represented in symbols as '*A* is *B*' it will not unambiguously show its universality. For 'Iron is found in Lancashire' might be represented by the same symbols, but is as clearly particular.

Women are jealous, A flower is a beautiful object, are therefore indefinite propositions.

At the same time, the words *all* and *none*, as signs of the universality of a judgement, have disadvantages of their own. For a judgement is really universal, when the subject is conceptual, and the predicate attaches to the subject (or is excluded from it) necessarily; but if it is found to attach to the subject (or to be excluded from it) in every existing instance without any necessity that we know of, we use the same expressions, *all* and *none*. Thus we may say that *No American poet stands in the first rank*, or that *All the French ministries are short-lived*; but neither of these is really an universal proposition. Each expresses a judgement made about a number of individuals: it states an historical fact, and not a scientific truth. It would be convenient to call such propositions **collective** [1] or **enumerative**; for they really collect in one the statements which may be made about every instance of a certain class, and make their assertion on the strength not of any conceptual necessity, but of an enumeration.

We must of course distinguish the question whether a proposition is meant as universal, in the strict sense, from the question whether we have a right to enunciate it universally. If instead of saying *All the French ministries are short-lived* (where the article *the* shows that I am referring to all of a certain number of things), I were to say *All French ministries are short-lived*, it might be contended that the proposition no longer referred primarily to individuals or instances, but affirmed a necessary character of French ministries as such. In truth the statement is not clear, and a man would have to ask me, whether I meant it as an historical summary, or an universal truth; but the ambiguity of the statement is the very point to be noticed; for the two interpretations indicate the difference between a merely enumerative, and a true universal, judgement. The difference is plain in suitable examples: contrast, for instance, 'All, all are gone, the old familiar faces', and 'All lovers young, all lovers must, like chimney-sweepers, come to dust.'

We have seen that there is a marked distinction between a singular judgement, whose subject is an individual, and an universal or particular judgement, whose subject is conceptually determined by a general or abstract term. The enumerative judgement (and

[1] Cf. Bradley, *Principles of Logic*, Bk. I, c. ii. §§ 6 and 45. In the Table of Contents he speaks of 'collective' judgements in this sense.

this is true in some degree of the particular also) approximates to the type of the singular rather than of the universal.[1] For though the subject of the proposition be a general term, and I predicate about all the members included under that term, yet I do so because I have examined them severally and found the predicate in them all, or at least, on good evidence or bad, believe it to attach to them all, not because of any necessary connexion between the predicate and the common character of these individuals which the general term signifies. *French ministry* is a general term; but (for all that I see) it is not because being a French ministry involves being short-lived, that all the French ministries are short-lived; I assert it because I have noted each case; just as it would be upon the strength of noting the individual case that I should assert the first ministry of M. Jules Ferry to have been short-lived. At the same time, the enumerative judgement, though thus approximating to the type of the singular, gives the hint of a true universal judgement. It suggests that the ground for the predicate may lie in the common character signified by the general term under which all these instances are collected. If I say *Luther was hated*, there is nothing to indicate what about him was hateful: with which of all the coincident attributes in Luther his hatefulness is universally connected. If I say *All reformers have been hated*, though that is as much an historical statement as the first, and therefore enumerative only, it suggests that the reason why all those men have been hated (Luther and Calvin, Cromwell and Gladstone —the statement implies a possible enumeration) lies in the fact that they were reformers. Thus from an enumerative judgement we may pass to an universal; from a study of individuals to the assertion of an universal connexion of characters. When we enunciate enumerative judgements, we are on that road: sometimes farther, and sometimes less far.

The difference between a true universal judgement and one merely enumerative is exceedingly important. The one belongs to science, the other to chronicle or history. An universal judgement concerns any and every instance, alike past, present and future, examined or unexamined. An enumerative judgement concerns only those instances which have been examined, or have existed, and which are summed up in the subject. *All reformers are hated*: if that is merely enumerative, it does not require me to

[1] Cf. Bradley, *Principles of Logic*, Bk. I. c. ii. § 45.

anticipate hatred if I undertake reform; it affords me no explanation of the hatred with which these men have been met. But if it is a true universal, it explains the past, and predicts the future. Nevertheless an universal judgement has nothing, as such, to do with *numbers* of instances; if the connexion affirmed in it be necessary, the judgement is still universal, whether there be a million instances of its truth, or only one [1]; so that the form 'All *A* is *B*' hardly does justice to it. An enumerative judgement contemplates a number of instances, and refers to all of them; and the form 'All *A* is *B*' or 'All the *A*'s are *B*' expresses it adequately.

The particular proposition may be interpreted as referring either to individuals not enumerated or to an universal not fully determined; and it will approximate more to the enumerative, or more to the universal, accordingly. If I say *Some women have ruled kingdoms*, I mean women whom I could enumerate—Semiramis, Cleopatra, Zenobia, Elizabeth, Christina, &c.: not women of such and such a type, but this and that woman. If I say *Some pigments fade*, I do not mean pigments that I could enumerate, but any pigments of a certain kind; and supposing that I could specify or determine the character of pigment, I could say that all pigments of that character fade. There is nothing in the verbal form of a particular proposition to show whether the speaker is thinking rather of individuals whom he does not name, or of conditions which he does not specify; though content and context will often guide us on this point.

It will be readily seen that there is the same sort of difference between the particular proposition interpreted of individuals not enumerated, and the particular proposition interpreted of conditions not fully specified, as exists between the enumerative and the true universal proposition. If the women vaguely referred to as *some* were enumerated, I could say *All the women on my list have ruled kingdoms*; if the pigments vaguely referred to as *some* were characterized, I could say *All such pigments fade*. The former is the enumerative, the latter the universal All. And this difference, whether between the two interpretations of the particular proposition, or between the enumerative and the universal, may be expressed by saying that in

[1] Or, as some logicians would add, none. Such a view makes the universal judgement, however, purely hypothetical: cf. Leibniz, *Nouveaux Essais*, IV. xi. 14; Bradley, *Principles of Logic*, Bk. I. c. ii. §§ 43–6; Bosanquet, *Logic*[2], vol. i. pp. 263–266; *v.* also Bradley, *Appearance and Reality*, p. 361.

the one case the subject of the proposition is interpreted in extension or denotation, in the other case in intension. The subject of a proposition is interpreted in extension or denotation, when we are thinking primarily of the various species or individuals included in the subject to which the predicate refers; it is interpreted in intension, when we are thinking primarily of the subject as of a certain kind, of the character implied by the subject-term, with which the predicate is connected. 'Some *A* is *B*' is interpreted in extension or denotation, if I think of this, that and the other *A*: in intension, if I think of *A*'s of a certain character. 'All *A* is *B*' is interpreted in extension or denotation, if I think of every one of the *A*'s: in intension, if I think of the character of *A*'s as such.

What has been said on the quantity of judgements and propositions may be summed up as follows. Categorical judgements may be made about either individuals or universals. If about individuals, these may be indicated either by a proper name or designation—and then the judgement is called singular—or by a general term. In the latter case, if the judgement concerns all that is included in the extension or denotation of its subject-term, it is called universal; but a distinction must be made between a true universal judgement, in which the predicate is affirmed (or denied), without respect of individuals, of whatever exhibits the subject-concept (or intension of the subject-term) and one only enumerative or collective, in which it is affirmed or denied of all of certain species or individuals, which might be enumerated, but which the subject-term enables us to indicate collectively. If the judgement concerns an unspecified part of the extension or denotation of the subject-term, it is called particular. Judgements about an universal are universal. Propositions are denominated after the character of the judgement which they express. A true universal judgement can be indicated by the same words (*All* and *None*) as an enumerative, and is often confused with it. A particular judgement is really incomplete; it may be an incomplete enumerative, or an incomplete universal judgement, according as we think rather of the instances we imperfectly denote, or the conditions we imperfectly specify, in the subject. We make particular judgements chiefly in opposition to the universal statements of others, to which we 'take exception', or in approach towards universal judgements; and their value for knowledge is subsidiary and instrumental. The subject of a categorical proposition may be viewed primarily in intension,

it the proposition affirms or denies a connexion of characters, or in extension or denotation, if it affirms or denies a certain character in individuals.[1] The former aspect predominates in the universal, the latter in the enumerative: in the particular, sometimes the former and sometimes the latter, according as we think more of the conditions imperfectly specified, or the instances imperfectly denoted: the singular proposition merely affirms or denies in an individual a certain character.[1] Sometimes these distinctions, though we are conscious of them in our thought, are not expressed in language; and for certain purposes of inference, it is enough to consider propositions simply as either universal or particular: universal, when the whole extension or denotation of the subject-term or when an individual is referred to, particular when a part of the extension or denotation is referred to only.

Judgements are distinguished according to **relation** into *categorical*, *hypothetical*, and *disjunctive*. We have been considering hitherto categorical judgements. A **categorical** judgement merely affirms or denies a predicate of a subject: *dogs bark*, *dead men tell no tales*. An **hypothetical** judgement connects a consequent with a condition which it does not, however, assert to be fulfilled: *if money is scarce, the rate of discount rises*. The condition is called sometimes the antecedent (in grammar, the protasis), as what is connected with it is called the consequent (in grammar, the apodosis). A **disjunctive** judgement affirms alternatives: *bees are either male, female, or neuter*.[2] The hypothetical judgement is sometimes called *conjunctive*, as conjoining the truth of the consequent with that of the antecedent: while the disjunctive disjoins the truth of one alternative from that of the others.[3] Both are sometimes called complex judgements, in contrast with the categorical, which is called simple.

In an hypothetical judgement, the antecedent and consequent may have the same, or different, subjects or predicates: the scheme of the proposition may be either 'If A is B, it is C' (*If corn is scarce, it is dear*), or 'If A is B, C is D' and (*If the dead rise not, we are of*

[1] The singular proposition whose predicate is a proper name does not assert a *character* of its subject.

[2] For any given bee, these are alternatives: for bees collectively, they are three forms which are all realized: cf. p. 188.

[3] The term *hypothetical* has also been used by some generically, with *conjunctive* and *disjunctive* to denote the species.

all men most miserable), or 'If *A* is *C*, *B* is *C*' (*Si tu et Tullia valetis, ego valeo*). Again, antecedent and consequent may be either negative or affirmative: but these differences make no difference to the character of the judgement as hypothetical: it still affirms the dependence of a consequent on a condition: hence the alternative of affirmative and negative, though applying to the antecedent and consequent severally, does not apply to the hypothetical judgement as a whole.

It is sometimes said that hypothetical propositions can be reduced to categorical. So far as the verbal form is concerned, this can often be done. 'If *A* is *B*, it is *C*', in which antecedent and consequent have the same subject, may be written '*A* that is *B* is *C*'; *If corn is scarce, it is dear* becomes *Scarce corn is dear*: *If that dog is teazed, he bites* becomes *That dog bites when teazed*. Even where antecedent and consequent have different subjects, a little ingenuity may produce from the hypothetical a proposition categorical in verbal form; *If no war is toward, the temple of Janus is closed* might be written *The temple of Janus is closed in peace time*: *If men are not free, blame is unjust* might be written *Men who are not free are not justly blamed*. But whether a judgement is hypothetical or categorical cannot be determined merely from the verbal form of the proposition in which it is expressed. The hypothetical judgement asserts the qualification of the subject by the predicate of the consequent to be dependent on a condition expressed in the antecedent; as has been said, it does not assert this condition to be fulfilled. But where this condition is known to be at times fulfilled, it may still be expressed by an *If* (as well as by a *When* or *Whenever*); and if it can also be expressed as a qualification of the subject or predicate of the consequent, then that predicate may be asserted of the subject so qualified, or the predicate so qualified of that subject. Now the first three of the above examples are of this sort. Corn is sometimes scarce, that dog is sometimes teazed, Rome is sometimes (though rarely) free from war. And the proposition *If corn is scarce, it is dear*, regarded as a statement about scarce corn, must be ranked as categorical, notwithstanding its form; while *Scarce corn is dear*, regarded as a statement about corn whose scarcity it leaves in doubt, must be regarded as hypothetical. We are so well aware that corn is from time to time scarce, that we naturally interpret categorically in this instance. But in an argument containing the proposition *Men who are not free are not justly blamed* we

might see that it was not intended to deny that all men are free, but only to point out a consequence that would follow from denying it; and then the judgement remains hypothetical. Otherwise, and if taken as implying the existence of men not free, the categorical proposition is not the equivalent of an hypothetical in which their existence is not implied.[1] The reduction to categorical form is only justified when the hypothetical proposition is meant merely to affirm a connexion of one character with another existing in a given subject or in all subjects of a certain kind.

The difference between the two types of judgement—between affirming or denying a predicate of a subject, and affirming the dependence of a consequent on a condition not asserted to be fulfilled—becomes clear where the judgement concerns an individual situation, and particularly if it contains an unfulfilled condition, in past or future time. *If he is insane, he cannot make a will* implies, no doubt, a connexion between insanity and testamentary incapacity, but not between the individual and either. *If I had served God as diligently as I have done the king, He would not have given me over in my grey hairs:* no doubt this implies the categorical judgement *God does not forsake those who serve Him diligently*; but it cannot be reduced to this; for it implies also *Therefore He would not have forsaken me, if I had served Him diligently*; and we cannot eliminate this hypothetical judgement. Κροῖσος Ἅλυν διαβὰς μεγάλην ἀρχὴν καταλύσει, *If Croesus crosses the Halys, he will ruin a great power*[2]; here it is not stated whether Croesus will cross the river or not; so that, as the fulfilment of the condition upon which the event in the consequent depends is left in doubt, there is nothing but a dependence categorically asserted.

It may be urged that, as this at least is asserted categorically, the hypothetical judgement is categorical after all. And against any one who attempts to abolish the distinction between the two kinds of judgement by saying that all judgements are really hypothetical, it is a good answer to point out that the hypothetical thus involves the categorical. But that does not invalidate the distinction between them; for the distinction rests on the difference between asserting

[1] The form 'Men who are not free *would not be* justly blamed' retains in the *would* the expression of hypothetical judgement, and cannot be regarded as a categorical proposition.

[2] More literally, *Croesus by crossing the Halys will ruin a great power*, which might be taken to mean that Croesus will cross the Halys and ruin a great power. So taken, the oracle is categorical; and the line well illustrates how the grammatical form is no sure guide to the logical character.

a dependence of consequent upon condition not asserted to be realized, and asserting (affirming or denying) a predicate of a subject. If it be granted that the hypothetical judgement asserts the former, though it does so categorically, yet it differs from the categorical.

It has been said [1] that the very reason just given for maintaining the essential difference of these two types of judgement excludes the consideration of that difference from Logic. For both assert; they differ in what they assert; the difference is therefore in the matter and not the form of judgement. We have the same form, *A is B*, whether for *A* we write *Croesus*, and for *B a king of Lydia*, or for *A the destruction of a great power*, and for *B must follow on Croesus crossing the Halys*. But it will be readily admitted that the distinction between categorical and hypothetical assertion is formal in the sense that it is illustrated in our thought about all kinds of subjects; and to exclude it from Logic on the ground that, as compared with the common form of assertion in both, it is material, only shows the impossibility of making Logic a *purely* formal science. It is claiming to consider the genus, and refusing to consider the species: a procedure which would be tolerated in no other subject, and cannot be tolerated in Logic.

[There is however a difficulty about the meaning of saying that a consequent depends on a condition, when that condition is unfulfilled. *If Hannibal had marched on Rome after Cannae, he would have taken it.* This proposition makes an assertion; in doing so, it asserts something about the real, for it claims to be true. But what does it assert about the real, and what historical fact (as we may put it in such an instance) does it affirm? Not that Hannibal marched on Rome after Cannae, for he did not; nor that he took Rome, for he did not; nor therefore that one event was due to the other, for neither happened. How then can we say that one depended on the other? In the sense, it may be answered, that if he had marched on Rome at that time, he would have taken it. But this is the original proposition whose meaning we are trying to discover. And it does not state a fact in Hannibal's history, or in the history of Rome, but what is called an unfulfilled contingency; and how can that be asserted of the real? Every hypothetical judgement presents this problem. For its truth does not require that either condition or consequent be realized, and yet, if true, it is true of reality; and reality, we may urge, is actual. What then does it affirm to be actual in the real? Mr. F. H.

[1] Cf. Mansel, *Prolegomena Logica*, pp. 232, 251.

[Bradley [1] replies that it ascribes to reality a character which is the *ground* of the connexion stated in the hypothetical judgement. Rome *was* in such a state that it could not have resisted Hannibal after Cannae. This is true ; but it still leaves us with the question, How can there be the ground in the real universe of something which nevertheless does not happen ? Or we may put the problem a little differently by asking how there can be a dependence between a consequent and a condition that do not exist.

Professor Cook Wilson holds that an hypothetical proposition affirms the dependence of the solution of one problem upon the solution of another. 'And they sent the coat of many colours, and they brought it to their father ; and said, This have we found : know now whether it be thy son's coat or no. And he knew it, and said, It is my son's coat ; an evil beast hath devoured him ; Joseph is without doubt rent in pieces.' [2] Here the hypothetical proposition is implied, *If this is thy son's coat, he has been killed* ; and this means that the determination in the affirmative [3] of the question whether it is his coat involves the like determination of the question whether he has been killed. Where the condition is an unfulfilled condition in past time, this dependence of the solution of one question on that of another is affirmed not to exist now, but to have existed. The determination of the question whether Rome would fall directly after Cannae did depend on that of the question whether Hannibal would march on Rome. And such a proposition implies also the assertion that the condition was not fulfilled.

What Professor Cook Wilson points out seems true and important. An hypothetical proposition does assert the solution of one problem to be connected with the solution of another. But the particular difficulty before us is not removed by that doctrine. For neither problem need be solved or by us soluble, and yet the hypothetical proposition may be true, as in the instance *If the* 'Phaedo' *is historical, Socrates believed in the immortality of the soul.* Now our difficulty concerned the affirmation that a ground exists for what yet does not happen, or that there is a connexion between terms which yet do not exist. And there is the same difficulty in asserting one solution to depend on another, when neither is made. It concerns the meaning of saying that something is possible, which is not actual.

It is a partial answer to say that connexions, or principles of connexion, exist in a different way from the particular things and events in which they are displayed or illustrated. They *are*, even when they are not *exemplified*. 'If you put a match to that powder,

[1] *Principles of Logic*, Bk. I. c. ii. §§ 50–53. [2] Gen. xxxvii. 32, 33.

[3] The determination in the negative of the question in the antecedent leaves the question in the consequent undetermined, unless the antecedent is the sole condition of the consequent. Cf. *infra*, p. 338.

[it will explode.' Why ? because of a connexion between rise of temperature and detonation in compounds of a certain kind, which is not dependent for its being upon the actual process of those changes. The explosion then is possible, because there are certain connexions, and some of the things, given which these connexions are exemplified, exist These connexions, which somehow are, even though the conditions for their display do not exist, are the 'ground' which the hypothetical judgement 'affirms of the real' But the connexions whose affirmation is implied need not be such as are repeatedly illustrated. Sometimes the reason why the solution of one problem carries the solution of another lies in principles of connexion displayed in situations that are repeated ; so it is with the connexion between rise of temperature and detonation in gunpowder. But sometimes the conditions are apparently unique, and we cannot resolve them into an assemblage of repeatable elements ; so it is often with complex historical situations where all seems to turn on the action of a great personality. And in some hypothetical propositions the connexion between the solutions of two problems seems to be the only connexion affirmed, as in the instance already given. 'If the *Phaedo* is historical, Socrates believed in the immortality of the soul' ; though doubtless this implies an assertion of certain particular facts about that dialogue of Plato.[1]]

The disjunctive judgement may be expressed schematically in the forms '*A* is either *B* or *C*' (*Every man at forty is either a fool or a physician*), 'Either *A* is *B* or *C* is *D*' (*He either fears his fate too much, Or his desert is small*,[2] Who dares not put it to the touch, To gain or lose it all), 'Either *A* or *B* is *C*' (*Either the Pope or the King of Italy should retire from Rome*). As the hypothetical judgement always affirms a dependence of consequent on condition, so this

[1] The reader must not suppose that these paragraphs deal at all completely with the problems raised by hypothetical judgement. Nothing, for example, has been said about distinctions of quantity in them. It has been urged by some that they are all universal. But though without necessary connexion in the real no hypothetical judgement would be true, such implied connexion may be remote from the actual hypothetical judgement made. Again, some hypothetical judgements are concerned with certain individual consequents and conditions, some with any of a certain kind ; or the condition may be of the former sort and the consequent of the latter, or vice versa. These differences however are not of first-rate importance.

[2] This might be equally expressed 'He either fears his fate too much, or deserves little' : indeed in sense the alternative predicates are predicated of the same subject, not (as in the proposition *Either Tacitus was a slanderer or Tiberius a villain*) of different subjects. This affords another example of the fact that the logical character of a judgement cannot always be inferred from the grammatical form of the proposition.

always affirms a disjunction, whether the alternatives themselves be given affirmatively or negatively. So far as the nature of the disjunction goes, there is no difference between '*A* is either *B* or *C*', and '*A* is either not *B* or not *C*': between 'Either *A* is *B*, or *C* is *D*', and 'Either *A* is not *B*, or *C* is not *D*': between 'Either *A* or *B* is *C*', and 'Either *A* or *B* is not *C*'. But it should be noted that 'Neither . . . nor' is no disjunction at all, but a conjunction of negations. On St. Paul's voyage to Rome 'neither sun nor stars in many days appeared'; there is no choice between alternatives here, but two statements—the sun did not appear, and the stars also did not.

There may be any number of alternatives in the disjunction; but that clearly does not alter the character of the judgement.

It is not always clear in a disjunctive proposition whether the alternatives offered are meant to be mutually exclusive. If *A* is either *B* or *C*, then it cannot be neither; but may it be both? The question concerns the right interpretation of a form of speech, rather than the nature of disjunctive judgement. Sometimes from the nature of the case we may know that the alternatives exclude each other: as if we are told that Plato was born either in 429 or 427 B.C. Where this is not so, it is perhaps safer to assume that they are intended as mutually exclusive, unless the contrary is stated; a legal document is careful so to write it, where '*A* or *B* or both' is meant, or to write '*A* and or *B*' with that signification.

If has been suggested that the disjunctive judgement is in reality a combination of hypotheticals; that '*A* is either *B* or *C*' means 'If *A* is not *B*, it is *C*; if *A* is not *C*, it is *B*; if *A* is *B*, it is not *C*; if *A* is *C*, it is not *B*'. Doubtless these four propositions are involved (supposing *B* and *C* to exclude each other): but we do not therefore get rid of the peculiar nature of the disjunctive judgement. For they are not four independent hypothetical judgements; and their force is not appreciated, unless it is seen that together they make up a disjunction, that they offer us a choice between alternatives. Thus disjunctive judgement at once includes and goes beyond hypothetical, in the same sort of way as hypothetical judgement includes and goes beyond categorical. An hypothetical proposition makes an assertion, like a categorical; but what it asserts is a relation of a consequent to a condition. A disjunctive proposition involves hypotheticals, which it presents as true together, but it asserts the truth of one (or, if they are not mutually exclusive,

of at least one), without specifying which one, among alternative categoricals.

The disjunctive judgement also raises a metaphysical problem, when we ask what real fact corresponds to it. 'Plato was born either in 429 or 427 B.C.' cannot state the actual fact about Plato: he was born definitely in one year, not merely in one or other; it is because *we do not know* in which, that we state an alternative, and there was no alternative in the event. Here, therefore, the disjunctive proposition seems rather to express the state of our knowledge, than the state of the facts. On the other hand 'Number is either odd or even' seems to express a disjunction in the facts[1]; and the species of the same genus are a kind of real disjunction. If a colour is to exist, it must be blue, or red, or some other colour, and if it is one, it can be none of the others. We come back here upon the same truth which met us in considering negative judgements, that a thing is definitely this or that by not being something else; we have to recognize also that there is often a limited number of possibilities, in the way, for example, of colour, or of animal species, but why or how there should be a limit to what is possible in the universe is a hard question.[2]

We come next to the distinctions of **modality** in the judgement. In respect of modality, categorical judgements are distinguished as *assertoric*, *problematic*, and *apodeictic* (or *necessary*); the first is sometimes opposed as *pure* to the other two as *modal*; but we shall find that if judgements are divided into pure and modal, the assertoric can be regarded as a form of modal judgement. Propositions of the form '*X* is *Y*', '*X* is not *Y*' are **assertoric**—'the train is late', 'the train is not late'; of the form '*X* may be *Y*', '*X* may not be *Y*', **problematic**—'the train may be late', 'the train may not be late'; of the form '*X* must be *Y*', '*X* cannot be *Y*', **apodeictic**—'the train must be late', 'the sun cannot be late'. The distinctions are also expressed by adverbs: *X* actually, possibly, necessarily is (*or* is not) *Y*.

In the sense of the word to which we have so often called attention, these distinctions are clearly logical: i. e. they belong to no

[1] Of course there is a disjunction in the facts, in the former case as well, so far as that the 429th and the 427th years from any point of time whence we choose to begin our reckoning are distinct years.

[2] For the fuller treatment of this form of judgement also the reader is referred to more advanced works.

special science, but recur in our thought about all kinds of subject. Whatever X and Y may be[1], we may find ourselves asserting that X is, that it may be, or that it must be Y.[2] But their logical character is specially manifest in this, that they raise a fundamental question about the nature of the thinking activity, viz. that of the difference between opinion and knowledge[3], just as the distinction of judgements according to quality raises the question of the difference between affirming and denying. And as the latter difference cannot be reduced to a difference in the predicate affirmed, by combining the negative with the predicate, so neither can the former. Still, we found a ground for the existence of the two 'qualities' of judgement in a certain fact about the being of things, viz. that each is positively what it is by exclusion of all else, by difference. It is not so easy to find a ground for the existence of the 'modalities' of judgement in the being of things.

Let us take three judgements differing in modality and expressed in propositions of the form 'X is Y', 'X may be Y', 'X must be Y'—'the train is late', 'the train may be late', 'the train must be late'. We can express the same judgements by saying that the train is actually, or possibly, or necessarily late. But it is clear that we have not here three judgements with the same subject, *the train*, and different predicates, *actually late, possibly late, necessarily late*; for those are not three kinds of lateness. The modality of a judgement cannot be something qualifying its predicate. 'Norman mouldings were possibly coloured': 'Norman mouldings were actually coloured'; the adverbs do not express a mode of colouring, as if we said that the mouldings were brilliantly coloured, or coloured blue. 'Water runs down hill': 'water must run down hill'; these are not different ways of running, like running fast or running slowly. Grammarians tell us that adverbs qualify verbs and adjectives, but adverbs of modality seem to be an exception.[4]

[1] Except so far as in some subjects, like arithmetic, a judgement is nearly always made with consciousness of its necessity: cf. *infra*, p. 196. Even here however I might say, before I had made the calculation, that 37596 may be a square number.

[2] For the sake of brevity, I shall not throughout consider negative as well as affirmative judgements. It should be noted that the problematic affirmative 'X may be Y' is not contradicted by the problematic negative 'X may not be Y', but by the apodeictic 'X cannot be Y': and similarly the problematic negative by the apodeictic affirmative.

[3] Cf. p. 160, *supra*.

[4] Unless indeed they qualify the copula, the verb *to be*, as some have said. Cf. next page.

Again, it is not the judgement, in the sense of the act of judging, that the modal words qualify; if I judge 'the train may be late', my judging is actual; it is the lateness of the train that is possible. That, however, as we have just seen, does not mean that its lateness is a certain sort of lateness, as if we said that the lateness of the train is scandalous.

Once more, we cannot say that the modal words qualify the matter judged. I judge that the train may be late, or that the window may be open; the judgements have the same form, which I can express symbolically in the formula 'that *X* is *Y* is possible'; the assertoric and apodeictic may be similarly expressed—'that *X* is *Y* is actual', or 'necessary'; or more compendiously, instead of the words 'that *X* is *Y*' I can write '*X Y*'. But *X Y* is certainly not the matter judged; for when I judge it possible that the train is late, I do not judge *X Y*, that the train is late, at all. The matter judged is that which is judged to be, the subject qualified by the predicate. *X* and *Y* are not the subject and predicate in these three judgements, as indeed the formulae in which the modal words are predicates indicate. The affirmative and negative judgements, '*X* is *Y*' and '*X* is not *Y*', can have the same subject and predicate, but differ in quality; so we are apt to speak as if the assertoric, problematic, and apodeictic judgements could have the same subject and predicate, but differ in modality. The analogy is false. The true analogy is rather this, that as in a negative judgement the matter judged is 'that *X* is not *Y*', and is therefore different from the matter judged in the affirmative, so in the modal judgements the matter judged is that *X* is actually, or possibly, or necessarily *Y*. But here what actually, possibly, or necessarily is, as there what is not, is said to be. Hence as Plato asked what is meant by saying that not being is, so we must ask what is meant by saying that possible being, or actual or necessary being, is.

To ask this is the same as to ask whether modality can qualify the copula. We use the verb *to be* as the sign of judgement, because the predicate expresses some further being of the subject than is expressed already by the subject-term. I look up and say 'the window is open', because that is of the being of the window. But whatever the window is, it is actually, and not possibly; and perhaps what it is actually, it is necessarily. If so, what is possible being, and how can we distinguish actual from necessary being?

The modal words cannot indicate different ways in which *X is Y*,[1] any more than differences in *Y*. What then do we mean by them, and why do we use them ?

We use them to mark the distinction between knowledge and opinion, and the differences in the certainty with which we hold an opinion. This is not a complete answer, because the modal words are used in divers senses ; but the difference in the modality of judgements is the difference between knowledge and opinion, and between certainty and uncertainty in opining ; and so far as these words are marks of modality, they mark that. It is no objection to this view, but rather a confirmation of it, that men often use the modal forms expressive of knowledge or certainty, when they do not really know, or are not certain. They may assume a virtue, if they have it not ; and unless these forms had such meanings, there would be no motive to use them. But we must turn to a closer examination of their use.

In the history of thought the assertoric form, '*X* is *Y*', seems to come first. Certainty or conviction precedes doubt, and precedes the reflective consciousness of knowledge. What Bain called primitive credulity cannot make us know, but it can make us assert. Our early assertions, however, are made without reflection ; we do not ask whether they are consistent with others that we have made, or whether it is possible to doubt them. When we ask such questions, we may find that different assertions which we have made are inconsistent, and that they cannot all be true, though we do not know which are false ; or we realize that we can doubt one, but not another. Our assertoric thinking is thus displaced by problematic thinking, or by necessary thinking—i. e. the apprehension of necessity, or knowledge.

But the assertoric proposition itself, '*X* is *Y*', may express two different mental attitudes.[2] We may hold and express an opinion without doubt before question has been raised ; after question has been raised, we may still hold and reassert the opinion as confidently as before, although we have not been able to prove or see into the necessity of the fact asserted. There are several kinds of example that may be given of this. It occurs in regard to sensible facts.

[1] Or *is not*. I have not complicated the discussion by taking also negative examples.

[2] We shall see that it is also often used where the judgement expressed is apodeictic : v. *infra*, p. 196.

A man walking up Eskdale in a fog and having lost his way says that he hears Cam Spout ; if challenged, he may listen again, and say that he is sure he hears the sound. Or his opinion is asked about a proposed act, and he condemns it ; another dissents and asks his reasons ; and he replies that he cannot give any reasons, but is sure that the act is wrong. Or again (and we shall find this very common in the inductive sciences) we assert as a fact something which we cannot explain or understand, because we have had experience of events that seem only explicable if it is true. *Some men detect water with the divining-rod. That is very extraordinary ; how do you account for it. I can't, but they detect it* Here the assertoric judgement is challenged ; events are recalled which seem inexplicable unless there is this power ; and so it is reasserted. On the second assertion, the word *detect* would be emphasized in speech ; or the emphasis could be given in writing by the words 'they do detect it', or 'they actually detect it' : and language has other idioms for expressing this assertoric confidence.

The difference between the two mental attitudes just noted lies in this, that whereas in both we feel confident, in the former this confidence is unreflecting, in the latter it is felt in the face of suggested doubt, and so is reflective. It might perhaps be best to call a judgement pure, rather than modal, which is made without any reflection upon the question of its truth ; and assertoric when, upon reflection, we can give no proof of it, nor see the necessity of the fact asserted, but are confident of it. The word *actually* would mark a judgement as assertoric, not pure ; but the ordinary categorical form can also express it ; we are considering the nature of the acts of judgement, but can only contemplate these by the help of propositional forms.

A consideration of the problematic and apodeictic judgements will throw further light upon the assertoric. When an opinion is challenged, we commonly try to justify it by producing grounds for it, though we cannot always do this, and our pure judgements, as just observed, are apt to be displaced by problematic or apodeictic. The apodeictic may be taken first ; it is a judgement made with a consciousness of the necessity of the fact asserted. But we often use the apodeictic form of proposition, '*X* must, *or* cannot, be *Y*' ('*X* necessarily is, *or* is not, *Y*'), when we do not apprehend a necessary connexion between *X* and *Y* ; and there are two classes of case to be distinguished when we do apprehend it, viz. those in

which we need, and those in which we do not need, in order to see the connexion, to look beyond the content of the judgement *X Y*.[1] Both are important, because in both we have knowledge.

A boy may believe and assert, because he has been taught it, or because he remembers to have seen no others, that all lines are either straight or curved; if the assertion is questioned—it matters not whether the question comes from himself or another—and he asks himself what ground he has for making it, he will realize that it belongs to the nature of linearity that every line must be straight or curved. Put symbolically, the ground for the judgement '*X* is *Y*' is seen to lie within the nature of *X*.[2] We call such a judgement *self-evident*. There are self-evidently necessary negative judgements, as well as affirmative, e.g. 'the difference between two degrees of quality is not a quality'.[3] What is self-evident need not be evident at once, or to everybody; the intelligible is intelligible only to the intelligent. In calling anything self-evident we mean not that it is evident without need for understanding, but that we need consider nothing but the terms of the judgement, to see its necessity.

[Logicians of two different schools have denied the existence of the self-evident. The one school are the Empiricists, who, rightly insisting that there is no knowledge without experience, wrongly suppose that we cannot by thinking discover the nature of anything that we have not perceived. The child learns the multiplication-table by counting marbles, or what not; but it comes to understand that the equality of two groups severally of 3 and 4 marbles to two severally of 5 and 2 marbles is independent of the units being marbles, or the day Monday, or the place London, or itself the person counting—that $3+4=5+2$ universally; nor does it need, nor could its judgement be increased in certainty by, experimenta-

[1] We may symbolize thus the categorical propositions whose subject and predicate are *X* and *Y*, and which are so far 'materially' the same, but whose 'formal' character—modality, quality, quantity—may differ; remembering however that in the problematic proposition '*X* may be *Y*', *X* and *Y* are not the terms of any judgement made, but of a suggested judgement which is not made. Cf. *supra*, p. 190, and *infra*, pp. 196–197.

[2] In Aristotle's language, the predicate belongs to the subject *καθ' αὑτό*, or *per se*—in virtue of itself.

[3] A gallon and a quart are two quantities. I can take a quart from a gallon of water, and I shall have a certain quantity (three quarts) left. The difference between two quantities is in this sense a quantity. But suppose two qualities differing in degree, say a darker and a lighter blue, or a more and a less intense pain: it is meaningless to say that the quality of lower degree can be taken from that of higher, and leave another quality which is the difference of those degrees. This self-evident fact has an important bearing on the so-called calculus of pleasures and pains.

[tion with further particulars; and from henceforth it sees this principle to be as true for countable things of which it has not had experience as for those of which it has. It has thus obtained by thinking knowledge about things of which it has had no experience (though it could not have done so without some experience of countable things). The Empiricist, however, denies this, and holds that the proposition '$3+4=5+2$' is a mere generalization from experience, entertained so confidently not because it is seen to be necessary, but because it is verified in so many instances. He is however herein using an argument—'*because* this equation holds good in so large a number of examined instances, *therefore* it holds good in the unexamined'. Either the conclusion of this argument follows necessarily from the premise, or it does not. If it does not (and in fact it does not), he cannot justify our confidence in any process of arithmetical thinking; if we have put 3 shillings into an empty purse, and then 4, and have taken out 2, we ought not to say there are 5 left until we look, or to be surprised if we find more or fewer. If, on the other hand, the conclusion does follow necessarily from the premise, then here at least is an instance of our discovering by thinking a fact about things which we have not learnt by experience. Empiricism breaks down over the validity of inference; if it allows that, it gives away its case; if it disallows it, it cannot argue.

The other objection to self-evident truth is more serious. It is said that all things are interconnected; that their relations to each other are not 'external', i.e. that relations cannot change without a change in the nature of the things related, being really an expression of their nature; that we cannot know anything in all its relations, and that a predicate Y which we ascribe to a subject X must be conditioned by what we do not know of the subject as well as by that which we have indicated of its nature when we call it X. Even so simple a fact as that $2+2=4$ is part of the whole system of numerical relations; it could not still remain, if *per impossibile* other numerical relations were different from what they are, e.g. if $2+3=7$ and $314+228=56$. As it is connected with all these, it can only be fully known through them, and when seen in its relations to the rest of the system. No particular truth, on this view, is absolutely true; only in apprehending everything could we know anything as it is. This doctrine is maintained forcibly, and even scornfully, by Mr. F. H. Bradley (e.g. *Essays on Truth and Reality*, c. vii) and by Mr. H. H. Joachim in his book *On the Nature of Truth*, c. iii; its most famous advocate in modern times is Hegel. Something like it was known to Aristotle, and criticized by him in the *Posterior Analytics* (*β*. xiii. 97ª 6–22).[1] It rests on meta-

[1] Professor G. F. Stout has put well the argument against it, in the first Essay in *Personal Idealism* (on *Error*).

[physical considerations which cannot be lightly dismissed; and they might seem to require us to deny that any truth is *self*-evident, because nothing can be understood except through the whole. But, even if they are sound, we must still acknowledge that we make some judgements with a consciousness of their necessity, and others not; we cannot abolish the distinction between knowledge and opinion. Such transformation as complete knowledge would effect in the thought expressed by a self-evident proposition must be of the same nature as occurs when a geometrician comes to realize that a proposition which he has demonstrated about one species of figure is but a special case of a much wider proposition capable of a general proof. His original proposition is not thereby shown to be false, though his insight into the facts was incomplete.]

More often, however, when we use the apodeictic form of proposition, the fact asserted in seen to be grounded in some other fact or facts, not stated in that proposition, which we should assign as a reason for it. Water must rise in the common pump, when the piston is raised: why *must*? because of the pressure of the atmosphere. Mere observation would lead us to affirm assertorically that it does rise; it is the consciousness of the connexion of that fact with the pressure of the atmosphere in a machine constructed as a pump is, that makes us affirm it apodeictically. But are we sure, it may be asked, that the atmosphere must have weight? for unless we are, we can only assert that the water must rise *if* and *when* the atmosphere has weight. We cannot here discuss the grounds on which we regard the general principles of science as established; that belongs to a consideration of inductive reasoning. But two things are clear: first, that if the grounds of the judgement XY can only be affirmed assertorically, XY itself is necessary only upon the condition that those grounds are as we assert; secondly, that, even so, the connexion between those grounds and the fact XY may be seen to be necessary. We may call the necessity of a judgement, which we see to follow from certain grounds, but whose grounds we cannot affirm apodeictically, an hypothetical necessity; when the grounds can be affirmed necessarily, then perhaps we may say that the judgement is apodeictically necessary. Thus, if two straight lines falling on another straight line and making the internal angles on the same side of it together equal to two right angles can never meet, the angles of any triangle, however large or small, are equal to two right angles; to one who regards the axiom of parallels as self-evident, the judgement that the angles of a triangle are equal

to two right angles will appear apodeictically necessary; to one who does not, hypothetically.

It will be seen from this, that there is a close connexion between the hypothetical and the apodeictic judgement. But we cannot say that all hypothetical *propositions* are apodeictic, for we often use them when we do not see the consequent to be necessarily involved with the antecedent, e.g. a public speaker says that if a certain measure is carried, certain results will ensue. This is only another illustration of the fact that a propositional form which is intended to express a certain kind of judgement may be used when we do not really make the judgement which it should express. Every deliberate falsehood illustrates this, and every false apodeictic proposition, whether deliberately intended to deceive or not. And it often happens that the assertoric form of proposition is used to express necessary truth, the apodeictic to express doubtful opinion. In mathematics every step is seen by the mathematician to be necessary[1], insomuch that it is often summarily said that mathematics deals with 'necessary matter'. There is consequently no need to distinguish apodeictic from other judgements in mathematics, and they are all, as a rule, expressed assertorically; we say '2×2 is 4', not '2×2 must be 4': 'the interior angles of a triangle are'—not 'must be'—'equal to two right angles'. And contrariwise we use the apodeictic form of proposition to hide our doubts, perhaps even from ourselves; we are conscious of grounds for a judgement and grounds against it, but we look to those only which enforce the side we wish to take, and in reference to them make our assertion apodeictic. 'It must be so: Plato, thou reasonest well' does not express the same confidence as if the speaker had said 'It is so'. 'Methinks the speaker doth protest too much.' The apodeictic formula, *X must be Y*, thus covers in use diversities of thinking; but it always implies that the speaker has reflected upon the question of the truth of his judgement.

The problematic judgement, *X may be Y*, similarly involves reflection; but it does not, like the apodeictic, involve the judgement

[1] Many mathematical statements are made without seeing into, or realizing, their necessity at the time, and the thinking is then assertoric; but because their necessity *can* be seen, we may call them apodeictic. There are a few, which mathematicians have believed to be true, but found false—e.g. general formulae for the finding of prime numbers, which have finally broken down. If it had been seen that the formula must yield a prime for any value, it could not have broken down.

that X is Y. He who judges that straight lines making equal angles with the same straight line cannot meet judges also that they do not ; he who judges that Mars may be inhabited does not judge that it is. It involves reflection, therefore, like the apodeictic judgement : but reflection upon something suggested, as it were in an attempt to judge, which we cannot find sufficient grounds either to affirm or to deny. It is an expression of uncertainty.

The problematic is the most difficult of the three modalities of judgement, and its consideration is complicated by the fact that the formula X *may be* Y is sometimes used when there is nothing problematic in our thinking, because there is no uncertainty. When a genus G has divers alternative species S^1, S^2, S^3, we say that a G *may be* either S^1, S^2, or S^3: a triangle may be equilateral, isosceles, or scalene—currants may be black, white, or red. So long as such propositions are general, they express knowledge of the divers forms in which it is seen that a genus must be, or found that it is, realized ; and they are not problematic. They are only problematic, if the subject is a definite individual of the genus—'that currant-bush may be white, black, or red ' ; for then, they express our ignorance as to which it is. Again, we use the problematic formula, X *may be* Y, when we know that, under certain conditions P, the subject X does, or must, exhibit the character Y, but we either do not now desire, or are unable, to state the conditions. In this sense we say ' Water may boil below 212° Fahrenheit '—the condition omitted being a diminution of the ordinary atmospheric pressure; or ' A man may die of joy ', the condition here being one which we could not state precisely, though no doubt it is connected with the condition of his heart. Disregarding these uses for the present, we must turn to those which really express problematic thinking.

The plainest examples occur where the judgement concerns individuals, e.g. ' Rain may fall to-morrow ' (this is concerned not with a particular thing or person, but still with a particular day); or again ' The Sultan may behead his vizier to-morrow '. It is clear that such judgements imply uncertainty in the speaker. But is uncertainty only a state of the mind, or is it also a state of the facts ? A necessary judgement is really an apprehension of necessary connexion in the facts : is a problematic judgement an apprehension of a less than necessary connexion in them ? There is a sense in which we may intelligibly maintain this. Given that things of the same kind behave differently in different relations,

and given a complex system containing many things of divers kinds, whose kaleidoscopic interactions bring different things of the same kind into different situations, then we can say that there is no necessary connexion between being of the kind *X* and behaving in the way *Y*. This varying collocation of things is the basis, as we saw, of the relation of *accident* or of the 'coincidental' between predicate and subject; and a system of things subject to such changes may be called not 'necessary matter' but 'contingent'.[1] Yet in a given situation, as we saw in discussing the accidental, we commonly think that what happens is necessary. Is this opinion a mistake? There is one region in which men have been disposed to think so—that of voluntary action. It has been thought that the freedom of the will implies that a man's action does not issue necessarily from his character and circumstances, so that no knowledge of these, however complete, would enable one to say that he must act thus or thus. If this is so, there is a ground for the problematic character of any judgement about the future actions of a voluntary agent in the intrinsic uncertainty, or real contingency, of the event. But this uncertainty can only belong, if at all, to *future* actions. If I say 'The Sultan may have beheaded his vizier yesterday', I imply no more uncertainty in the facts than if I say 'Rain may have fallen yesterday'; the same is true of the judgement 'The Sultan may now be beheading his vizier', just as much as of 'Rain may now be falling'. All these alike are problematic only in virtue of my uncertainty about the facts, not of any uncertainty in the facts themselves. And the same character belongs to problematic judgements which are not concerned with an individual but a kind of thing. 'Cancer may be incurable' means that though cancer either is incurable or not, and we are aware of certain facts inclining us to think it is, we have not sufficient grounds for a decision.

Waiving that case, however, a problematic judgement implies by the form *X may be Y* our belief of certain facts which are not sufficient ground for the judgement *X is Y*, though we believe that along with other facts they would be. We do not in practice make such judgements in the absence of all knowledge. 'The grandfather of

[1] In Aristotle's phrase, *ἐνδεχόμενα ἄλλως ἔχειν*, 'things that can be otherwise': but Aristotle does not make it very clear how far he thought their variability depended on shifting collocations, and how far on a 'real contingency', which he did not altogether reject.

Pocahontas may have died of diabetes': that is possible, because any man may; but as we do not know that in this particular case any of those special facts were present which, with others, cause a man to die of diabetes, we should never so judge. A problematic judgement is provoked by knowledge; it is problematic because of ignorance.[1]

It follows that further knowledge would lead to its supersession by an apodeictic or assertoric judgement, according as our doubts were removed by a discovery of conceptual connexions or of historic facts. A genuine example of cancer being cured would refute the judgement 'Cancer may be incurable'; so also would such an understanding of the nature of the disease as enabled us to see how it could be cured. But though further knowledge would lead us to abandon the problematic judgement, we do not, when we make it, know whether *X* is never *Y*, or always *Y*, or sometimes *Y* and sometimes not. In this there is a difference between a genuinely problematic judgement and those expressed in propositions of the same form which we noticed and set aside. For in them we imply that there are conditions, whether we can fully state them or not, under which *X* is *Y*.[2] These quasi-problematic propositions have therefore an affinity with certain particular propositions. In the particular proposition 'Some *X* is *Y*', we saw that we might either be thinking of individuals of the kind *X*, not separately enumerated, which are *Y*, or of some general determination of *X*, not stated, which would involve its being *Y*; the former sort is rather of the nature of the singular proposition, the latter is on the way to the universal. In the latter, the conditions, given which any *X* is *Y*, may be either known but not stated, or, though unknown, shown to exist by examples of *X* being *Y*: 'Some triangles have the square on one side equal to the squares on the other two'—viz. right-angled triangles; 'Some children are taller than their parents'—doubtless in virtue of certain physiological conditions, but we do not know them. Particular propositions like these have been called 'modal particulars', because of their close similarity to the quasi-problematic propositions just considered. The judgements can indeed be as easily expressed in the form '*X* may be *Y*' as in the form 'Some *X* is *Y*'; each form implies that under certain conditions, not specified, though perhaps known, *X* is *Y*; but there is this difference between them, that the latter implies that the

[1] Cf. Bosanquet, *Logic*[2], vol. i, pp. 315–318, on 'real possibility'.

[2] Or, if the judgement were '*X* may not-be *Y*', under which it is not.

conditions are sometimes actually fulfilled, the former does not do so.[1]

We may sum up what has been said of the modality of judgement as follows. In every judgement I intend to assert truth, but not necessarily about the particular reality to which the subject of my proposition refers; the truth I assert may be that I am unable to discover the truth about this reality. I may judge without pausing to question what I assert; and in such case my judgement is called assertoric, and expressed in the form '*X* is (*or* is not) *Y*'; it can, however, also be called pure, as being pure or free of any reflection upon the question of its truth. On the other hand, I may reflect on this question, and if I see the judgement to be true in virtue of the very nature of its terms, or if I find that what it asserts is involved in what I already know, or take, to be fact, my judgement is called apodeictic, and expressed in the form '*X* must (*or* cannot) be *Y*'. Those apodeictic judgements which are grounded in facts not forming part of what they themselves affirm have a different logical character according as these facts can be affirmed apodeictically or only assertorically: if the latter, the judgement resting on them is not strictly apodeictic, for only the sequence can be affirmed apodeictically. If I find that what I attempt to assert in a suggested proposition is connected with conditions, some of which I know to exist, while I am ignorant whether the others required are realized or not, I assert it to be possible; such a judgement is called problematic, and expressed in the form '*X* may (*or* may not) be *Y*'. The problematic proposition does not imply that particular events are unnecessary in their happening, though, when general, it does often imply that an event of a certain kind depends on a conjuncture, or contingency, which is not universally necessary. It is possible that when reflecting on the question of a judgement's truth, we cannot find any ground for asserting what we assert, except that we perceive or remember the fact stated, or have had it on good authority; though this may be reason enough to convince us of the truth of our assertion: then the content of the judgement is affirmed to be actual, and the judgement called assertoric, and expressed in the form '*X* is (*or* is not) *Y*', with an emphasis perhaps on 'is', or the addition of the word 'actually'.

[1] e. g. 'A man may call at every public-house from John o' Groats to Land's End.'

This assertoric judgement, being not a bare unreflective assertion, but expressing besides our mental attitude towards a suggested doubt, is different from the assertoric judgement, above called also pure, that contains no such reflection; and as involving such reflection, this is modal.

These distinctions of modality do not then express differences in the necessity with which elements connected in reality are connected[1]; yet they do express this, that whereas some connexions in reality are seen to be necessary, others, and the existence of such elements, and their distribution in time and place, are not. Many philosophers have felt it impossible not to believe that the existence of all things, and their distribution, and every feature of their interaction, are as necessary as those matters which are asserted in our really apodeictic judgements; and if their belief could pass into clear vision, judgements at present problematic or assertoric would be replaced by apodeictic.

[Some further questions connected with modality, and in particular with the problematic judgement, deserve attention.

In the first place, in a problematic proposition, do we really judge? In the assertoric or apodeictic, we judge that *X* is *Y*, though there is a great difference between thinking, however confidently, what we do not see to be necessary, and knowing. But in the problematic, we do not at any rate judge that. Is it more than an expression of doubt, and of our inability to 'make up our mind'? It certainly is an expression of doubt. But we do not utter such propositions *in vacuo*, and out of relation to any question which we desire to answer; and if a man were asked whether there really is telepathic communication, and replied after consideration 'It may be so', he would mean rather more than that he did not know. He would mean that there were certain facts preventing him from denying it, though insufficient to prove it; that there was some reason for thinking yes.

If there were no assertion of fact in a problematic proposition, we should not judge one event to be more probable than another. The whole mathematical treatment of the probability of events rests upon the assumption of a limited knowledge of their conditions. If I say that there is more probability of throwing 7 with the dice than 12, it is because I know that there are six ways of throwing 7 and only one of throwing 12. In complete ignorance of a subject I could not say that anything was probable regarding

[1] Hence we cannot accept such a definition as Aldrich offers of modality: 'Modalis, quae cum *Modo*, h. e. vocabulo exprimente quo modo praedicatum insit subiecto.' *Artis Logicae Rudimenta*, c. ii. § 2. 1 (Mansel's 4th ed., p. 47).

[it.[1] But the attempt to estimate degrees of probability raises a difficulty which the problematic judgement of itself does not raise. I say that something may happen on a given occasion, because I know (or believe) that some of the conditions required for its happening exist; but if I say that one event A is more probable than another B, I do not mean to assert that the conditions necessary for the occurrence of A are more completely realized than those necessary for the occurrence of B; for that implies that the conditions requisite for B are incomplete, and if I know (or believe) that, I shall call B not less probable, but impossible. More conditions necessary for A than for B are known to me; but as the rest is unknown, it may turn out that the conditions requisite for B are really complete, and those for A incomplete. Anyhow, if one or other event must occur, one will occur, and the other not; one is necessary, the other impossible; more and less probability do not attach to the events. We say therefore that they attach to our judgements; the judgement that A will happen is more probable than the judgement that B will happen. But one is true, the other false, and we do not know which is which; is it not foolish to prefer one, as the more probable? It is the more probable judgement, comes the answer, because there are more grounds for it, i.e. there are more grounds for thinking that A will occur than that B will. But what does this mean? the grounds for thinking that A will occur are the facts, or a knowledge of the facts, which necessitate A. Less than this is no ground for thinking that A will occur, but only that it may occur; and similarly with B. The real situation then is that there are grounds for thinking that A or B may occur, but not for thinking that the one will occur rather than the other. An example will make the point clearer. Suppose I am to draw from a box, in which there are 5 black balls and 1 white, and to bet on the result. I shall be told to bet on drawing a black ball, because it is more probable; yet all the time perhaps only the white ball is within reach, and my drawing a black is impossible. How can that be the more probable judgement which leads me to act upon the expectation of an event which is impossible? The usual answer is, that with the knowledge available it was more reasonable to bet on black; but would it not have been more reasonable not to bet at all? And indeed, is not that the only reasonable course? did I know enough to bet reasonably? And if not, can we defend the statement that the one event was more probable?

I think we may partially solve this difficulty as follows; but not wholly. We must distinguish between what is reasonable if we are to act many times, and if only once. If I am going to draw from the box many times, the ball being replaced and the

[1] Cf. *infra*, pp. 423–424.

[box shaken after each draw, then supposing I always bet on black, I shall win more frequently than if I always bet on white. In saying this I know that there are five times as many black balls as white, and I believe that the movements of shuffling and the direction of the thrust of my arm are favourable about equally often to each ball. But what is reasonable to do on each of a number of occasions is no longer reasonable when there is only one occasion of acting. The real meaning of the statement that drawing a black ball is 5 times as probable as drawing a white is that in a large number of trials black will be drawn about 5 times as often ; but I cannot transfer the ratio to the event of a single trial. If there are twice as many boys as girls in a village, it is not because each child is $\frac{2}{3}$ boy and $\frac{1}{3}$ girl ; and it is the same here. Those who call it reasonable to ' follow the chances ' in an isolated action are like people who think that an average or percentage is displayed in each of the items from which it is obtained. An excellent example of the difference is provided by life insurance. An insurance company, knowing that, out of a great number of persons who have lived to 55, so many have died at 56, so many at 57, &c., but that the average length of life beyond 55 has been (say) 15 years, and believing that the circumstances favourable or unfavourable to longevity will continue much as hitherto, offers an insurance policy to persons of 55 at a premium based on the assumption that they will live to 70. It does not matter to the company that it loses on X and gains on Y, provided it makes the calculated profit on the average. But it matters very much to X whether the company is going to lose or gain on his insurance. If he dies next year, he will have made a very good bargain ; if he lives to 90, a very bad one. What is reasonable for the company to offer is not reasonable for him to accept, if he regards life insurance as a speculation. If he insures his life for the sake of the security of his family, the question is whether this security is worth the price asked for it. The proper price to ask of him may be settled by applying the theory of probability ; but you cannot so settle whether a thing is worth its price.

And yet many a man faced with the question, whether it is worth his while to pay the premium asked, would take into account his so-called ' expectation of life '—how long it is ' probable ' that he will live. If he lived all the lives of all the insurers, this would be reasonable ; as he lives only his own, is it reasonable ? I think, applying the same considerations as hitherto, we may justify him a little further.

We allowed that a man making repeated draws from the box will draw black more often than white, because black will be more often under his hand ; and therefore, though he does not know on any given occasion which will be under his hand, he will act reasonably if he always bets on black. These occasions of action

[are all of the same kind. But life requires us to act in all sorts of situations, with very imperfect knowledge of the conditions affecting the event in each, wherein it is said that we should follow the more probable judgement, or take the course more likely to succeed. Our difficulty was to discover what is meant by calling one judgement (or event) more probable than another ; both seemed equally problematic. But we were considering an isolated judgement. What is meaningless when our judgement is to guide our action only once may not be so when very many actions are in question, and we judge always on one principle. Suppose that the ratio of the *known* circumstances favourable to one event to the *known* circumstances unfavourable to it, or favourable to another is roughly the same—or even, more nearly directly than inversely the same—as the ratio of the *existing* circumstances favourable to the one event to the *existing* circumstances unfavourable to it, or favourable to another : then if men always acted as if that event would happen, for which they *knew* more circumstances favourable, they would more often succeed than fail. That is what is meant by adopting the course, or acting on the judgement, for which there are the better grounds. And the reasonableness of so acting is not disproved by the fact that all men fail sometimes, and some men even most times, when they act on this principle : any more than the reasonableness of betting repeatedly on the black ball is disproved by the fact that every man sometimes loses, or that some men lose on the whole, within the run of their draws. Indeed the reasonableness is greater in the former case ; for men need not bet, but they must act ; and if they must act, and act in the absence of the knowledge which would enable them to secure success in each case separately, they must act upon a rule which will enable them to secure the most success upon the whole, and leave its distribution to fortune.[1]

Yet I do not think the above a complete solution of the problem. A rule that can be applied without a fresh exercise of judgement in each case is only possible in matters like drawing balls from a bag, or throwing dice, where the factors whose existence we take into account can be treated quantitatively. We may grant that the man who acts most prudently on a given occasion may fail on that occasion, and the imprudent man succeed, and yet that the man who always acts prudently will succeed more than men who always act imprudently ; but that does not explain what acting prudently on a given occasion is. No doubt it requires study of the ascertainable facts ; but it also involves an estimate of their importance ;

[1] So, because we do not know enough of the 'merits' of every case to decide each 'upon the merits', we are often compelled in administration to adopt a rule which suits most cases, and acquiesce in its sometimes failing, and our not knowing when it will fail.

[and this is something quite different from the thinking which discovers that out of all possible throws with the dice six give 7 and only one 12. We speak of a man having a sound judgement, and the collect prays for ' a right judgement in all things '. The exercise of such a judgement is not knowledge, and not mere guess-work. It is better than tossing up, yet it cannot be justified to another. Though a man is prepared to act on his judgement, he is not prepared to enunciate it assertorically. It is here, as it seems to me, that the real puzzle of the problematic judgement lies. We hold that one man is wiser than another, and that, not only in reaching opinions on which action is to be based, but also in the study of matters that do not admit of demonstration, e.g. in historical inquiry or anthropological. And this wisdom does not *consist in* either the advantage resulting from acting as if its opinions were true, or their confirmation by subsequently discovered fact ; though these things may be evidence of it. And yet how is one man wiser than another, when neither knows ?]

[I have spoken frequently of the *grounds* for a judgement, and in the previous edition of this book it was said (p. 171) that what gave modality to a judgement was the presence of the thought of grounds for what is alleged, though I think it better to say that a modal judgement expresses reflection upon the question of the truth of what is judged, or suggested ; for an assertoric, or a self-evident apodeictic judgement, has not grounds in the same sense as an apodeictic judgement deduced from others, or a problematic judgement. By the grounds of a judgement are commonly meant grounds given in other judgements ; but they are not these other judgements, i.e. the acts of our minds in judging; they are the facts which in them we assert. And these alleged facts are only grounds of our present judgement in the sense that we see a connexion between them and the fact which it asserts. The relation, however, may be of several kinds. The grounds may be facts whose existence is seen to account for that of the fact grounded on them ; this occurs in causal explanations. Or they may be facts which make intelligible the fact grounded on them, though there is no causal relation, as in mathematics. Or, thirdly, they may be facts which do not make intelligible to us what is said to be grounded on them, but which we think could not exist, but for that ; this occurs in inductive reasoning, or when we argue from an effect to the existence of its cause ; if I have known water to be found by men with apparently no other means of discovering its presence, I may infer that the divining-rod informs them of it, though I do not thereby understand its action. There is an old distinction between *ratio essendi* and *ratio cognoscendi*, a reason for the being of a fact, and a reason for acknowledging its being, which expresses the difference between grounds of a judgement in the

[first of these three senses, and in the other two. The grounds which justify an apodeictic judgement must be either *rationes essendi*, or such *rationes cognoscendi* as we get in mathematics; for we can only judge apodeictically if we have insight into the necessity of the fact alleged.]

[There are a few other adverbs (besides *possibly*, *actually*, and *necessarily*) which may be introduced into a proposition in order to express that we have reflected upon and made our estimate of its truth: e.g. *probably*, *truly*, *falsely*, *really*: although all but the first of these may also be used merely to qualify some term in the judgement; a truly virtuous woman, for example, meaning a woman virtuous in a particular way, or a falsely delivered message, one not delivered as it was received, whereas a probably dangerous undertaking does not mean an undertaking involving a particular kind of danger. Such adverbs (if used to express our attitude as to the truth of the proposition reached by omitting them from that in which they are used) may be called modal, and judgements modal, in which they are used. But no adverbs of any other kind make a judgement modal, and no qualification of the content, but only of the unreflecting directness with which, in a 'pure' judgement, the content is affirmed. Differences of tense, for example, must not be reckoned to affect the modality of a judgement[1]; they

[1] As by J. S. Mill, *Logic*, I. iv. 2, who rightly rejects the view of those who would make every adverb the ground of a modal difference in the proposition where it occurs. The distinctions of modality descend from Aristotle, *de Interp.* xii. 1, 21ᵃ 34–37, and *Anal. Pri.* a. ii. 1. 25 1 *sq.*, but the word τρόπος (= *modus*) is said to occur first in the Commentary of Ammonius; *v.* Ammonius *in Ar. de Interp.* 172ʳ, (quoted in part Prantl, vol. i. p. 654)=Berlin ed. p. 214 Τρόπος μὲν οὖν ἐστι φωνὴ σημαίνουσα ὅπως ὑπάρχει τὸ κατηγορούμενον τῷ ὑποκειμένῳ, οἷον τὸ ταχέως, ὅταν λέγωμεν "ἡ σελήνη ταχέως ἀποκαθίσταται", ἢ τὸ καλῶς ἐν τῷ "Σωκράτης καλῶς διαλέγεται", ἢ τὸ πάνυ ἐν τῷ "Πλάτων Δίωνα πάνυ φιλεῖ", ἢ τὸ ἀεὶ ἐν τῷ "ὁ ἥλιος ἀεὶ κινεῖται". ἀριθμὸς δὲ αὐτῶν φύσει μὲν οὐκ ἐστιν ἄπειρος, οἱ μὴν δὲ περιληπτός γε ἡμῖν, ὥσπερ ὁ τῶν καθόλου ὑποκειμένων ἢ κατηγορουμένων, ἀναριθμήτων δὲ αὐτῶν ὄντων. τέτταρας δὲ μόνους ὁ Ἀριστοτέλης παραλαμβάνει πρὸς τὴν θεωρίαν τῶν μετὰ τρόπων προτάσεων, τὸν ἀναγκαῖον τὸν δυνατὸν τὸν ἐνδεχόμενον καὶ ἐπὶ τούτοις τὸν ἀδύνατον. . . : '*Mode* is a word signifying how the predicate belongs to the subject, e.g. "quickly", when we say that "The moon waxes quickly", or "well" in "Socrates argues well", or "much" in "Plato loves Dion much", or "always" in "The sun always moves". The number of them is not infinite in the nature of things, but is beyond our computation, like the number of universals that can be subjects or predicates, though they cannot be numbered. Aristotle, however, brings into his consideration of modal propositions four modes only, the necessary, the possible, the contingent, and further the impossible. . . .' This statement about Aristotle is based on *de Interp.* xii, and the modalities were often enumerated as these four, sometimes with the addition of the true and the false. The same wide definition of τρόπος is given by Michael Psellus (*v.* Prantl, ii. 269), but he singles out for discussion only those which 'determine the connexion' of subject and predicate, i.e. the modalities proper. Cf. Buridanus (Prantl, iv. 22), who explains that the qualification which is to make the proposition modal must attach to the

[merely affect the predicate, and not our attitude towards affirming the predicate of the subject; and past, present, and future verbs may all occur (as we have seen) in judgements of any modality. No doubt differences of tense are a somewhat peculiar affection of the predicate. If I say *Jehu drives furiously*, I predicate a different action from what I predicate if I say that he drives slowly; but the action predicated is the same, whether I say that Jehu has driven, is driving, or will drive, and only the time of the action differs. This, however, merely amounts to saying that the predicates of judgements differing in tense differ thereby in the category of time, and not in another category. Time is a very peculiar feature in the existence of things, but still it is a feature in their existence, and gives rise to a great variety of modifications in their predicates. There is no more reason for reckoning as modal these differences in time, than there is for so reckoning the differences in degree, or in place, of which a predicate is susceptible. *The plague raged last year: it is raging now: it is raging here: it is raging in Calcutta.* If the plague can exist in different times, so also can it exist in different places; and if judgements do not differ in modality by connecting its existence with different places, neither do they differ in modality by connecting its existence with different times.]

There are a few other distinctions drawn among judgements, which ought to be noticed. We may deal first with a series of antitheses whose force is sometimes too readily considered to be the same: these are *analytic* and *synthetic*, *essential* and *accidental*, *verbal* and *real*.

'In all judgements,' says Kant,[1] 'wherein the relation of a subject to the predicate is cogitated (I mention affirmative judgements only here; the application to negative will be very easy), this relation is possible in two different ways. Either the predicate B belongs to the subject A, as somewhat which is contained (though covertly) in the conception A; or the predicate B lies completely out of the conception A, although it stands in connexion with it. In the first instance, I term the judgement analytical, in the second, synthetical. Analytical judgements

copula, and not to the subject or predicate. The word *modus* is of course a term of wide signification, but Logic is concerned with certain *modi propositionis*; and it is obviously wrong to suppose that any adverb will make the proposition in which it occurs modal; nor can differences of tense do so, though they express a modification of *the predicate*.

[1] *Kritik of Pure Reason*, E. T. (Meiklejohn), p. 7. The translator uses *conception* as equivalent to *concept* (cf. *supra*, p. 22).

(affirmative) are therefore those in which the connexion of the predicate with the subject is cogitated through identity[1]; those in which this connexion is cogitated without identity, are called synthetical judgements. The former may be called *explicative*, the latter *augmentative*[2] judgements; because the former add in the predicate nothing to the conception of the subject, but only analyse it into its constituent conceptions, which were thought already in the subject, although in a confused manner; the latter add to our conception of the subject a predicate which was not contained in it, and which no analysis could ever have discovered therein.' Kant's example of an analytic judgement is 'all bodies are extended': for our conception of body is *extended substance*, and therefore, in order to make the judgement, we need only analyse the conception. 'All bodies are heavy', on the other hand, is a synthetic judgement; for it is not contained in the conception of bodies, that they gravitate towards one another.

Kant's statement of the distinction between analytic and synthetic judgements has been much discussed and criticized. He himself attached to it great importance. For he thought that analytic judgements could be enunciated universally in advance of experience under guarantee of the law of contradiction; because the predicate was contained in the subject-concept, it could not be denied of the subject without self-contradiction. Since I mean by calling anything a body that it is an extended substance, I can say that all bodies are extended without waiting to examine everything that falls under that denomination. With synthetic judgements it is otherwise. It is no part of what I mean by calling anything a body, that it is heavy; and I need experience to assure me that whatever falls under the denomination *body* has weight. But there are some synthetic judgements which we know to be true universally without appeal to experience; and how that is possible Kant conceived to be the fundamental question of metaphysics.

But we never make judgements analytic in Kant's sense—i.e. guaranteed by the mere identity of the predicate with an element in the subject-concept. To do so would be tautology; and to

[1] In speaking of the connexion between the predicate and subject as cogitated through identity, Kant means that the predicate-concept is identical with some part of the subject-concept: where it is cogitated without identity the two concepts are quite distinct.

[2] Or *ampliative*.

utter a tautology is not to judge, for in all judgement we advance to the apprehension of a new element in the being of a subject already partially apprehended. Suppose the constituent elements of the concept *A* to be *BCD*, as those of body are solidity and extension. Yet the judgement '*A* is *B*' (*all bodies are extended*) is not the equivalent of '*BCD* is *B*' (*all extended solid substances are extended*). This proposition does merely repeat in the predicate something contained in the subject-concept; and inasmuch as the subject is already conceived as uniting in its being elements whereof the predicate is one, the proposition only goes over old ground. But that judgement picks out in the unity of what we call a body an element which it recognizes as combined with others to constitute a body. And the difference is fundamental. '*A* is *B*' means 'to the constitution of *A*, *B* must go with *CD*'; *all bodies are extended* means 'to the constitution of body extension must go with solidity'. Kant himself tells us that until the judgement is made, the predicate *B* is only covertly contained in the subject-concept *A*; so that it is really the work of the judgement to recognize *B* (as an element along with other elements) in the nature of *A*. And it is this recognition of the necessary implication of different elements in one nature, not the law of Contradiction, which allows us to enunciate the judgement universally. Suppose that we did not see that a substance could not be solid without being extended: then (1) if we meant by *body* merely a solid substance, we should see no self-contradiction in the statement that a body need not be extended: while (2) if we meant by the word a solid extended substance, the statement would indeed be self-contradictory, as is the statement 'a body need not be a body'; but the so-called analytic judgement *all bodies are extended* would be as uninstructive as the tautology *bodies are bodies*.

In all judgements then—even in those which Kant calls analytic—we assert a relation of distinguishable elements. Yet his antithesis of analytic and synthetic judgements is not baseless. That *cats purr* is a statement not made on the strength of seeing that to purr is necessarily connected with other elements in the being of a cat; and we may think of a cat without including in its nature purring. This then he called a synthetic judgement. But he also called synthetic such judgements as '$5+7=12$', or 'Two straight lines cannot enclose a space' (in which the connexion of the predicate with the subject is seen to be necessary),

because in them too the subject can be thought of without the predicate—whereby is meant not that we can conceive the subject to lack the predicate, for we cannot conceive what cannot be,[1] but that without thinking of the predicate at all, we can still in a measure conceive the subject. Hence the predicate-concept was no part of the subject-concept, and, not being included in it, could be denied of it without self-contradiction; and so, since we know the judgements to be true universally, without examining every instance, we have knowledge of things not guaranteed by the law of Contradiction before experience of them. This, to Kant's mind, was the great problem, which he expressed by asking how synthetic judgements *a priori* are possible.[2]

But the difference between the two classes of judgements is misrepresented when it is said, that in the analytic the predicate is merely part of the subject-concept, and the necessary truth of the judgement therefore obvious: in the synthetic the predicate is no part of the subject-concept, and the necessary truth of a synthetic judgement therefore a problem. No judgement is analytic in the sense of asserting of anything in the predicate what in the subject-concept we have already realized or indicated it to be. What Kant has really done is to distinguish those judgements in which the predicate is part of the definition of the subject from those in which it is not. The distinction we may mark by the antithesis **essential** and **accidental**, if *accident* be taken, as by

[1] ἀγνωσία ἐξ ἀνάγκης ἐπὶ μὴ ὄντι ('Of what cannot be we can only be ignorant'). Plat. *Rep.* v. 477 A.

[2] To know anything *a priori* (ἐκ προτέρων) means to know it by derivation from something prior; and a general principle is said to be prior to the facts, or subordinate principles, that exemplify it; to know anything *a posteriori* means to know it by derivation from the particular facts exemplifying or dependent on it. Thus I know *a priori* that 5 men in buckram and 7 men in buckram are 12 men in buckram, for it follows from the general principle that $7+5=12$: I know *a posteriori* that cats purr, through observation of many cats. Analytic judgements might in Kant's view be known *a priori*, because their truth followed from the law of Contradiction; but there was no principle from which self-evident synthetic judgements could be derived. Kant spoke of knowing these also *a priori* in the sense of knowing them not *a posteriori*, i.e. not on the evidence of their repeated confirmation in experience; properly, they are the *priora* from which we derive knowledge about unobserved particulars. Thus to know *a priori* came to mean to know in advance of experience; and his problem comes to this, viz. how, in advance of experience of that very thing, and therefore merely by thinking, can we know more about anything than what is guaranteed by the so-called 'laws of thought'? Cf. on the meaning of the antithesis *a priori* and *a posteriori*, pp. 436, 437, *infra*.

Aristotle in the phrase *καθ' αὑτὸ συμβεβηκός*[1] (= essential accident, or accident *per se*), to include attributes belonging to any subject of a certain nature in virtue of that nature, as well as those coincident in it with that nature.[2] Thus the accidental judgement might be in Kant's sense synthetic either *a priori* or *a posteriori*. And we might fairly oppose these, as 'ampliative' or 'augmentative', to essential judgements as 'explicative', because a subject and a property or accident of it are not one, as it and the definition of it are. But the opposition of *analytic* and *synthetic* is misleading, since that insight into the nature of a subject which definition expresses, though it may be called an analysis, is also an apprehension of the connexion of elements in an unity, and the necessity of this connexion cannot be derived from the law of Contradiction. That law is that contradictory propositions cannot both be true; but to know this is not to know which of two given contradictories is true.

Doubtless a man cannot without contradiction deny of a subject anything which by the subject-term he means that it is. But how has the subject-term come to have its meaning? If through insight into a necessary connexion of elements in the subject, then the so-called analytic judgement expresses this insight. Only if definitions were quite arbitrary, mere statements of the meaning of a name, would the truth of Kant's 'analytic' judgements rest merely on the law of Contradiction. If I choose to mean by *body* a solid extended substance, it is self-contradictory to say that a body is not extended. But equally, if I choose to mean by *body* a solid extended and heavy substance, is it self-contradictory to say that a body is not heavy. And Kant has forgotten to ask why we regard extension as belonging to the definition of body rather than weight.

We saw indeed, in discussing Definition, that we often have to settle arbitrarily what elements shall be included in the intension of a term, and therefore implied about those subjects to which we apply the term. Let us take an example of a subject in whose definition the elements are thus arbitrarily put together.[3] In the Elementary Education Act of 1870, § 3, an elementary school is

[1] Cf. e.g. Ar. *Post. An.* *a.* vii. 1, 75[a] 39–[b]2. *συμβεβηκός* in this sense includes properties, which are distinguished from accidents in sense of the *Topics* by being *καθ' αὑτό*.

[2] e.g. two straight lines, in virtue of their straightness, cannot enclose a space: to be heavy is coincident in bodies (so far as we can see) with their nature as bodies.

[3] Arbitrarily, not because there is no motive, but because the elements, though compatible, are not necessarily implicated together.

by definition 'a school, or department of a school, at which elementary education is the principal part of the education there given, and does not include any school or department of a school at which the ordinary payments in respect of the instruction, from each scholar, exceed ninepence a week'. To say therefore that an elementary school charged less than 10*d.* per head per week in fees was to make an analytic judgement from the standpoint of the Education Department in 1870; but only because it had been arbitrarily settled that none charging 10*d.* or over should rank as an elementary school, and not because we have such a knowledge of what an elementary school must be, as to see that it could not be elementary and charge a fee so high. The proposition then is true just because it has been agreed what *elementary school* shall mean; and while that agreement is adhered to, it cannot be denied without self-contradiction. But if I say that a triangle has sides, that is true not just because it is agreed to call nothing a triangle which has not, but because I see that lines can be put together into the unity of, and are required in, a triangle. Kant's account of analytic judgements ignores this difference. It implies that all definition is arbitrary, and that judgements whose predicate is part of the definition of the subject are necessarily true, only because what we mean by a name we mean by it.

Some propositions are indeed true universally by mere convention as to the meaning of names, because they give us information about the convention. These may be called **verbal**, and to them we may oppose as **real** all which are intended to give information about the nature of things. But verbal propositions are in Kant's sense synthetic. 'Elementary schools charge a fee below 10*d.*' meant that schools *called* elementary did so; and to charge a fee below 10*d.* is not part of being called elementary, but of what was meant by being so called. A proposition about the meaning of a name is clearly instructive, and ampliative. It is only inadvertently that we make about things statements, whose truth rests just on the meaning of words; and when we discover that we have done so, we acknowledge that we have really said nothing. Suppose that some one had argued in 1870 that a particular school which he knew to give mainly elementary instruction had a fee below 10*d.*, because it was an elementary school; clearly he would have wasted his breath, unless he knew that it had a right to be called so within the meaning of the Act; and

he could not have known this till he knew that its fee was below 10*d.*; and then the argument would have been superfluous.

There is another objection to Kant's division of analytic and synthetic judgements. In speaking of analytic judgements, he had in mind only universal judgements, in which, as he held, we analyse a concept; but there are judgements in which we may be said to analyse the sensible object before us, as when I look up and say 'the sky is starlit'. These have been called[1] 'analytic judgements of sense'; they clearly distinguish in a subject an element which they assert to be combined with others in the unity of that subject, and so far they are equally analytic with those which Kant called so; but yet they differ greatly. They are singular, not universal; they rest on perception, not conception; and by no possibility could their truth be made to seem dependent barely on the meaning of names.

Analytic judgements then may be analytic either of a sensible individual or of a concept: in neither case is their truth guaranteed by the law of Contradiction, but they rest on our apprehension of the connexion of elements in the unity of one subject. So far they do not differ from judgements called by Kant synthetic. But those analytic of a concept are essential, where without the predicate the other elements in the subject could not form a conceivable unity, whereof the predicate could be regarded as a further attribute. Judgements called by Kant synthetic, whose subject is something which can be thus conceived before the attribution of the predicate, may be called accidental (though not in the sense of that word in the doctrine of Predicables) or ampliative of their subject. They include both analytic judgements of sense, and all judgements about the meaning of names. Verbal propositions are therefore not analytic, and real propositions may be either analytic or synthetic. Essential judgements are true by the nature of things, not *ex vi termini*; or, if we call essential those judgements whose predicate is part of the arbitrary[2] definition of their subject, they will be essential in a different sense, and instructive only as statements about the meaning of a name; intended

[1] F. H. Bradley, *Principles of Logic*, p. 48: cf. Sigwart, *Logic*, § 18. 4 (E. T., Helen Dendy, vol. i. p. 108).

[2] Arbitrary (though not therefore settled without good reason) because what we are defining is something of our own institution, or because our so-called definition is a compromise of the nature explained pp. 99–102, *supra*. In the strict sense of definition, none is arbitrary: things are what they are.

otherwise the propositions are mere tautologies, and not expressive of any real act of judgement at all. It will be seen therefore that the three antitheses, of analytic and synthetic, essential and accidental, verbal and real, are by no means equivalent; they are neither made on the same *fundamentum divisionis*, nor do they respectively bring together and keep apart the same individual judgements.

[Some further points deserve notice in regard to the distinction of *analytic* and *synthetic* judgements.

1. The terms suggest that we in judgement pick to pieces or put together the object of our thought. And some who use the terms hold that in the last resort this is true; that mind by its activity constitutes its objects, though not perhaps as individual mind, yours or mine.[1] But whatever be the ultimate relation of mind to its objects, what the individual means to assert in judging is a relation of elements in the real that holds irrespectively of his present judgement. A judgement then is analytic in so far as it recognizes the distinct elements in what the judger starts by envisaging as an unity; synthetic in so far as it recognizes the union—whether by way of necessary connexion or of empirical conjunction—of elements which the judger starts by envisaging as distinct.

2. But hence, because the judger does not lose sight of his starting-point, it has been said[2] that all judgements are at once analytic and synthetic. In the sense that in all judgements we assert a diversity in unity, a many in one, this is true. But the relation of the elements, their mode of combination in the unity, is not always the same.

3. It has also been said that the same judgement may be analytic to one person, and synthetic to another: that, e.g., a judgement analytic to a teacher stating what he already knows is synthetic to a learner receiving information new to him; and similarly that a judgement may be synthetic at one time and analytic at another to the same person, and that to any one omniscient all judgements would be analytic. But this is an error. The view rests on the following consideration, that if, e.g., I learn for the first time that diamonds are combustible, I make a synthetic judgement, because to be combustible was no part of what I understood by the word diamond; but having learnt it, I include that in what I mean by the word, and henceforward, when I judge that diamonds are combustible, my judgement is analytic. Now, were this so, it is clear that the name *diamond* would have come to be used by me with a different meaning, i.e. the subject-concept would be different, in the judgement afterwards expressed by the words 'diamonds are combustible', from what it had been in the judgement expressed

[1] Cf. e.g. Bosanquet, *Logic* [2], vol. i. p. 84, vol. ii. p. 237, and Bk. II. c. x.
[2] e.g. Bosanquet, *Logic* [2], vol. i. p. 91.

[by the same words before. The earlier synthetic and the later analytic would not therefore be the same judgements, though expressed in the same proposition. Thus at best the view would involve a confusion between the judgement and the proposition.[1] But it is not even true that, when I know that diamonds are combustible, the meaning of the word diamond must change for me. The judgement is synthetic because combustibility is not something without which the nature of a diamond would cease to be conceivable. That fact is not changed by my learning that diamonds are combustible. What I know or think once I may know or think again; and the nature of a judgement is not altered by my having made it before. We must, however, acknowledge that there are certain differences in the state of mind of one who makes a judgement for the first time and one who repeats it; there are emotional accompaniments in the former case, or a preceding attitude of expectation, not present in the latter.]

Two comparatively unimportant classes of proposition, **exceptive** and **exclusive**, may be mentioned before closing this chapter. An exceptive proposition is one which excepts from its application a certain part of the extension of the subject[2]: as in Clough's satirical version of the Second Commandment—'No graven images may be Worshipped, except the currency.' An exclusive proposition is one which confines the application of the predicate to the subject of which it predicates it: as in Elijah's exclamation, 'I, even I only, am left.' Within a given whole, it clearly makes no difference whether a predicate is affirmed of one part only, or denied of all but that: *Only the brave deserve the fair* would mean the same as the poet's actual line *None but the brave deserve the fair*. The scholastic logicians treated these and some other forms of proposition under the head of **Exponibilia**, i.e. statements whose full meaning could only be expounded in more propositions than one. Thus 'None but the brave deserve the fair' or 'Only the brave deserve the fair' implies two propositions, that the brave (or some of them) deserve the fair, and that those who are not brave do not. The infinite proposition was also an exponible; for if I say that Parliament is not-in-session I imply that it is not in session, and is in some other state instead.

[1] *v.* L. Nelson, *Ueber das sogenannte Erkenntnisproblem*, pp. 36–40.

[2] In strictness, of what would otherwise be the subject: as the part excepted cannot be called part of the subject of a judgement which expressly does not apply to it.

CHAPTER IX

OF THE DISTRIBUTION OF TERMS IN THE CATEGORICAL JUDGEMENT: AND OF THE OPPOSITION OF JUDGEMENTS

We saw in the last chapter that all categorical judgements,[1] in respect of their quality, were either affirmative or negative; and in respect of quantity, might be treated as either universal or particular. The latter division indeed strictly applies to those judgements only whose subject is a general term, and therefore not to singular judgements; but for the purposes for which these can be reckoned with universal judgements the division is exhaustive. These purposes are the determining the *distribution* of terms, together with what depends on that. A term is said to be **distributed**, when it is used in reference to its whole extension, or to all that it can denote: **undistributed**, when not so used.[2] Now the subject of a singular judgement [3] denotes one individual only, and the judgement [3] refers to that; the subject of an universal judgement [3] is general, and may denote any number of individuals, but since the judgement is universal, it applies to them all. Therefore

[1] By judgement in this chapter will be meant categorical judgement.

[2] We have already seen, in discussing the extension, or denotation, of terms, that confusion may arise between the relation of a generic concept to the more specific concepts included under it and the relation of the universal to the individual, and that, properly speaking, a singular term has no extension, but only denotes. But in considering the distribution of terms, it is not always necessary to bear in mind this distinction. I may therefore say indifferently that a term is used with reference to its whole extension, or to all that it can denote, even if we reserve the latter expression (denotation) to signify the individuals of which a term can be predicated.

[3] More strictly, of a proposition expressing a singular, or an universal, judgement. It is terms verbal that are distributed or undistributed, according as the term of thought, what they cause or help us to think of as subject or predicate in a judgement, is or is not all that they can denote. For this reason it might seem more proper to speak only of the distribution of terms in a proposition. But since it is the act of thought or the judgement that gives to the terms of the proposition in which it is expressed their distribution, we may also speak of the distribution of terms in a judgement; and because it is important to bear in mind that terms have distribution only through our use of them in judging, not through their presence in a sentence, I have spoken thus.

in both singular and universal judgements, all that the subject can denote is referred to, or, in other words, the subject is distributed; and, in considering the distribution of terms in a judgement, we may accordingly rank the singular with the universal.

As every judgement has both quantity and quality, and in each respect there are two alternatives, there are four varieties of judgement in respect of these two characters combined. An affirmative judgement may be universal or particular: a negative judgement may be universal or particular. It is customary in Logic to indicate these four forms of judgement by the first four vowels, thus:—

an universal affirmative judgement is indicated by the letter *A*;
an universal negative „ „ „ „ „ „ *E*;
a particular affirmative „ „ „ „ „ „ *I*;
a particular negative „ „ „ „ „ „ *O*.

Thus the affirmative judgements are *A* (universal) and *I* (particular): the negative judgements are *E* (universal) and *O* (particular); and this may be remembered by noting that *A* and *I*, which indicate the universal and particular affirmative judgements, are the first two vowels in the verb '*affirmo*': *E* and *O*, which indicate the universal and particular negative judgements, the vowels in the verb '*nego*'.

All universal judgements (*A* and *E*) distribute their subject: all negative judgements (*E* and *O*) distribute their predicate. No particular judgements (*I* and *O*) distribute their subject: no affirmative judgements (*A* and *I*) distribute their predicate. Thus:—

in *A*, the subject is distributed, the predicate undistributed;
in *E*, „ „ „ distributed, „ „ distributed;
in *I*, „ „ „ undistributed, „ „ undistributed;
in *O*, „ „ „ undistributed, „ „ distributed.

It is important to understand and become familiar with these characteristics of a judgement.

A term, as was explained just now, is said to be distributed when it is used with reference to all that it can denote.[1] The term 'book' is distributed, when used as subject in a proposition that refers to all books: undistributed, when so used in a proposition that does not

[1] i. e. denote univocally: an equivocal term is to be regarded as a different term in each sense.

refer to all books. It is obvious that an universal proposition about books (whether affirmative or negative) refers to all; and that a particular proposition does not: *all books are written before being printed*: *no book was printed before* 1450[1]: *some books are published unseen*: *some books are never published*. That the subject of universal propositions is distributed, and of particular propositions undistributed, needs no further illustration. Two cautions, however, may be offered.

1. The subject of a proposition is the whole subject-term; if I say *all modern books are printed from movable type*, the subject is not *books*, but *modern books*; it is true that my judgement does not refer to all books, but it refers to all modern books, and so the subject is still distributed; while it is undistributed in the proposition *some modern books are printed from stereotype plates*. But I may restrict a general term like *book* not by words which leave it still general (e.g. *modern book*, *book printed by Elzevir in Leyden*), and therefore capable of being either distributed or undistributed, but by a demonstrative pronoun, or other words which destroy its generality (e.g. *that book*, *these books*, *the first book which I ever possessed*). In the latter case, the term becomes a designation, and is therefore singular, or (like 'these books') a singular collective; and the proposition should rank with universals. Nevertheless the general term which is restricted, by a demonstrative or otherwise, to the designation of a particular individual, is not distributed, since it does not refer to all that it can denote. 'Book' therefore is undistributed, but 'this book' is distributed, in the proposition 'This book wants rebinding'; for 'book' might be used of other books, but 'this book' is already used of the only book of which, so long as I mean the same by 'this', it can be used.

2. In speaking of the distribution of terms, we are inevitably led to view judgements in extension rather than intension: and indeed as referring (ultimately) to so many *individual* subjects, rather than asserting a connexion between universals. Now we have seen that a judgement may refer to individuals, but need not; and that in a judgement properly universal, there is no express thought of individuals. In saying that a triangle has its angles equal to two right angles, I am not thinking of all the particular triangles that have ever existed or may exist; I am thinking of their

[1] The proposition must be taken to refer to European books and movable type: the first dated examples being of 1454.

common character as triangles; this is one and the same in them all, and so I use the indefinite singular, *a* or *any* triangle.[1] It may therefore appear erroneous to say that such a judgement distributes its subject, if to distribute a term is to use it with reference to all that it can denote; for of all the individuals which the term triangle can denote I am not thinking. But it is true in this sense, that whatever particular triangle you choose to take, my judgement holds good of that. We must avoid supposing that in every universal judgement we are expressly thinking of all the different individuals of which the subject-term is predicable; but we must recognize that our judgement holds of them all.

The distribution of the predicate in a judgement is not generally so readily understood as that of the subject; for the extension of the predicate is not naturally before us. The rule is that negative propositions distribute their predicate; affirmative do not: and this equally whether they are universal or particular.

All preachers praise virtue: some practise it. It is easy to see here that I refer in one case to all and in the other only to part of what the term *preacher* can denote. The subject therefore is distributed in one case, and not in the other. But what of the predicate? That is distributed or undistributed not as it refers to all or only some preachers; for a term is distributed or undistributed when it is used in reference to the whole or to a part only of *its own* extension, not of the extension of the subject of which it is predicated. Now the extension of the terms '*praiser of virtue*' and '*practiser of virtue*' includes everything which can be said to praise or practise virtue. Preachers may do so, but so may others who are not preachers; these also therefore are included in the extension of the predicate; but what is thus included is not predicated of preachers. In the judgement 'X is Y', I predicate Y of X; but I might predicate it also of Z; X and Z are both included in the extension of Y, or in what Y can denote; but when I affirm Y, I do not affirm it in its whole extension; for then in saying 'X is Y', I should mean that it is X and Z, and in saying 'Z is Y', I should mean that it is Z and X. The predicate therefore is not used in reference to its whole extension, i. e. is undistributed.

The predicate of an affirmative judgement in fact cannot be

[1] I do not deny that a particular 'representative' triangle must be considered in making the judgement.

thought in extension at all. The subject of which it is predicated forms part of its extension ; but in the predicate, as opposed to the subject, I am thinking of a character or attribute belonging to that subject. A great deal of the difficulty which hangs about the doctrine of the distribution of terms arises from the fact that a term is said to be undistributed both when it is used with explicit reference to a part only of its extension, and when it is used without explicit reference to its extension at all. The subject of a particular judgement is undistributed in the former sense ; when I say that *Some preachers practise virtue*, I am explicitly confining my statement to a part of the extension of the term *preacher*. The predicate of an affirmative judgement is undistributed in the latter sense. When I say that *All preachers praise virtue*, though it is true that preachers, even all of them, are only part of the extension of the predicate, yet I am not thinking in the predicate of its extension but of its intension. The extension of a term consists of all the alternative species, or different individuals, in which its intension is manifested. It is impossible to predicate all the alternative species of the same subject, or to say of anything that it is so many different individuals. 'An ellipse is a conic section.' The extension of the predicate *conic section* is hyperbola, parabola, ellipse, circle ; I cannot say that an ellipse is all of these ; I do not want to say that it is an ellipse ; I am thinking of the common character in them all, i.e. using the predicate in intension. Still, it is only part of the extension of the predicate which is referred to in this judgement, and therefore the term is said to be undistributed in the judgement, though in the predicate extension is not considered at all.

In a negative judgement, on the other hand, the predicate is necessarily denied in its whole extension. *Caesar is not ambitious ;* there are a thousand varieties of ambition among mankind ; but if I deny ambition of Caesar, I deny all these. It is the same whether the judgement is universal or particular. *No Mussulman fears death.* Whether we look to the forms which fearing death may take, or to the individuals in whom it is exhibited, if I deny the predicate of Mussulmans, I deny all forms of it, or deny that they are any of those individuals in whom it is exhibited. But again, *Some marine animals are not vertebrate* ; of those animals I do not merely deny that they are dogs or cats, plaice or salmon, all of which form part of the extension of vertebrate ; vertebration in every form is denied of them ; a negative judgement denies its predicate *in toto.*

In an affirmative judgement, the subject is necessarily part of the extension of the predicate; in a negative judgement it is as necessarily no part thereof. And to say that the subject is no part of the extension of the predicate is to say that the predicate is denied in its whole extension.

But here again it is primarily the intension of the predicate which is in my mind. When I say that 'Brutus is an honourable man', the only individual referred to is Brutus, though 'they are all honourable men that have slain Caesar'; when I say 'Caesar was not ambitious', I need not be thinking of any one who was. It is an attribute which I affirm in one case and deny in the other. Nevertheless, whereas if I do attend in affirmative judgements to the extension of the predicate I cannot affirm the whole, and do not want to affirm the only part—viz. the subject of the same judgement—which is referred to, for that would be mere tautology, in a negative judgement, if I attend to the extension of the subject, I can deny the whole. 'A cycloid is not a conic section'; if I remember that *conic section* includes hyperbola, parabola, ellipse, and circle, I can say that a cycloid is neither an hyperbola nor a parabola nor an ellipse nor a circle.

We are not thinking primarily of the extension of the predicate in a negative judgement; but if we do think of it, we must deny it *in toto*, or else our proposition will not mean what we intend it to mean; therefore the predicate is distributed. 'The Tenth don't dance'; we are not thinking of those who do; but bears dance, and so are part of the extension of the predicate, and if the predicate were not denied in its whole extension, it would be compatible with the truth of that proposition to say that the Tenth Lancers were bears; or if the predicate were used only in reference to the ursine portion of its extension, the proposition would mean no more than that the Tenth were not bears.

[Sometimes the device of circles, representing the extension of the subject and the predicate, is used in order to explain the distribution of terms. Collect the mammals in one circle, and the snakes in another: then if no snakes are mammals, snakes will lie outside the whole mammal-area: and if some craniates are not mammals, some part of the craniate-area will lie outside the whole mammal-area; whereas if some craniates are mammals, some part of the craniate-area will coincide either with the whole or with a part

[only of the mammal-area; and if all mammals are craniates, the mammal-area will fall completely within the craniate-area. But all the objections which lie against representing in this figurate way the logical relation of a larger to a smaller class within it lie equally against so representing the distribution of terms. We may say that the negative proposition *snakes are not mammals* excludes snakes from the whole class of mammals, and not merely from a portion of it (say men): but we must not think of the class as an area cut up into districts called species, or as a collection of which the species are component groups. And if we ask what is meant by saying that a larger class *craniates* is partially coincident with the whole of a smaller class *mammals*, we must answer that the relation is not that of one superficies partially coincident with another, but of an universal character exhibited in a certain kind of subjects; in fact, the logical relation must explain the diagram, and cannot be explained by it.]

[Any one who realizes that the predicate of a proposition is not thought in extension will see that there can be no truth in the doctrine of the *Quantification of the Predicate*. But the doctrine has the support of distinguished writers, among others of Sir William Hamilton, who invented it, and of Stanley Jevons; and it ought perhaps to be examined here. It may be easily shown to be false; and the conscientious student haply stumbling upon the mass of intricate technicalities based upon it may be glad to feel excused from the labour of mastering them by the knowledge that they are built upon a worthless foundation.

By quantification of the predicate is meant affixing a mark of quantity to the predicate as well as the subject of a judgement. Thus instead of the four forms of judgement, *A*, *E*, *I*, *O*, we get eight, as follows:—

U. All *X* is all *Y*. All organisms are all mortals.
A. All *X* is some *Y*. All men are some mortals.
Y. Some *X* is all *Y*. Some mortals are all men.
I. Some *X* is some *Y*. Some men are some (things) fleet of foot.
E. No *X* is any *Y*. No snakes are any mammals.
η. No *X* is some *Y*. No men are some mammals [e.g. not monkeys].
O. Some *X* is no *Y*. Some mammals are not any quadrupeds.
ω. Some *X* is not some *Y*. Some quadrupeds are not some mammals [e.g. not cows].

In defence of this mode of stating propositions it is urged that as the proposition whose predicate has *all* before it, and the corresponding proposition whose predicate has *some* before it, do not

[mean the same thing, and we must know which we mean when we judge, we ought to express it. It is strange, if that is the case, that no language ever has expressed it; and it may be confidently asserted that of these eight forms of proposition only *E* and *O* express anything that we ever really mean when we make a judgement (though others express, in 'portmanteau' fashion, what we mean when we make two judgements); and that the reason why we ought not to express in our proposition whether we mean *all* or *some* before the predicate, is that we mean neither.

Let us take an *A* proposition. It used to be stated 'All *X* is *Y*'; we are told to state it 'All *X* is some *Y*'. *All men are some mortals*: which mortals are they? the horses? the grass of the field? clearly not, but only the men. Yet it can hardly be meant by the proposition, that all men are men; it is something about men that the proposition tells us. What about them? that they die, and not which kind they are among the kinds of things which die; we know that they are men already, and that need not be repeated in the predicate.

But there is a difference between saying that all men are all mortals, and saying that all men are some mortals; the first implies that the terms are commensurate, that there are no mortals but men: the second that men are mortal, but an undetermined range of things (cats and dogs and horses and asses and what not) are so besides. Ought not this difference to be expressed?

Doubtless, but it requires another proposition; *All men are mortals—some mortals are not men*. In recognizing that men die, we do not judge that things of any other kind die; and though we may be aware of it when we say that men die, it is no part of the judgement *Men die*. *All men are some mortals* is not one judgement, but a 'portmanteau' proposition—two judgements expressed in what (in respect of its grammatical form) is one sentence.

It is true that in some judgements we expressly think the predicate and the subject to be commensurate. In a definition, we must do this. *Momentum is the product of mass into velocity: wealth is that which has value in exchange*; in these cases, it is included in our thought that the product of mass into velocity is momentum, or that which has value in exchange, wealth. But such judgements are ill expressed in the form 'All *X* is all *Y*'. We do not think of all momenta, all samples of wealth, but of wealth and momentum each as one thing. Again, the formula 'All *X* is all *Y*' makes us think of *X* and *Y* as different things: whereas the whole force of a definition is to assert that the subject and predicate, the thing defined and the definition of it, are the same thing.

There are propositions whose terms are known to be commensurate, but which are not definitions, such as *all equilateral triangles are equiangular*. These also we are told to represent in the form

['All *X* is all *Y*', and to say that *all equilateral are all equiangular triangles*. But this does not correctly express the true meaning of the other proposition. For granted that in enunciating it we are aware that the terms are commensurate: what we wish to assert is the mutual implication of two attributes in any triangle. It follows from this that every triangle exhibiting one exhibits the other; but those which exhibit one are not a different set of triangles from those that exhibit the other. By putting a mark of quantity before the predicate as well as before the subject, we make it appear as if the extension of one term was affirmed of the extension of the other, and (if we consider individuals) as if the individuals denoted by one term were affirmed of the individuals denoted by another. But that is either impossible, if the individuals are different, or tautologous, if they are the same.

'All' can be no part of any predicate, except where (as in *these are all the apostles*) the subject is collective. If the universal judgement 'All living things reproduce their kind' is true, then it is true of any living thing and therefore of peas. I may introduce 'perfectly' into the predicate, and then it will be meant that peas reproduce their kind perfectly. But I cannot introduce 'all' into the predicate. For then, since all living things are all things that reproduce their kind, peas and even a single pea would be said to be all things that reproduce their kind; and that is nonsense. The predicate of a judgement is affirmed distributively of each that falls under the subject; the predicate quantified by all could be only true of the subject collectively. No equilateral triangle is all equiangular triangles; how then can they all be? The proposition only means that all equilateral triangles are equiangular and vice versa. As before, it is a 'portmanteau' proposition, and not a single judgement.

The *U* form of proposition has been considered at some length, because it is in a way the most plausible member of the series. Universal judgements whose terms are commensurate do differ from those whose terms are not, and do form a very important class of judgements; and there is no special recognition of them in the ordinary fourfold classification of judgements (*A*, *E*, *I*, and *O*). It has been wrongly alleged that Aristotle ignored such judgements; on the contrary, he recognized their great importance in science. To remedy this supposed omission the doctrine of the quantification of the predicate offers us an entirely false analysis of them, and one which Aristotle himself exposed.[1] The analysis overlooks altogether

[1] *De Interp.* vii. 17^{b} 12 ἐπὶ δὲ τοῦ κατηγορουμένου καθόλου κατηγορεῖν τὸ καθόλου οὐκ ἔστιν ἀληθές· οὐδεμία γὰρ κατάφασις ἀληθὴς ἔσται, ἐν ᾗ τοῦ κατηγορουμένου καθόλου τὸ καθόλου κατηγορεῖται, οἷον ἔστι πᾶς ἄνθρωπος πᾶν ζῷον. (ἄνθρωπος, man, is an universal: when I say 'All men are animals', I predicate of an universal universally; when I say 'Some men are white', I predicate of

[the *intension* of terms. Professing to complete what is defective in the current recognition of different kinds of proposition, itself leaves important differences unrecognized. We have seen that a proposition of the form 'All X is Y' represents two kinds of judgement essentially different in thought, according as it is really universal, meaning 'X as such is Y', or only enumerative, meaning 'All the X's are Y'. Of this difference, whether in universal judgements whose terms are commensurate (U) or not (A), this doctrine takes no note; but sets up instead two kinds which misrepresent our thought by the sign of quantity prefixed to the predicate.

The particular affirmative propositions may be dismissed briefly. We are told that 'Some X is Y' should be written either 'Some X is some Y' or 'Some X is all Y'. Take the former, 'Some X is some Y': we ask immediately, which X are which Y?; and the only answer is that the X that are Y are the Y that are X. *Some sowers reap*; if that means *some sowers are some reapers*, this can only mean that the sowers who reap are the reapers who sow. Take the latter, 'Some X are all Y'; *some animals are all the pigs* (for it does not mean, *are all of them pigs*: as we might say that some families all squint, meaning that all the members of some families squint). Which animals are all the pigs? surely only the pigs themselves. If it be said that the proposition means that there are more animals than pigs, then the real subject of the judgement is the other animals (which are not pigs), and not (as this form pretends) the animals which are pigs. If, again, it be said to mean that all pigs are animals and some animals are not pigs, we have as before two judgements packed into one sentence. What is one judgement, and what is the character of a judgement, are questions to be determined by considering our thought, and not the verbal devices we adopt to express it. To *think* that all pigs are animals, and some animals are not pigs, is to judge not once but twice, even though we were to write such a pair of judgements in the form *some animals are all pigs*.

an universal particularly, or in part. Aristotle goes on to say, in the words quoted, that the predicate cannot be similarly taken universally [i.e. not 'as an universal', but 'in its whole extension']. 'But in the case of the universal which is predicate, it is not true to predicate universality; for no affirmation is true when universality [in extension] is assigned to the predicated universal, e.g. All men are all animals.' Cf. Ammonius in loc. f. 82, who points out that then each man would be all animals.) *Anal. Pri.* *a.* xxvii. 43^{b} 17 *αὐτὸ δὲ τὸ ἐπόμενον οὐ ληπτέον ὅλον ἔπεσθαι, λέγω δ' οἷον ἀνθρώπῳ πᾶν ζῷον ἡ μουσικῇ πᾶσαν ἐπιστήμην, ἀλλὰ μόνον ἁπλῶς ἀκολουθεῖν, καθάπερ καὶ προτεινόμεθα· καὶ γὰρ ἄχρηστον θάτερον καὶ ἀδύνατον, οἷον πάντα ἄνθρωπον εἶναι πᾶν ζῷον, ἢ δικαιοσύνην ἅπαν ἀγαθόν.* ('But the attribute must not be taken to be attributed *in toto*, I mean for example animal as a whole to man, or science as a whole to music, but just simply to follow on the subject, as our premiss says; for the other is both useless and impossible, e.g. that all men are all animals, or that justice is all good.')

[To the negative judgement also the quantification of the predicate does violence. The universal negative is to appear in the two forms 'No X is any Y' (E) and 'No X is some Y' (η). The former may stand; for as we have seen, if X is not Y, it is not any case or kind of Y. The latter may well puzzle us. It denies of X some part of the extension of Y; *pig*, for example, is part of the extension of animal, and sheep are not pigs; hence sheep are not some animals; but this is quite consistent with their being animals. 'No X is some Y' is therefore consistent with 'All X are Y', and what it means is that 'Some Y are not X'; whether any X are Y or not it leaves doubtful. There remain the particular negatives, 'Some X is not any Y', and 'Some X is not some Y'. Again the former will stand; but what does the latter mean? It does not mean that some X is not Y at all, e.g. that some animals are not pigs at all, but are something quite different (say sheep or cows); for that is expressed by the form 'Some X are not any Y'. It can only mean that there are some Y's distinct from some X's: i.e. that though some X may be Y, they are not every Y. 'Some murderers are not caught' is sense; but 'Some murderers are not some caught', if different from that, and sense at all, is only true because fish and cricket-balls are also caught, and some murderers are not these; so that if the proposition were to be false, they would have to be fish and cricket-balls and everything else that is ever caught; it is the contradictory of the impossible judgement that those X are every Y. But as we never make that judgement, we never want to contradict it; yet these are forms of judgement which those who would quantify the predicate condemn Logic for hitherto ignoring.[1]

Thus all the eight forms of proposition with quantified predicate have been found vicious, except E and O; and these are so interpreted as to lay undue stress on the aspect of extension in the predicate. The truth is that if we prefix to the predicate of a proposition a mark of quantity, *all* or *some*, we are bound to think of the various individuals (or species) characterized by the predicate, not merely of the character, or 'universal': we are bound to take the predicate in extension, and that we cannot or do not wish to do. We cannot affirm of one term the extension of another. If a set of individuals, or of species, forms the subject of an affirmative

[1] We might make them a present of certain forms which they appear to have overlooked. If the extension of Y be p, q, r, then 'No X is any Y' means 'No X is either p or q or r'. But the parts of the extension are taken disjunctively: why should they not be taken together? Then we should have the form 'No X is all Y'—meaning that no X is both p and q and r. So we might have 'Some X are not all Y'. It is true these forms are useless; and in that they resemble the affirmative forms 'All X are all Y' and 'Some X are all Y'. But they have the advantage over those of being true.

[judgement, another set cannot form the predicate. 'All X is some Y' is meaningless. 'Some Y,' we are told, means 'part of the class Y'; but which part is X? Let the class Y be divided into two parts, X and Z; we do not need to say that X is the former part; it is false to say that it is the latter. And in a negative judgement, unless the predicate is a proper name, which has no extension, what we wish to deny of the subject is having the predicate character, not being those individuals which have it.

Still, it is urged, the judgement compares the extension of two classes. 'All X is all Y' means that the class X and the class Y are co-extensive; 'All X is some Y' means that the class X is included in the class Y, which extends beyond it. But if the class X and the class Y are co-extensive, how are they two classes? Taken strictly in extension (as the doctrine of the quantification of the predicate takes its terms) the class X and the class Y are not the common character X and Y realized in many things, but the set of things in which this character is realized. If the class X is the things in which the common character X is realized, and Y is realized in the same things, then there is only one class or set of things, and not two co-extensive classes; so that, after all, we have the class X, and predicate the character Y of them, i. e. we do not take Y in extension. And if the class X is included in the class Y, what does that mean? Suppose that all Y's were collected in one place, all X's would be found in the crowd; then, when we said that all X is some Y, we should mean that all X were included in the crowd of Y's. But now our predicate is no longer Y, and has become 'included in the crowd of Y's'. We must quantify that if all predicates are to be quantified, and state whether all or part of what is included in the crowd of Y's be meant. Clearly part; so that our judgement will run 'All X are some things included in the class Y (or crowd of Y's)'. But which things so included are they? as before, themselves, the X's. If this answer be not accepted, and it be said that *some* means 'included in the class of', then our new judgement must run 'All X are included in the class of things included in the class Y'. But now the last eleven words become the predicate, and it must again be quantified; we must say 'All X are some things included in the class of things included in the class Y'. So the process goes on *ad infinitum*. You cannot predicate of one class the whole or part of another. You may compare the size of two classes: e. g. when we say that male infants are more numerous than female; but then one class is not predicated of another; female infants do not include male infants and extend beyond them. You may predicate a genus of a species, and the genus as compared with the species has a wider extension; but it is not the extension of the genus which you predicate of the species, nor any part of it.

It may be thought that in discussing the quantification of the

[predicate we have been belabouring errors too trivial for notice. No one, of course, really supposes that the act of judgement means any of these absurdities. But many people have supposed that a judgement compares the extension of two terms, or includes a subject in or excludes it from a class; and they think of a class as *so many* things or kinds of thing. Such views imply the absurdities that have been dragged to light; and the custom of elucidating the relation of terms in a judgement by the relative position of circles on paper, outside each other, one inside the other, or with a common segment, tends, as has been said before, to make us think wrongly about a judgement precisely in the direction of these absurdities. It is of great importance, in speaking of the distribution of terms (as we shall have to do frequently when examining the syllogism), not to suppose that the terms of a judgement are all taken in extension, and that we are always identifying and distinguishing all or part of what our terms denote. The doctrine of the quantification of the predicate flourishes upon this mistake, and a thorough examination of that doctrine is a good prophylactic measure. Moreover, many of the developments of Symbolic Logic [1] are based on the extensional implications of propositions. If I say that all mammals are craniates, I imply that there are not fewer craniate animals than mammals; hence I may write, for 'X is Y', '$X = XY$', and substitute XY for X elsewhere in my equations. If all organisms are mortal, and every mortal an organism, I may write '$X = Y$', and substitute accordingly. When symbols are carefully devised, we can represent propositions symbolically, and operate with our symbols without realizing their meaning, and so reach results which we can retranslate into propositions whose meaning we realize, and whose truth follows from that of the premisses which we put into symbols at the outset. But the success of such operations does not show that we mean by our categorical propositions to assert numerical equality between classes, but only that, if what we mean is true, then, whether we determine our class by the subject character, or by it and the predicate-character together, we shall take the same things, and so the same number of things. We are not always thinking of classes and their numerical relations when we judge. Hence, as it seems to me, the error of representing either all thinking as a kind of mathematics, or all thinking as class-thinking, and mathematics as merely a special sort of class-thinking.[2]]

We may pass now to the *opposition* of propositions or judgements.

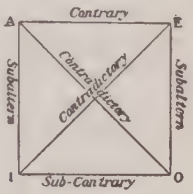
Propositions having the same subject and predicate, but differing in quantity, or quality, or both, are said to be **opposed** to one

[1] e.g. Jevons's *Equational Logic*.
[2] Cf. Mr. Bertrand Russell's *Principles of Mathematics*.

another. The four forms of proposition *A*, *E*, *I*, *O* admit four kinds of opposition among them.

1. *A—E*. Where the propositions differ in quality, and are both universal, they are called **contrary** to each other: *everything in Aristotle is true, nothing in Aristotle is true* are contrary propositions.[1]

2. *I—O*. Where they differ in quality, and both are particular, they are called **sub-contrary**: e. g. *some things in Aristotle are true, some things in Aristotle are not true.*



3. *A—O, E—I*. Where they differ both in quantity and quality, they are called **contradictory**: e. g. *everything in Aristotle is true, some things in Aristotle are not true: no Mussulman fears death, some Mussulmans fear death.*

4. *A—I, E—O*. Where they differ in quantity but not in quality, they are called **subaltern**: e. g. *everything in Aristotle is true, some things in Aristotle are true: no Mussulman fears death, some Mussulmans do not fear death.*

Contrary and *contradictory* are terms in common use, though sometimes treated as equivalent; the origin of the terms *subaltern* and *sub-contrary* may be seen in the above-given, and ancient, 'diagram of opposition'. *I* is placed under *A*, and *O* under *E*, for the same reason that in setting out a classification we place the species under the genus: the wider includes the narrower under it: *A* and *I*, *E* and *O* are called subaltern, because in each pair one is subordinated to the other: *I* and *O* are called sub-contrary, because they are subordinated to the contraries *A* and *E*, their respective universals.

It will be observed that in order to overthrow an universal proposition, affirmative or negative, it is only necessary to establish

[1] Contraries are what stand furthest apart upon a scale of some kind—*τὰ μάλιστα διεστηκότα ἐν τῷ αὐτῷ γένει*: as white and black on the scale of illumination, highest and lowest on the scale of elevation, or of pitch, &c. Contrary propositions are those which stand furthest apart on the scale of quantity: one asserting that to be true of all which the other asserts to be true of none. The notion of contradiction belongs properly to judgements only, and not to terms, though sometimes transferred to the latter, *A* and not-*A* (blue and not-blue, &c.) being called contradictory terms. (Cf. Ar. *de Interp.* 20ª 31–36.) But we have seen that mere not-*A* is no term at all: there must be some positive meaning. (See however Bradley, *Principles of Logic*, p. 119, for the view that all disparate or incompatible terms should be treated as contraries: e. g. blue and red. 'In logic the contrary should be simply the disparate.')

the particular negative or affirmative; that everything in Aristotle is true is refuted by showing something in his writings false; that nothing in Aristotle is true, by showing something true. We contradict the affirmation 'All men are liars' by saying 'not all', not by saying 'all not'. But of course the greater includes the less, and we refute a proposition by establishing its contrary, as well as by establishing its contradictory. In common speech therefore we are said to contradict a proposition when we advance another whose truth is inconsistent with that of the first, whether it be the contrary or the contradictory; and since the contrary imputes more error than the contradictory (for if a man tells me that all animals reason, I impute more error to him by replying that none do, than that some don't) it may in a sense be said to contradict more fully. It is, however, convenient to have different words to mark the relation of *A* and *E* to each other, and their relations to *O* and *I* respectively; and Logic confines the title of contradictory opposition to the latter.

Given the truth or falsity of any proposition, we can see at once which of the opposed propositions must be true, which false, and which (upon the information given us) remain doubtful. For contrary propositions cannot both be true, and therefore if *A* is true, *E* must be false, and vice versa: but they may both be false (for it is not necessary that either all babies should be disagreeable, or else none of them), and therefore if one is given as false, the other remains doubtful. Contradictory propositions cannot both be true, but neither can they both be false; and therefore if *A*, *E*, *I*, or *O* is true, *O*, *I*, *E*, or *A* must respectively be false, and vice versa. Subaltern propositions may both be true, or both false, or the particular may be true while the universal is false; but the particular cannot be false while the universal is true, for the greater includes the less; hence given the truth of *A* or *E*, *I* or *O* is true, and given the falsity of *I* or *O*, *A* or *E* is false; but given the falsity of *A* or *E*, *I* or *O* remains doubtful, and given the truth of *I* or *O*, *A* or *E* remains doubtful. Sub-contrary propositions cannot both be false (for in that case their respective contradictories, which are contrary to one another, would both be true); but they *may* both be true, just as contraries may both be false; hence given the falsity of *I*, *O* is true, and vice versa; but given the truth of *I*, *O* remains doubtful, and vice versa.

Of two contrary or of two contradictory propositions one may

be advanced against the other, i. e. we may deny one, and advance the other in its place; and of two subaltern propositions, the particular may be advanced against the universal. If any one said 'Some animals reason', we could not answer 'No, but all do'; but if he said, 'All animals reason', we could answer, 'No, but some do'. Sub-contrary propositions, on the other hand, cannot be advanced one against the other. 'Some animals reason': we cannot retort, 'No, but some don't'; 'Some animals don't reason': we cannot retort, 'No (i. e. that is false), but some do'. We may indeed, to the statement that some animals reason, reply, 'Yes, but some don't'; and to the statement that some animals do not reason, 'Yes, but some do'. In these cases, however, the particular proposition 'Some don't reason', or 'Some do reason', is advanced not against its sub-contrary, 'Some do reason' or 'Some don't reason', but against the universal proposition 'All reason' or 'None reason': which it is feared we might otherwise be supposed to allow, when we admit that some reason, or that some do not. Hence it has been urged that we ought not to speak of sub-contrary propositions as opposed,[1] nor include them in a list of the forms of opposition; but if they are not opposed, they are anyhow contrasted, and that may justify their continued inclusion. Given the truth or falsity of any proposition, the step by which we pass to the perception of the truth, falsity or doubtfulness of its several opposites is in the strictest sense formal. It depends in no way upon the special content of the proposition, but solely upon the necessary relations, according to their quantity and quality, in respect of truth and falsity, between propositions having the same subject and predicate. And since no other information need be given, except whether the one proposition is true or false, in order that we may determine the truth, falsity, or doubtfulness of the remaining three, the process of inference (if inference it is to be called) is *immediate*.

[1] Aristotle notices this in *Anal. Pri.* β. xv. 63^{b} 27 *τὸ γὰρ τινὶ τῷ οὐ τινὶ κατὰ τὴν λέξιν ἀντίκειται μόνον* ('For *some are* is only verbally opposed to *some are not*').

CHAPTER X

OF IMMEDIATE INFERENCES

INFERENCE is a process of thought which, starting with one or more judgements,[1] ends in another judgement whose truth is seen to be involved in that of the former. This judgement, which, in relation to the judgement or judgements from which the process starts, is called a conclusion, must, as compared with them, be a new judgement; to repeat in fresh words our original statement is not inference, any more than translation is inference. For the most part a new judgement is only got by putting together two judgements, and as it were extracting what they yield. But there are a few conclusions which we appear to draw not from any 'putting together' of two judgements, but simply from the relation to one another of the terms in one judgement. This is called *immediate* inference, etymologically because (in contrast with syllogism[2]) it proceeds without the use of a middle term: but, to put it more generally, because we seem to proceed from a given judgement to another, without anything further being required as a *means* of passing to the conclusion.[3]

It was mentioned at the end of the last chapter, that when we infer, from the truth or falsity of a given proposition, its various opposites to be true, or false, or doubtful, we perform an act of immediate inference. We have now to consider other forms of immediate inference, of which the fundamental are *Conversion* and *Permutation* (or *Obversion*).

A proposition is **converted,** when its subject is made the predicate,

[1] Or, more generally, elements, if we allow (with Bradley, *Principles of Logic*, pp. 370–373) that, e.g., $2+2=4$ is inference. But the above is not intended as a final definition of inference. Cf. *infra*, p. 244.

[2] For the function of the middle term in syllogism, cf. *infra*, c. xi.

[3] All inference is immediate in the sense that from the *premisses* we pass without the help of anything else to the conclusion; but this is called immediate in the sense that from the given relation of two terms in a *single* proposition we pass without the help of anything else to a different proposition. It is doubtful, however, whether, so far as there is any inference in it at all, it is really in this sense immediate. Cf. the discussion pp. 240 sq.

and vice versa, its quality (affirmative or negative) remaining unchanged: as, for example, when from 'No true Mussulman fears death' we pass to 'No one who fears death is a true Mussulman'. The original proposition is called the *convertend*, and the new proposition its *converse*.

Whether, and in what way, a proposition can be converted, depends on its form, *A*, *E*, *I*, or *O*[1]: because the process of conversion is invalid, unless it conforms to the following rule, that *no term may be distributed in the converse, which was not distributed in the convertend*.[2] An *A* proposition is converted *by limitation*: an *E* or an *I* proposition *simply*: and an *O* proposition not at all except through first *permuting* it.

A proposition is said to be converted **simply**, when the quantity of the converse is the same with that of the convertend. In an universal negative proposition (*E*) both terms are distributed; in a particular affirmative proposition (*I*) both are undistributed. Therefore their mutual substitution in the process of simple conversion does not distribute any term that was not distributed before. Thus *E*, 'no *X* is *Y*', becomes *E*, 'no *Y* is *X*': e.g. 'no lawyers are parsons'—'no parsons are lawyers'; 'no true poet admires Macaulay's Lays'—'no one who admires Macaulay's Lays is a true poet[3]'; 'no snakes suckle their young'—'no mammals are snakes[4]'; 'Chatham is not the younger Pitt'—'the younger Pitt is not Chatham'.

Again, *I*, 'some *X* is *Y*', becomes *I*, 'some *Y* is *X*': e. g. 'some diamonds are black'—'some black stones are diamonds': 'some evergreen shrubs flower brilliantly'—'some brilliantly flowering

[1] The matter of some judgements renders their conversion unnatural, even where the form allows of it: e. g. 'Civilization spreads by the extermination of lower races'. Cf. pp. 235–237, *infra*.

[2] Another rule for conversion is sometimes given, to the effect that the terms (or the subject and predicate) of the converse must be the same as the terms (or the predicate and subject) of the convertend. But this is not a rule to observe in converting; it explains the process of conversion itself.

[3] *v.* M. Arnold, *Lectures on Translating Homer*, Popular Edition, 1896, p. 171: the question before us is not whether the proposition may be rightly contradicted, but how it may be rightly converted.

[4] When the predicate of the convertend is not a substantive or substantival term, we must either substitute for it in the converse a substantive, if there be one of equivalent meaning (as in this case), or import some substantival expression like 'one who' (as in the previous example) for the original predicate, now introduced into the subject, to qualify. We often choose the genus of the subject about which we are speaking, as in the first example of the conversion of *I*; but so far our procedure is not formal.

things are evergreen shrubs ' ; ' some victories are more fatal than defeat '—' some events more fatal than defeat are victories '.

A proposition is said to be converted **by limitation**, or **per accidens**, when, it being universal, its converse is particular. In an universal affirmative proposition Y is predicated of all X ; but it may attach to other subjects equally, P, Q, and R ; therefore what is Y need not be X, and we can only say that some Y is X, not that all Y is X. To use the language of *distribution*, the subject is distributed, the predicate not : if we merely substituted each for the other, the original predicate, become the subject of an universal proposition, would be distributed ; for ' all roses are deciduous ' we should have ' everything deciduous is a rose '. We must therefore *limit* the extent to which we affirm our original subject *rose* of our original predicate *deciduous* ; and hence such conversion is called ' conversion by limitation '. So A, ' all X is Y ', becomes I, ' some Y is X ' : ' all men are mortal '—' some mortals are men ' ; ' all Roman priests are celibate '—' some celibates are Roman priests ' ; ' all isosceles triangles have equal angles at the base '—' some triangles with equal angles at the base are isosceles '.[1]

In the last example, any one who knows geometry will be tempted to convert *simpliciter*, and say that all triangles with equal angles at the base are isosceles. He would not be wrong as a geometrician ; but he would need a knowledge of geometry, and not merely of logic, to justify him. In conversion, we look solely to what is justified by the *form* of the proposition to be converted, be it A, E, I, or O ; in this respect ' all isosceles triangles have equal angles at the base ' is indistinguishable from ' all isosceles triangles have angles equal to two right angles ' ; the geometrician knows that it does not follow from the latter, that all triangles having angles equal to two right angles are isosceles ; neither therefore does it follow *logically* from the former, that all triangles having equal angles at the base are isosceles. The *form* of proposition ' all X is Y ' only justifies a conversion to ' some Y is X ' ; in order to convert to ' all Y is X ' we must know that X and Y necessitate each other, or that there is nothing *accidental* in the relation between them ; this is not implied merely in the one being predicable of the other, because the relation of a predicate to its subject may be either accidental or essential. It must at the least be accidental, and therefore from its bare form, we are entitled to convert an A proposition as

[1] With this paragraph, cf. *supra*, pp. 223–224.

if Y were an accident of X; but we are not entitled to do more. For this reason, conversion by limitation is called conversion *per accidens* (κατὰ συμβεβηκός); if Y is an accident of X, i. e. coincides in the same individual subject with X, then X is predicable of a subject which Y characterizes, and we may say that some Y is X.[1]

In a particular negative proposition (O), the subject is undistributed, the predicate distributed; if here we substituted each for the other, the original subject, become the predicate of a negative proposition, would be distributed in the converse. And since the predicate of a negative judgement cannot, like the subject of a judgement, be limited by a sign of 'particular' quantity, an O proposition is not convertible, except *by negation*: a process which will be explained later (p. 238). This is not always realized, when we use symbols, and forbid the passage from 'some X is not Y' to 'some Y is not X'; for it is quite possible that both of these propositions may be true at once: e. g. some freemasons are not freethinkers,[2] and some freethinkers are not freemasons. But although 'some X is not Y' and 'some Y is not X' may be true at once, yet we are not justified by the form of the one in passing to the other; and this becomes obvious by comparing such an example as the last (where both propositions are true) with another, where the converse is manifestly false: e. g. 'some men are not monks'—'some monks are not men'. In form the two propositions ('some freemasons are not freethinkers' and 'some men are not monks') are the same; and therefore formally the conversion must be invalid in the former case, since it is invalid in the latter.

It is indeed impossible, in converting a proposition, to treat the terms quite like symbols, and to proceed solely by the con-

[1] Even when the predicate is known to be of the essence of the subject, we must convert *per accidens*, if the predicate is the genus: e.g. 'all men are animals'—'some animals are men'. We cannot call animal an accident of man, but we may say that it is an accident that an animal should be a man, in the sense that an animal may or may not be a man. The term *accident* is not wholly suitable, because, though the conditions necessary for the generation of an animal may exist without those necessary for the generation of a man, they cannot exist except in a form involving the generation of an animal of some species, nor can the conditions necessary for the generation of a man exist without those necessary for the generation of an animal: there is no coincidence of *independent* series, as when one series of events brings a train to a point whither another series has brought a flood and washed away the metals, and the result is a 'railway accident'. But the usage is analogous.

[2] Though certain persons on the Continent seem to believe otherwise.

sideration of the distribution of the terms in the convertend, without considering what the terms are. In an *E* proposition, for example, if both terms are proper names, the act of conversion is felt to be different from what it is where the subject is a general concrete term and the predicate attributive: in passing from 'no judge has any right to meddle in politics' to 'no one who has any right to meddle in politics is a judge', the character of the judgement alters in a way that it does not, when we pass from 'Chatham is not the younger Pitt' to 'the younger Pitt is not Chatham'. It is not natural to say 'no one who has any right to meddle in politics is a judge'; and though it is natural enough to say 'no one who meddles in politics has any right to be a judge', this is not the converse of the proposition with which we started. It is equally natural to say 'Chatham is not the younger Pitt' and 'the younger Pitt is not Chatham', according as we are discoursing about the one or the other; for two individuals stand as it were on the same level in thought, and each may indifferently be distinguished from either. But our rights depend upon our position, and not vice versa; so that it is natural to deny certain rights to a man filling a certain position, but not to deny the position to a man possessed of those rights. Other examples of the same thing might be given. A proposition both terms of which are singular is called an *A* proposition, but it cannot be converted *per accidens*: 'Chatham is the elder Pitt' can only become 'the elder Pitt is Chatham'. If the subject is and the predicate is not a singular term, conversion is a form without meaning; 'Chatham was eloquent' becomes 'an eloquent man was Chatham', and however we may write it, the latter means just the same as the former; we cannot predicate Chatham of 'an eloquent man', for this is a general term, and that a singular.[1] Again, 'Demosthenes and Cicero were the greatest orators of antiquity' becomes 'the greatest orators of antiquity were Demosthenes and Cicero'; we cannot say 'some greatest orators of antiquity were Demosthenes and Cicero' without altering the force of the term 'greatest orators' from comparative to positive. 'Some men are Christians' is a proper, 'some Christians are men' an improper mode of speech; religion can belong only to men, and we do not predicate of an attribute *partially* the subject presupposed by it. A difficulty arises again in a proposition not universal where some

[1] We can say 'That eloquent man was Chatham', but here the subject is a singular term.

measure is given of the extent to which the predicate characterizes the subject, e.g. by using such words as 'many' or 'few'; 'most great men have been of obscure origin' converts to 'some men of obscure origin have been most great men'; but no one would ever say this, for the measure 'most' applies to 'great men' as taken in extension, and therefore cannot be predicated of 'men of obscure origin'.

It would be absurd to say that as conversion is a strictly formal process, we must therefore convert propositions by its rules, according to their form as *A*, *E*, or *I*. Logic investigates the actual nature and procedure of our thought; and when we find that our thought is not governed by the bare form of a judgement irrespective of its content, it is no use to pretend otherwise. The conversion of propositions may be studied formally, with symbols for terms; but when real terms replace the symbols they must affect the judgement, and our treatment of it in conversion; for example, symbols, like *X* and *Y* in the proposition 'no *X* is *Y*', are always regarded as general terms, but the actual terms need not be general. This is said, not in order to discredit the abstract and formal treatment of conversion, which is sound within its limits; but in order to emphasize the fact that the form and matter (or the form and content) of thought are not capable of separate consideration, like the mould and the pudding: what from one point of view is form is from another matter, and the same form in different kinds of content is not altogether the same, any more than is the same genus in different species. The importance of this fact must excuse the reiteration of it; meanwhile in a text-book of Logic, as of any other science, we must consider typical cases, with a general *caveat* that the subject is thereby artificially simplified.

In conversion, the subject and predicate were transposed, but otherwise unaltered, and the quality of the proposition remained the same. In **Permutation**, or (as it has been also called) **Obversion**,[1] there is no transposition of terms, but the quality of the proposition is changed, and the predicate at the same time replaced by its contradictory. It consists in fact of substituting for an

[1] Jevons, in his *Elementary Lessons*, calls it Immediate Inference by Privative Conception. Earlier writers dealt with it under the head of Equipollency of Propositions: cf. Sanderson, II. 6 'Aequipollentia communiter sumpta est duarum propositionum, verbo tenus, quoquomodo discrepantium omnimoda in sensu conspiratio'. Aristotle, *de Interpr.* x. 20ª 20–26, notices the equivalence of a proposition and its obverse, but gives no name to the change.

affirmative or negative proposition an equivalent negative or affirmative of opposite quality, by means of negating the predicate.

Thus—

A, All *X* is *Y*, becomes *E*, No *X* is not-*Y* : All right angles are equal, No right angles are unequal ; Barkis is willin', Barkis is not unwillin'.

E, No *X* is *Y*, becomes *A*, All *X* is not-*Y* : No dogs allowed, All dogs forbidden ; Lear is not mad, Lear is not-mad.

I, Some *X* is *Y*, becomes *O*, Some *X* is not not-*Y* : Some stretches of the road are level, Some stretches of the road have no gradient.

O, Some *X* is not *Y*, becomes *I*, Some *X* is not-*Y* : Some learned theories are not sense, Some learned theories are nonsense ; Some swans are not white, Some swans are not-white.

Further transformation of a given proposition may be effected by a combination of Conversion and Permutation. The process of permuting and then converting is called **Conversion by Negation.** The conclusion so obtained may be permuted again, and this process of permuting, converting, and permuting is called **Contraposition.**

All forms of proposition except *I* can be converted by negation ; the process is inapplicable to *I*, because it becomes *O* by permutation, and a particular negative, as we have seen, cannot be converted. For the same reason *I* cannot be contraposed.

In *conversion by negation*—

A becomes *E* : All *X* is *Y* ∴ No *X* is not-*Y* ∴ No not-*Y* is *X*. All acids turn blue litmus-paper red ∴ No acids do not turn blue litmus-paper red ∴ Nothing that does not turn blue litmus-paper red is an acid.

E becomes *I* : No *X* is *Y* ∴ All *X* is not-*Y* ∴ Some not-*Y* is *X*. No stimulant nourishes ∴ All stimulants are innutritious ∴ Some things innutritious are stimulants.

O becomes *I* : Some *X* is not *Y* ∴ Some *X* is not-*Y* ∴ Some not-*Y* is *X*. Some sea-animals are not vertebrate ∴ Some sea-animals are invertebrate ∴ Some invertebrates are sea-animals. Some things necessary to life have no market-value ∴ Some things that have no market-value are necessary to life.

This is the only way in which a particular negative can be converted.

In *contraposition* [1]—

A becomes *A* : All *X* is *Y* ∴ No not-*Y* is *X* ∴ All not-*Y* is not-*X*. All Arabs are hospitable ∴ All who are not-hospitable are not-Arabs.

E becomes *O* : No *X* is *Y* ∴ Some not-*Y* is *X* ∴ Some not-*Y* is not not-*X*. No unfriendly man is happy ∴ Some who are not happy are not friendly.

O becomes *O* : Some *X* is not *Y* ∴ Some not-*Y* is *X* ∴ Some not-*Y* is not not-*X*. Some reformers are not radicals ∴ Some who are not radicals are not not-reformers (are not opposed to reform).

The above processes, when worked in symbols, might be supposed to be equally applicable to all judgements. But when we apply them to concrete examples, we see at once (as with Conversion) that it is not so. It is indeed often convenient in discourse to make what was predicated of a subject itself the subject and starting-point in our predication, or to lay stress on the affirmative value of a negative, or the negative value of an affirmative statement. But the use of these processes is limited in part by the idiom and vocabulary of the language, in part by the logical character of the terms in the judgement. The permutation of *I* to *O* looks almost ridiculous in symbolic form ; but where there exist two terms, the affirmation of one of which is equivalent to the denial of the other, there the process is in practice perfectly natural. No one would pass from ' Steam is invisible ' to ' Steam is not not-invisible ' ; but he might naturally pass to ' Steam is not visible '.

Contraposition, as involving the largest number of steps, and employing permutation twice, may seem to lead to the least natural modes of expression. For permutation introduces ' infinite ' terms, not-*Y* and not-*X* ; and infinite terms do not ordinarily figure in speech ; so that unless we can substitute a term that is not infinite in form, our result seems fantastic. But we may see that the process of thought involved in contraposition is a common one (although the mode of expression may be awkward), if we look at it under the forms of the hypothetical proposition. Given that all lovers are jealous, it is possible to infer that all the not-

[1] What has here been called the converse by negation is by some writers called the contrapositive (e.g. J. Wallis, *Logic*, II. 7) ; and what has here been called the contrapositive, the obverted contrapositive. And the converse of the obverse of the converse of a proposition has been called its inverse.

jealous are not-lovers. No one would, however, express himself thus. But the original proposition, if it is a true universal, states a necessary connexion between the predicate and the subject; it involves the proposition that if any one is a lover he is jealous. Therefore, if any one is not jealous, he is not a lover; and this is an inference quite naturally expressed. 'If anything is *X*, it is *Y* ∴ if it is not *Y*, it is not *X*'; we have here precisely the same inference as in the contraposition of *A*, 'All *X* is *Y* ∴ All not-*Y* is not-*X*'. We may interpret in a corresponding way the contraposition of *E* and *O*, if we bear in mind the modal or problematic force which may belong to the particular judgement. 'No *X* is *Y*' will mean, 'If a thing is *X*, it is not *Y*': from this we cannot, however, infer that if it is not *Y* it is *X*; if a man is insufficiently fed, he cannot do a proper day's work; but it does not follow that if he cannot do a proper day's work, he is insufficiently fed; this may or may not be so. Hence we can only infer that 'If a thing is not *Y*, it may or may not be *X*': and that is the force of 'Some not-*Y* is not-*X*', regarded as a modal particular. Similarly with *O*; 'Some *X* is not *Y*' will mean, 'If a thing is *X*, it may or may not be *Y*'; from which it follows that 'If a thing is not *Y*, it may or may not be *X*'.

[The operations whose formal character has been considered in this chapter are called Immediate Inferences; but we have seen that one of them, Permutation, used to be regarded as belonging to the subject of Equipollency of Propositions, and J. S. Mill [1] is not alone in so regarding them all. In his view we have been dealing merely with equivalent propositional forms; the processes are 'inferences improperly so called'; and indeed they have once or twice been called transformations in the course of the text. Thus conceived, they would belong rather to a study of language than to Logic. We must therefore consider whether there is really any inference involved in them or not.[2]

The question is by no means easy, involving as it does that of the nature of inference generally. There is no inference where there is no movement of thought; but the movement of thought must spring from a perception of connexion in the objects of thought, not from subjective conditions in the mind of the thinker; it must involve an advance to the apprehension of a fresh object of thought, and be more than a mere playing as it were upon the same object. It is not inference, e.g., if the sight of a stormy sea leads one man

[1] *System of Logic*, II. i. 2.
[2] Cf. Bradley's *Principles of Logic*, Bk. III. Pt. I. c. ii. §§ 30–37.

[to reflect that steam has reduced the terrors of navigation and another that England owed much to the winds in 1588. Nor, if a fact involves two terms in a common relation, is it inference to pass from a statement that makes one term the subject standing in relation to the other to a statement making the second the subject standing in relation to the first. For the difference of subject and predicate, as Professor Cook Wilson insists, is subjective ; it belongs to the order of our approach to the complete act of judgement, in which we think the whole fact, and makes no difference to what, in that act, we think the fact to be. When Achilles was sought, and found playing with the maidens, the seekers were surprised to find Achilles their companion, the maidens that their companion was Achilles ; but both became aware of the same complex fact. I may live by the Atlantic mouth of the Panama Canal, and learn one day that it is west of the Pacific mouth, or by the Pacific mouth, and learn that it is east of the Atlantic mouth ; but in either judgement I should be aware of the same fact, and there is no inference from one to the other.[1] Again, there is no inference from an universal proposition to its subaltern, though they are not the same, because what is thought in the latter is only part of what is already thought in the former ; there is no advance to the thought of something not thought of, though bound up with what was thought of at the outset. On the other hand, the *obviousness* of a transition is no ground for denying that it is inference, though lack of obviousness might be taken as a sign that inference is present ; for if in thinking the premiss we had also thought what is stated in the conclusion, it could not come to us as a surprise, that we had committed ourselves to the latter. Neither again is the fact that the conclusion is *implied* in the original statement a ground for denying the presence of inference ; for all premisses imply their conclusion.

We must bear in mind also that the same propositional form may express different thoughts, and whether there is inference will depend on the thought which the words express. It is particularly important to remember this when working with symbols. Symbolic notations will often enable us to operate more rapidly than with words, and without realizing in the process what is meant ; and when we translate into words the result reached, it is sometimes one which we should not very readily have seen to be involved in what we started from, but sometimes also one not warranted thereby. Thus we may argue in symbols, converting and obverting, 'No X is Y $\therefore$ No Y is X $\therefore$ All Y is non-X $\therefore$ Some non-X is Y'. The original proposition might be 'Things made of asbestos do not burn', and the final conclusion 'Some things not made of asbestos burn' ; and this arouses no suspicion. But let the original proposition be

[1] Such restatements have nevertheless been sometimes called immediate inferences.

['No man dies twice', and we can hardly accept the conclusion 'Some who are not men die twice'. We might hesitate even about the simple converse, 'Nothing that dies twice is a man', as implying an admission that dying twice does occur. Such paradoxes arise because in working out symbolic sequences we are considering only what relations of subject and predicate are excluded and what left possible by the information given; and the inference to 'Some non-X is Y' is intended to mean not that there exist things not X which are Y, but that the fact that nothing which is X is Y does not exclude their existence. But propositions in significant terms commonly imply the existence of instances of their subjects. Not however always; and when we pass from a premiss implying the existence of its subject to a conclusion only asserting compatibility of attributes, or such connexion between them that if there were an instance of one it would also be an instance of the other, then, and also vice versa, there is inference which would not equally exist if both propositions were understood in the same sense. Such inference however may involve the use of some other premiss besides the convertend expressed; and *mutatis mutandis* the same would be true in obversion.

A categorical proposition commonly implies the existence of instances of its subject, and therefore, if it is affirmative, of its predicate also.[1] But in making it we may or may not have determinate instances in mind. We found that the form 'All X is Y' is sometimes used to state a fact about all members of the group or class X, sometimes to state a connexion between being X and being Y; in the former case, it might be said to be intended *historically* (e. g. 'all the ruminants part the hoof'), in the latter *scientifically* (e. g. 'all rivers run down hill'). But if intended scientifically, the proposition need not be intended to assert the existence of instances of the subject; e. g. 'a perfect fluid is frictionless' may be intended only as a statement of what would be the character of a perfect fluid if it existed; and then, though categorical in form, it is intended only *hypothetically*. And a particular categorical might be said to be intended historically when we make it with instances in mind, e. g. if we said that 'some garrison towns are important civilly', thinking of Winchester, York, and Canterbury: and scientifically when we wish rather to affirm the compatibility of the subject and predicate characters (or, if the proposition be negative, the possibility of their disjunction). In the latter case, however, we more commonly use the modal form 'X may be Y', than the categorical particular 'Some X is Y'.

Let us now consider the simple conversion of an I proposition. Any one starting from the judgement that 'some garrison towns are important civilly', whether he has in mind definite instances or not, must know or believe the fact stated in the converse, that

[1] This is sometimes called its existential import.

[' some places civilly important are garrison towns '. The fact, of which Winchester, York, and Canterbury are instances, is the same, whichever way it is put: whether the logical subject be 'some garrison towns ', or ' some places civilly important '. There is therefore here no real inference. There could be inference only if from a judgement in which we are thinking definitely of certain towns, though not naming them, we passed to one asserting general compatibility. But here in effect we should be passing from the proposition that Winchester, York, and Canterbury are important civilly, and to the proposition that some towns civilly important are garrison towns. This is inference, but syllogistic, not immediate; and we should not express it by such verbal variation as is symbolized in passing from ' Some X is Y ' to ' Some Y is X '. The conversion of I then is not a process of inference.

The conversion of the universal affirmative A has more show of inference, because it proceeds by limitation; and it might be urged that there is inference in seeing that I am not entitled to infer that, since all the ruminants part the hoof, all the cloven-footed animals ruminate. But surely I know from the outset that in affirming Y of X, I do not confine the predicate to that subject; and to realize that Z also may be Y is to realize that what is Y need not be X. It can hardly be called inference to realize that information about X does not extend beyond X, nor to refrain from asserting what I know that I have no right to assert.[1] And I must in the original proposition, whether understood historically or scientifically, if I implied the existence of instances of the subject at all, have meant that these were also instances of the predicate; and therefore I must have realized that some things exhibiting the predicate character exhibit also the subject character, which is what is stated in the converse. So far, therefore, in the conversion of A there is no real inference.

But the universal affirmative, intended scientifically, does not always imply the existence of instances of its subject. *Tout savoir est tout pardonner*; I might translate this by saying ' Those who know all pardon all ', not implying that any of us does know everything, but only that, if he did, he would pardon everything. Now if I convert this and say ' Some who pardon all know all ', I shall probably mean that there are persons who both pardon and know everything. Here then there will have been inference; but again, it does not lie in the conversion. It lies in combining the thought of the general connexion with the thought that there are some who know all about some situations; and so concluding that there are some who pardon all in some situations. The inference involves a premiss not expressed. To pass from the merely hypothetical sense of an universal affirmative to the categorical involves inference, but

[1] Cf. Bradley, loc. cit.

[hypothetical inference[1], not conversion. To pass from meaning it historically to meaning it scientifically is inference, but it is induction.[2] It is more difficult to say whether, if we mean it scientifically, but categorically, there is inference in passing to a purely hypothetical meaning: suppose, e. g., that I judge that 'all rivers run down hill', meaning that by their nature as running water they must do so, is it inference to pass to the thought that any other rivers, if they did exist, would also run down hill? I think not; in the necessary judgement there is really inference from the outset; it is essentially inference to see that if a condition X is realized, Y must be realized too; I advance herein by mere thinking from X to Y. But if I have realized this in considering existing instances, there is no further inference in seeing that it would hold in others.

The last point needed notice in relation to the conversion of the universal negative, E. 'No X is Y' converts simply to 'No Y is X'. The convertend implies commonly the existence of instances of X, but not necessarily of Y; the converse however does imply the existence of instances of Y. Now if in the convertend it be meant that there are instances of both X and Y, the thought that the latter are not the former hardly seems separable from the thought that the former are not the latter; and there seems to be no inference from 'No fish are mammals' to 'No mammals are fish'. If however this be not meant in the convertend, and in the converse it be meant that there are instances of Y, then there is inference, but it involves another premiss. I might judge that 'nothing inductive is self-evident', while doubtful whether anything is self-evident; if I proceed to judge that 'nothing self-evident is inductive' meaning that there are self-evident propositions, the judgement that these are not inductive comes by help of the convertend, but that they exist at all is independent of it. Still, I cannot reach the universal 'nothing self-evident is inductive' without realizing that if anything were self-evident, it would not be inductive; and this connexion of condition and consequent is not the same as what is realized in the universal negative from which I started; that was, that if anything were inductive, it would not be self-evident. From 'if X, then not Y' to 'if Y, then not X' does seem to be inference, the condition being different in the two. It is true that it may easily be shown that I cannot repudiate this conversion without self-contradiction; if a thing might be Y and still be X, then since, if X, it is not Y, it might be Y and not be Y. But though it is impossible to affirm the convertend and deny the converse without contradiction, inference is involved in realizing this, and the converse is not actually thought in thinking the convertend. Only then if an E proposition be intended as a statement that two groups of instances exclude each other (or that the individuals indicated by two singular terms are different), is its conversion not inference.

[1] Cf. *infra*, c. xv. [2] Inductive syllogism in Fig. 3. Cf. *infra*, p. 319.

[As the conversion of *O*, the particular negative, is impossible without first permuting, or obverting, it to *I*, we must ask next whether there is inference in Permutation. The process of Permutation involves the use of the infinite or negative term not-*Y* in the predicate in lieu of *Y*. Now we have seen that an infinite term has not any meaning at all unless it has some positive meaning; *not-Y* must mean *something else than Y*.[1] We have seen also that the disjunctive judgement '*A* is either *B* or *C*' does not always imply that it cannot be both. But Permutation rests upon disjunction; *Y* and *not-Y* are alternatives, and it is assumed that if *Y* is affirmed or denied of any subject, *not-Y* can be denied or affirmed accordingly. Bearing in mind these considerations, we shall find that there is a certain difference in different cases, in respect of the presence of any real inference in permutation, according to the meaning attached to the negative term.

It is unnecessary here to separate universal and particular propositions. If we are told that *X* is not *Y*, and *Y* and *not-Y* are alternatives, one of which must attach to it, then since it does not exhibit *Y*, it must exhibit the other, *not-Y*. We thus reach the affirmative, '*X* is not-*Y*'; and the question is whether that is any way different from the negative with which we started.

Now we cannot deny that there is any inference in disjunctive reasoning at all. When I argue that since *A* is either *B* or *C*, and is not *B*, therefore it is *C*, there is clearly inference; and I could not argue that, because *A* is not *B*, it is *C*, unless I were given the disjunctive premiss, *A* is either *B* or *C*, as well. But in permutation, my alternatives are not two different positive terms, like *B* and *C*, but *Y* and not-*Y*. Is there any inference in saying that because *X* is not *Y*, it is not-*Y*?

It will be allowed that the conclusion would not hold unless *X* were either *Y* or not-*Y*. But it may be said that this, the 'principle of Excluded Middle', like the Principle of Contradiction, though true, is not a premiss of inference. No one knows what he means in saying that *X* is not *Y*, unless he sees that in that case it *is* not-*Y*: any more than he can know what he means in saying that *X* is *Y*, unless he sees that in that case it is not not-*Y*. If a proposition is true, its contradictory is false; but there is no step from the truth of the one to the falsity of the other, no movement of thought; since the truth of the one is not apprehended without apprehending the falsity of the other.

If the infinite term *not-Y* were purely negative, this view of the matter would demand assent. But *Y* and *not-Y* are in practice always alternatives within some definite limits. *Y* may be *blue*, and then *not-Y* will be *of some colour not blue*: or *Y* may be *English-speaking*, and *not-Y speaking some language not English*. And in

[1] Otherwise, the term is *Y*, and the form *not-Y* only shows that *Y* is being denied of some subject in a judgement.

[passing from one of these predicates to the other, there is inference, and we do not rely merely on the law of Excluded Middle. 'Noble blood is not blue ∴ it is not-blue': if this means 'of a colour not-blue', we require the further premiss that it is either blue or of some other colour. We thus pass from a determinate positive predicate to another predicate less determinate, but still positive.

If however there is no positive alternative meaning in the predicate *not-Y*, then indeed there is no inference, but only equipollency. 'Steam is not visible ∴ it is invisible' seems a mere substitution of one equivalent expression for another. It follows, that we cannot tell by the mere symbolic form whether the permutation of a negative proposition contains any real inference or not, but must look to the content [1]; and if it contains real inference, the inference is disjunctive.

The permutation of an affirmative proposition may, like this last, be no real process of inference. We pass here from '*X* is *Y*' to '*X* is not not-*Y*'. It is not always possible to find in this any other meaning than that from which we started. We cannot always interpret *not-Y* to mean 'possessed of some other of the range of alternatives to which *Y* belongs'; if a subject must display some one out of a given range of alternatives, and does not display *Y*, it will display one of the others; but if it does display *Y*, we cannot be sure that it may not display one of the others as well. If a man holds office in the Government, and does not hold an office that entitles him to Cabinet rank, he must hold an office that does not entitle him to Cabinet rank; but if he does hold an office that so entitles him, he *may* also hold one that does not. Equally, if *not-Y* is quite unlimited in range, and includes everything whatever except *Y*, it will not follow that because *X* is *Y*, it is not also not-*Y*; because we can predicate of a goose that it hisses, we are not precluded from applying any predicate but *hissing*. The only sense, therefore, in which it is true to say that *X* is not not-*Y*, is one in which we deny no alternative, but only deny the denial of *Y*; and that is just equivalent to the affirmation of *Y*, or at least can hardly be said to involve any inference from it. If however we have in mind a range of mutually exclusive alternatives among which *Y* is one, then permutation takes us from the affirmation of *Y* to the denial of the rest; and this is again disjunctive reasoning, wherein the conclusion will be more or less definite according to the definiteness of our knowledge of the alternatives to *Y*. But

[1] The reader may be reminded, that among the range of alternatives which the denial of a positive term leaves open, the corresponding negative term has often come to signify one only. *Not-blue* may cover all colours but blue; but *unfriendly* does not cover all the alternatives to friendly; it implies a definite degree of hostility which may be absent in those who are not positively friendly to us. But this is a matter of the interpretation of language rather than one of Logic.

[so far as there is inference here, there is no use of an infinite term; where *not-Y* is really infinite or unlimited, the only sense in which the permutation of an affirmative proposition is logically justifiable is one in which it involves no step of inference.[1]

If this is a just account of the nature of permutation, then any inference there may be, apart from disjunctive argument, in converting by negation, must lie in the converting. And the conversion of *O* by negation will no more be inference, if the permutation of it is not, than the simple conversion of *I*. Indeed no one believing that there existed things which are not *Y* could judge that 'some *X* are not *Y*', without at the same time thinking that some things which are not *Y* are *X*. Similarly the conversion of *E* by negation—'No *X* is *Y* ∴ some non-*Y* is *X*,'—is like the conversion by limitation of *A*; if there is anything new in the converse, it is the implied assertion that there exist instances of what is *X*, an assertion which the convertend, if intended hypothetically, did not contain. 'A perfectly wise man does no wrong, ∴ some who do no wrong are perfectly wise'; it is not converting by negation that would justify us here in passing from a sense of the convertend in which it does not imply that any one is perfectly wise to a converse that does. In converting *A* by negation on the other hand there is inference to the extent that there is in simply converting *E*. 'All *X* is *Y* ∴ No non-*Y* is *X*' involves the transition from 'If *X*, then *Y*' to 'If not *Y*, then not *X*', which may be indifferently expressed by 'No not-*Y* is *X*' or 'All not-*Y* is not-*X*'—i. e., the inference is in the conversion, not in the second act of permutation, by which some distinguish contraposition from conversion by negation.]

The immediate inferences which we have considered so far have all been of a more or less formal character; as is shown by the fact that they have been capable of explanation, up to a point, by using symbols and not real terms. There are certain kinds of inference, which have been called immediate, that cannot be exhibited by symbols at all, but only *in concreto*. One of these is known as *Immediate Inference by Added Determinants*: in which we add the same qualification to both subject and predicate in a proposition, and hold the result of our operation to be true, on the strength of the truth of the original proposition; e. g. 'A negro

[1] This is no doubt why Wallis (cf. p. 239, n. 1, *supra*) did not distinguish contraposition from conversion by negation. 'Hanc formulam locum habere docent in Particulari negativa. Atque huius potissimum causa videtur fuisse introducta: ut quae per neutram reliquarum converti possit. Puta. Aliquod animal non est homo: ergo, Aliquod non-homo non est non-animal; seu (quod tantundem est) Aliquod non-homo est animal; seu, Aliquod quod non est homo, est tamen animal.' loc. cit.

is a fellow creature ∴ a negro in suffering is a fellow creature in suffering'.[1] Another is called *Immediate Inference by Complex Conception*: in which the subject and predicate of a given proposition are used to qualify in some way the same term, and thus complex concepts are formed, that are made subject and predicate of a new proposition, e.g. 'Physics is a science ∴ physical treatises are scientific treatises'. The following examples, some of them sound and some unsound, but the sound identical in form with the unsound, will serve to show that the ground of the soundness of these arguments does not lie in the form of them :—

The horse is an animal ∴ the head of a horse is the head of an animal.

Horses are animals ∴ the greater number of horses is the greater number of animals.

A shark is not a mammal ∴ the anatomy of a shark is not the anatomy of a mammal.

A shark is not a mammal ∴ the food of a shark is not the food of a mammal.

A shark is not a dog ∴ the owner of a shark is not the owner of a dog.

It is not worth while multiplying arguments to show how entirely the validity of such inferences as these involves their content. It would not be possible to reduce them to a definite number of fixed types, though in considering generally which are valid, some of Aristotle's observations in the *Sophistici Elenchi*, especially those on what he calls the Fallacy of Accident, would be pertinent. But their mention here will serve to illustrate, what it is well to realize early, that inference is not a purely formal process; that arguments are not all built on the principle of American watches, with interchangeable parts,[2] so that terms from one may be transferred to another, without interfering with the working of the inference; and that the study of inference, like the study of life, is largely a matter of examining *types*: though there are a certain number of common forms, which recur identically in divers contents. One of the most famous of these common forms is the Syllogism, to which we must now proceed; it has often been regarded as the form of all inference whatever that is not 'immediate'; it is indeed highly general, and found in all kinds of subject-matter; though the nature even of it cannot be profitably studied altogether in the abstract, but is to some extent affected by the concrete character of its terms.

[1] Thomson, *Laws of Thought*, § 55.

[2] *v.* Marshall's *Principles of Economics*, Bk. IV. c. ix. § 4.

CHAPTER XI

OF SYLLOGISM IN GENERAL

ARISTOTLE, who was the first person to work out the theory of syllogism, though not, of course (as Locke maliciously suggests that his followers claimed), the first to reason syllogistically, defines a syllogism as follows: *λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῷ ταῦτα εἶναι*[1]: that is to say, 'discourse in which certain things being posited, something else than what is posited necessarily follows merely from them'.

This definition is too wide. It covers, as the word syllogism in its etymological signification itself covers, every argument in which from a consideration of two truths we infer a third—every argument in which (to use a homely phrase) we 'put two and two together', and find a certain conclusion necessarily following.[2] But neither by Aristotle, when he investigated in his *Prior Analytics* the various forms of syllogism, nor by the world, which has followed Aristotle, has the term been actually used so comprehensively. A syllogism is actually an argument in which, from the given relation of two terms, *in the way of subject and predicate*, to the same third term, there follows necessarily a relation, *in the way of subject and predicate*, between those two terms themselves.[3]

Example will best explain what is here meant by the words italicized. If A is equal to B, and B is equal to C, then A is equal to C. If a bullet travels faster than a horse, and a horse travels faster than a man, then a bullet travels faster than a man. Now here the terms are A, B, and C: or a bullet, a horse, and a man; but the relations between the terms are in the one case relations of quantity, in the other of velocity. A and B are not related as

[1] *Anal. Pri. a.* i. 24^{b} 18: cf. *Top. a.* i. 100^{a} 25, where the same definition recurs, with the substitution of *διὰ τῶν κειμένων* for *τῷ ταῦτα εἶναι*.

[2] 'Putting two and two together' is often a process which leads people to conclusions of a highly conjectural character. In such cases, their reasoning does not come under the Aristotelian definition: for it is expressly stated by him that the conclusion must be inevitable—*ἐξ ἀνάγκης*.

[3] Bradley's *Principles of Logic*, Bk. II. Pt. I. c. iv. § 10, *et alibi*.

subject and predicate, for I do not say of *A* that it is *B*, but only that it is equal (in quantity) to *B*; a bullet and a horse are not related as subject and predicate, for a bullet is not a horse; its asserted relation to a horse is in the way of travelling faster, not in the way of being a subject whereof horse is a predicate. No doubt it is a predicate of a bullet, that it *travels faster than a horse*, as it is a predicate of *A* to be *equal to B*; but then what I proceed in my argument to compare with *C* is *B* itself, and not that which is equal to it; what I say travels faster than a man is a horse, and not what travels faster than a horse. *A*, *B*, and *C*, a bullet, a horse, and a man, are the terms which I compare, the former in respect of quantity, the latter of velocity; and from the given relations of *A* and *C* to the common term *B*, in the way of quantity, I deduce a relation between *A* and *C* themselves in that respect; or from the given relations of a bullet and a man to a horse in the way of velocity, I deduce a relation in the way of velocity between a bullet and a man.

Now the relations between the terms of an argument *may* be in the way of subject and predicate; and *then* the argument is a syllogism. Let us for the present use the symbols *X*, *Y*, and *Z* to represent terms related in this way. Suppose that *X* is predicated of *Y*, and *Y* of *Z*; then *X* must be predicable of *Z*. For example, silver prints fade in the sun; and the photographs which I have bought are silver prints; therefore they fade in the sun. Here the term common to the two *premisses* (for such the given propositions are called, from which the *conclusion* is deduced) is *silver prints* (*Y*): that is predicable of *the photographs which I have bought* (*Z*), and of that is predicable *to fade in the sun* (*X*); hence *to fade in the sun* (*X*) is predicable of *the photographs which I have bought* (*Z*). Or again, *Y* may be a predicate affirmed or denied both of *X* and *Z*; in the Dreyfus affair, the French War Office frequently argued that the man who wrote the famous 'bordereau' was on the General Staff: Esterhazy was not on the General Staff, and therefore did not write it; here *Y* (*being on the General Staff*) is affirmed of *X* (*the man who wrote the 'bordereau'*) and denied of *Z* (*Esterhazy*); and hence *X* is denied of *Z*—Esterhazy did not write the 'bordereau'. Yet again, *Y* may be a subject of which both *X* and *Z* are predicates affirmed or denied; then *X* may be predicable of *Z*, or vice versa. The horse is strong, and is an animal that lives exclusively upon a vegetable diet; therefore an animal that lives exclusively upon a vegetable diet may be strong. Here we have two terms, *strong* (*X*) and *being*

an animal that lives exclusively upon a vegetable diet (*Z*), affirmed as predicates of the same term (*Y*), the *horse*; and we hence deduce that *X*, *strong*, is predicable of *Z*, *an animal that lives exclusively upon a vegetable diet*, not indeed necessarily and universally, but as a possibility in certain cases.

These examples may perhaps explain what is meant by terms being related in the way of subject and predicate, and how the relation of two terms in that way to a common third term may necessitate their relation in the way of subject and predicate to one another.

What is here called a relation in the way of subject and *predicate* may be also called a relation in the way of subject and *attribute*; as it is called, for example, by Mr. Bradley in his *Logic*, Bk. II. Pt. I. c. iv. § 10, and elsewhere. If the word attribute is used, it must be understood generally of anything predicated [1]; it is an attribute of Baal to be a god, to be talking, to pursue his enemies, to be on a journey, to be asleep, to need awakening, to have 450 prophets in Israel, to be worshipped by the Phoenicians; whatever can be affirmed or denied of him is an attribute affirmed or denied; the attribute may be in any category, of substance (as when we say that he is a god), of quality, time, place, state, relation, &c.; the only thing necessary is that it should be related to him as what can be predicated of it to a subject, not (for example) as an uncle to a nephew, as yesterday to to-day, as cause to effect, as here to there, as means to end, as more to less, &c.; all of these are relations in which terms may stand to one another, if we mean by terms distinct subjects of thought, and not merely the subject and predicate into which the judgement which affirms their relation is resoluble. Thus when I say that the Old Pretender was nephew to Charles II, he and Charles II may be called the terms placed (in this judgement) in a relation of consanguinity; he and 'nephew to Charles II' are the terms placed in a relation of subject and attribute. When I say that Edinburgh is west of Liverpool, Edinburgh and Liverpool are the terms placed in a space-relation; but Edinburgh and 'west of Liverpool' the terms placed in a relation of subject and attribute. Understanding the word in this comprehensive sense, we may say that the theory of syllogism is the theory of inference in the domain [2]

[1] i.e. in a wider sense than it is used in when the attributes of anything are distinguished from its substance or kind, and its relations.

[2] By a *domain* here is meant a certain order or system of relations, of

of subject and attribute, just as well as in the domain of subject and predicate. But it is important to remember that 'attribute' is being used in a wider sense than it usually bears; we should not ordinarily call it an attribute of Mr. Pickwick to have been once impounded; or of Becky Sharp to have thrown Dr. Johnson's Dictionary out of the carriage window; the word is not ordinarily understood to include actions, or the casual relations of one thing to another; but in its present use, it includes every predicate. The advantage of using it is this, that inference depends on perceiving relations in what is thought of, and in taking the word *attribute* instead of *predicate*, we take a word expressing a real for one expressing a logical relation. Blue is an attribute of the star-gentian really and always: a predicate, only when one *judges* that the star-gentian is blue. It is true that in the theory of syllogism we have to do with attributes only so far as they are predicated; but we think of our predicates as attributes.

It has often been held that the syllogism is the type of all reasoning, except the inferences called immediate.[1] No one has done more to dispel this illusion than Mr. Bradley, in his *Principles of Logic*; though perhaps the zeal of an iconoclast has prevented him from dwelling enough on the fact that the syllogism formulates reasoning which is very frequent in occurrence. But our present business is to become familiar with the theory of syllogism on its formal side. There is a precision and completeness about this theory, which have

a single kind: as we might call space a domain in which all material things are related, and time a domain in which all events are related. The domain of subject and attribute is far less unified than that of space and time. A thing related to one other thing in space, or an event related to one other event in time, is necessarily related in those ways to all others. But a term related to a second term in the domain of subject and attribute is thereby necessarily related in that way only to those further terms, if any, to which the second is related in that manner (and not necessarily to all of them). The domain of subject and attribute is, as it were, a system of relations embracing group after group of terms, but not necessarily connecting any of the terms of separate groups; whereas time and space, which connect group after group of events or bodies, necessarily connect also any two members of any two groups. The word *category* might have been employed instead of domain, in the Kantian sense of a *principle of synthesis or relation*. But it was employed on the last page in the Aristotelian sense of a *kind of predicate* (determined indeed, on Kant's view, by the principle, or principles, of synthesis employed), and has been generally employed in the text in that sense; and it would have introduced confusion either to employ it without notice in a different sense, or to interrupt the present subject in order to point out the distinction between them.

[1] e.g. Hobbes, *Art of Rhetoric*, Bk. I. c. i, 'all inferences being *syllogisms*': *v.* Molesworth's ed., *English Works*, vi. 423.

made logicians dwell on it with something of an artist's concentration; and the truth of science has sometimes been sacrificed to neatness of exposition.

The business of syllogism is to establish a relation in the way of subject and predicate between two terms, by means of their relations in that way to the same third term. But the proposition which relates two terms as subject and predicate may be universal or particular, affirmative or negative.[1] Moreover, we have seen that there are various ways in which the two terms that are to be brought together in the conclusion may be related to a common third term; both may be predicated of it, or it of both, or one of it and it of the other. Therefore the following general problem presents itself to us, —Writing S for any subject, P for the predicate which is to be brought into relation to it, and M for the third or middle term whose relations with S and P are to bring them into relation with each other, we may ask—What must be the quantity and quality of the propositions (or premisses) connecting S and P respectively with M, and in which relation, viz. subject or predicate, must M stand to S and P in these premisses, in order to establish in the conclusion a proposition whose terms are S and P, of the several forms A, E, I, and O? In other words, what forms of premisses will prove that all S is P, no S is P, some S is P, or some S is not P, by means of the relations, in the way of subject and predicate, of S and P respectively to M? Or, yet again, *what relations in the way of subject and predicate between two terms S and P respectively and a common third term M will establish what relations in the way of subject and predicate between those two terms themselves*? This is the question, put in its most abstract form, to which the formal part of the theory of syllogism is an answer.

[1] When it said that a judgement, or proposition, 'relates' terms, 'places' them in a relation, and so forth, it must not be understood that the terms of thought come to stand in such relations through that act of judgement. My judgement is my apprehending, or coming to believe, that they stand in such relations, and the proposition expresses this apprehension or belief, or asserts what is apprehended or believed.

CHAPTER XII

OF THE MOODS AND FIGURES OF SYLLOGISM

A. **Nomenclature.** 1. In any syllogism, there are two propositions taken as true, and another inferred or following from them. The latter is called the **conclusion** (Lat. *quaestio* or *conclusio*, Gk. *πρόβλημα* or *συμπέρασμα*) : the former the **premisses** (Lat. *praemissa*, Gk. *προτάσεις*).

It was said, that the premisses are *taken as true* : whether they *are* true or false, the conclusion which they yield is the same ; only that if they are true, it is true, and if they are false, it is probably false.[1] We are not concerned, therefore, in the formal theory of syllogism, with the truth or falsehood of our premisses or our conclusion, but only with *the validity of our reasoning* : we wish to know, if the premisses are granted, what must be granted as following from them. If our reasoning be correct, a man cannot consistently admit the premisses, and deny the conclusion. Suppose that a man admits that every restriction upon freedom of contract is mischievous, and admits that the marriage laws restrict freedom of contract, then he must admit the marriage laws to be mischievous.

It has been made a reproach to the theory of syllogism, that it looks only to the cogency of the inference, and not to the truth of the premisses. We need rules, it is said, by which to determine whether a proposition is actually true, and not merely whether it is true, upon the hypothesis that certain other propositions are so. The theory of syllogism is decried as a Logic of Consistency ; for the most that it can do is to furnish rules by which to judge whether different assertions are consistent with one another. In rivalry with the Logic of Consistency, some writers have projected a Logic of Truth, and offered it to the world under the name of Induction.[2]

[1] Not necessarily, because a true conclusion may follow from false premisses (cf. *infra*. p. 334). But a conclusion correctly drawn from false premisses implies ignorance in the reasoner, though not ignorance of reasoning.

[2] Cf. Mill, *System of Logic*, III. iii. 9.

But it has been unfortunately discovered that the 'Inductive Methods' that were to test the truth of the premisses, from which the doctrine of syllogism enquires what may be inferred, suffered from the same defect as the syllogism itself; for they also were processes of inference, in which conclusions were drawn from premisses; their conclusions were only true, if the premisses were true; they showed themselves quite unable to determine whether their premisses were true or not, though it was generally just on that point that disputes were most pronounced.

The fact is, that so far as reasoning can be reduced to fixed forms at all, and these forms studied in the abstract—whether or not the forms are syllogistic—we *must* disregard the truth of the premisses; for in expounding an abstract form of reasoning we may even use symbols for terms,[1] i. e. we do not trouble ourselves to ask what in particular the terms are at all; and hence we cannot be asking whether the judgement which connects them is true.[2]

Given then the premisses, the conclusion follows necessarily; but it may nevertheless be false, if the premisses are false. The premisses, however, need not in the first place be given, they may be *wanted*.

Supposing a man to have admitted that whatever discourages thrift and independence is evil; and to have admitted that an universal system of pensions in old age at the cost of the state discourages thrift and independence: then he must admit as a conclusion that such a system is evil. Here, and to such a man, the conclusion presents itself in the first place as a consequence of what is already granted or 'given'. But supposing a man to be in doubt whether an universal system of pensions in old age at the cost of the state is evil or not, and to be *wanting* some proof, one way or the other; and that a friend offers him the above 'premisses', as showing that it is evil: then, and to him, the 'conclusion' presents itself in the first place as a *question* or *problem*, about which he wants to know whether he is to affirm or deny it; and syllogism is a process of *finding proof*, rather than of *drawing consequences*.

[1] As J. S. Mill does in expounding his Inductive Methods: but his symbols are very inadequate.

[2] Yet inference is at bottom a perception of connexion among facts, and how can we perceive any in premisses that are not true? On this difficulty cf. *infra*, pp. 331–334.

It makes of course no difference to the form of premisses which will establish a particular form of conclusion, whether the premisses be first known, and the conclusion discovered as a consequence : or the conclusion raised as a problem, and the premisses discovered to settle it. And in either case alike, the premisses are 'given' in the sense of being admitted and not proved in the argument. But they are not always 'given' in the sense of being that with which a man begins : our thought is as often occupied in looking for premisses to establish what we believe or suspect, as in looking at premisses to see what follows from them. And that is why Aristotle used the expressions *πρόβλημα* and *προτάσεις*. For him, the conclusion was generally regarded as *something to be proved*[1] ; the premisses, as something *proffered* in proof of it ; and so he asked rather, 'What kinds of premisses are required to prove various kinds of conclusion (*A*, *E*, *I*, and *O*)?' than 'What kinds of conclusion follow from various combinations of premisses?' But so soon as he had answered his question, and said '*These* kinds of premisses prove the various kinds of conclusion', then other people could look at the matter from the side of the premisses first. To them, the premisses were something which, if given, necessitated a certain form of conclusion : rather than something which, if a certain form of conclusion were to be established, must be given.

2. The premisses are called respectively the **major** and **minor premiss**. This nomenclature is adjusted to that of the *terms* in the argument. There are, as we have seen, three terms in a syllogism : two, which form the subject and predicate of the conclusion, and one with which each of the former is brought into relation (in the way of subject and predicate) in one of the premisses. The subject and predicate of the conclusion are called respectively the **minor** and the **major terms** : the term common to the two premisses is called the **middle term**.[2] The major premiss is the premiss in

[1] Or rather, to be *proved or disproved* : it was a thesis, which might form the subject of debate between two parties ; one of them, the oppugner, 'held out' to the other, the upholder, various propositions, which he asked him to admit, in hope to obtain admissions wherefrom there followed syllogistically a conclusion contradictory of the thesis of the upholder.

[2] These expressions are based upon what occurs in the *first figure*, where the major term is commonly of greater extension than the middle, and the middle than the minor : and the major premiss, as compared with the minor, is a more general proposition. But being transferred to the other figures, in which they cannot any longer be so interpreted, they must be explained generally as in the text : cf. *infra*, pp. 259 sq., where this is explained at length.

which the major term occurs, and the minor premiss that in which the minor term occurs. Thus in the syllogism

All organisms are mortal
Man is an organism
∴ Man is mortal

the major term is *mortal*, and the major premiss *all organisms are mortal*; the minor term *man*, and the minor premiss *man is an organism*; the middle term, *organism*.

It will be noticed that each term in a syllogism appears twice: the major and minor terms each in its respective premiss and in the conclusion, the middle in both premisses but not in the conclusion.

In giving examples of syllogism, it is usual to write down the major premiss first; but in ordinary life and conversation, no particular order is observed; nor is it *necessarily* the major premiss that is written first in a logical example.[1] The only mode of determining the major premiss is to look for the premiss which contains the predicate of the conclusion.[2]

3. Syllogisms are said to differ in **figure** (*σχῆμα*) according to the position of the middle term in the premisses.[3] (i) The middle term may be subject of the major premiss, and predicate of the minor: in this case Aristotle called the syllogism of the *first* (or perfect) figure. The example just given belongs to the first figure, as also does the following:—

No insects have eight legs
Wasps are insects
∴ Wasps have not eight legs.

It is convenient to have a conventional symbolism, in which to represent syllogisms according to their form; we shall use the letters *P*, *M*, and *S*. *S* (= subject, of the conclusion) will always indicate the minor term, *P* (= predicate, of the conclusion) the major term, and *M* the middle. Thus the figure of both these examples (i. e. their form, so far as it depends merely on the position of the terms in the premisses) may be written

M P
S M
∴ *S P*

[1] Cf. Locke, *Essay*, IV. xvii. 8 (fourth or later edition).
[2] Except in the 'indirect moods' of Fig. 1. Cf. *infra*, pp. 262, 268-269.
[3] Cf. c. xi, *supra*, pp. 250-251.

If we wished to indicate in our symbols the character of the propositions which compose the syllogism (i. e. whether universal or particular, affirmative or negative), we should have to write our two examples differently. The former is of the type

All M is P
All S is M
$\therefore$ All S is P

the latter of the type

No M is P
All S is M
$\therefore$ No S is P.

(ii) The middle term may be predicate in both premisses, the figure of the syllogism being indicated as follows :—

$P\ M$
$S\ M$
$\therefore\ S\ P$

e.g. No insects have eight legs
Spiders have eight legs
$\therefore$ Spiders are not insects.

Syllogisms in which the middle term is thus placed were called by Aristotle of the *second* figure.

(iii) The middle term may be subject in both premisses, the figure of the syllogism being indicated as follows :—

$M\ P$
$M\ S$
$\therefore\ S\ P$

e.g. The Veddahs of Ceylon show great conjugal fidelity
The Veddahs of Ceylon are savages
$\therefore$ Some savages show great conjugal fidelity.

Syllogisms in which the middle term is subject in both premisses were called by Aristotle of the *third* figure.

(iv) Aristotle recognized only these three figures. But he pointed out [1] that the premisses of a syllogism in the first figure would sometimes justify you in concluding to a particular proposition in which the minor term was predicated of the major, even though no

[1] *Anal. Pri. a.* vii. 29^a 19–27 (cf. p. 281, n. 2, *infra*).

conclusion was possible that predicated the major of the minor. For example, from the premisses

Some parliamentary voters are freeholders
No women are parliamentary voters

it is impossible to determine whether any women are freeholders or not (for a reason which will be explained later); but we can conclude that some freeholders are not women.

Again, from the premisses

All persons who have the franchise are eligible to Parliament[1]
No woman has the franchise

we cannot conclude that women are not eligible to Parliament (for others might be eligible besides those who have the franchise); but we can conclude that some persons who are eligible are not women.

The famous physician Galen is said by Averroes to have referred arguments of this kind to a separate and *fourth* figure (sometimes called after him the *Galenian* figure), in which the middle term is predicate of the major premiss and subject of the minor: the figure being accordingly symbolized

$$P\ M$$
$$M\ S$$
$$\therefore\ S\ P.$$

The theory of syllogism has been much darkened by this addition.[2] For in erecting these arguments into a separate figure it is implied that the distinction between major and minor term depends merely on their position in the conclusion, and is in no way intrinsic to the terms themselves. The meaning of that distinction must be considered next.

4. We have said that the major term is the predicate of the conclusion, and the minor the subject. But why are they called major and minor? Did Aristotle merely want shorter names, to avoid the constant repetition of such cumbrous expressions as 'subject of the conclusion' and 'predicate of the conclusion'? Are the names chosen arbitrary? And would it have been equally appropriate to call the subject of the conclusion the major, and the

[1] If the premiss had to be true, the clergy must be excepted.

[2] In the second and third figures, where the middle term occupies the same position in both premisses, either premiss may be regarded as major, without affecting the situation of the middle term: and hence there is no possibility of erecting a separate figure bearing the same relation to them as the fourth does to the first.

predicate the minor term? Or, on the contrary, does the choice of names indicate a real feature of the relation between subject and predicate in a judgement? Is there a reason why the predicate should be called the major term, and the subject the minor?

Aristotle conceived that there was such a reason, not indeed in all judgements, but in most and especially in scientific judgements (i. e. judgements which really express knowledge). We shall do best to look first at judgements in which the distinction of major and minor term *is* arbitrary. 'Some scholars are statesmen' might be as well expressed by saying 'Some statesmen are scholars'; for here the two terms or concepts have no necessary relation: it is only as coincident in the same individual that statesman can be predicated of scholar, or vice versa; and there is no more reason for making one term subject than the other. 'Some poulterers are not fishmongers' is a judgement of the same kind: the two trades are frequently conjoined, but merely conjoined, and as there would be no more reason for making the sale of fish an attribute of a poulterer, than the sale of poultry an attribute of a fishmonger, so in the negative judgement, each term is with equal propriety denied of the other. But where the subject of a judgement is a concrete thing or person, and the predicate an attribute: or where, though the subject is an abstract term, yet the predicate belongs to *it*, and is not merely coincident with it in the same thing; there the two terms cannot equally well be predicated of each other. We say that Caesar was a great general; if we said 'a great general was Caesar', we should still be understood to make Caesar the subject, and to have merely inverted the usual order of words in the sentence. We say that diamonds glitter, rather than that some glittering things are diamonds; that blue is a colour, rather than that a colour is blue.[1] To say that a colour may be blue is natural enough; just as it is to say that a stone may be a diamond; but still we predicate the genus of the species or individual, and not the species or individual of the genus: it is not the genus colour, but colour in some particular case, not the genus stone, but some particular mineral that is blue or that is diamond. Commonly, except where they are merely coincident attributes,[2] the predicate is

[1] Unless a definite instance is meant.

[2] Terms, though they be general concrete terms, like *statesman* or *fishmonger*, may yet express only a special or 'abstract' aspect of the nature of the thing they denote, if they are not in the category of substance: cf. *supra*, p. 37, n. 1.

a wider term, or more *generic*, than the subject in judgement ; it is something which belongs to this and may belong to other subjects, not a part of the extension of the subject itself. It is natural to predicate the genus of the species, the attribute of the concrete thing. In science especially, whose judgements should be necessary and universal, the predicate, if not commensurate with the subject, must be the wider term. We cannot predicate universally of any term what is only part of its extension. If stone is a wider or more comprehensive term than diamond, other things besides diamonds are stones, and therefore that proposition must be particular in which diamond is predicated of stone. A diamond is a stone, a stone may be a diamond ; blue is a colour, a colour may be blue.

In calling the predicate of the conclusion in a syllogism the *major* term, then, Aristotle chose a name which was appropriate, both when the predicate is related to the subject as attribute to concrete thing, and when it is related to the subject as the more to the less generic. By the name *major* he wished to indicate that the predicate is the more comprehensive term : that it signified something characterizing the subject, but characterizing, or capable of characterizing, other subjects also—something therefore which might be regarded as an attribute of the subject (in a wide sense of the word *attribute*), but not as a subject characterized by it.[1]

[1] Cf. *infra*, pp. 379-380. In *Anal. Post.* β. xvii. Aristotle uses the word *παρεκτείνειν, to extend beyond*, of the relation of major to middle term. He forgets however there, and ignores in the *Prior Analytics* when he adopts the expressions *major, middle*, and *minor terms*, what in the *Posterior Analytics* he rightly recognizes as characteristic of science (though not of all reasoning), that it aims at demonstrating *commensurate* judgements. Still, there are many scientific judgements which have not that character, and even in those that have it, the predicate, considered apart from the demonstration, is conceived as what does belong to this subject, and might belong to others. It is only in the demonstration by which it is shown to belong to one subject, that we come to realize it can belong to that subject alone. If we see, for example, in proving that the angle in a semicircle is a right angle, that the proof hinges upon a feature which cannot belong to the angle in another segment (viz. that the subtending chord passes through the centre of the circle), then we see that the predicate is commensurate with the subject ; and then also the predicate (if I may so express myself) sinks into the concrete nature of the subject, and is conceived as a necessary part thereof. While a demonstration is still wanted by us, to show us that the angle in a semicircle is a right angle, we have no ground for supposing that that is not a property of angles in some other segments as well : so soon as we realize that it can be the property of none other, we have incorporated the demonstration with the subject-concept (of the angle in a semicircle) and major, minor, and middle terms have for us lost their isolation. Demonstration,

The *middle* term takes its name not simply from being a point of connexion between the other two, but from being really intermediate in comprehensiveness. This it is, however, only in the first figure. It is only there that the middle term is predicated of the minor, and the major predicated of it. In the second, it is predicate in each premiss ; in the third, the subject, of which both major and minor terms are predicated. But that which in the first figure is really a *middle* term between the major and minor serves equally in the others to be the means of establishing that relation between the major and minor which we prove ; and the nomenclature that is fixed by the first figure is extended to them all.

It follows that Galen was wrong in assigning to a fourth and separate figure syllogisms in whose conclusion the most comprehensive term is subject, and the least comprehensive predicate, as in the example

> What breeds rapidly has a short life
> Flies breed rapidly
> ∴ Some short-lived things are flies.

It is true that in them the middle term is predicate of the premiss containing the predicate of the conclusion, and subject of the premiss containing the subject of the conclusion ; but in respect of comprehensiveness the predicate of the conclusion is minor, its subject major ; and therefore such syllogisms are better treated as belonging to the first figure, but having an inverted or indirect conclusion. The distinction of major and minor between terms is primarily that of greater and less comprehensiveness, and this is not altered by making the more comprehensive the subject, and the less the predicate, in the conclusion.

But the fourth figure has been taught for so many centuries among the ' moods and figures ' of the syllogism, that for the sake of the history of Logic we cannot altogether ignore it, even while we recognize the error in which it had its birth.[1]

5. The last paragraph spoke of *moods* and figures of the syllogism. The difference of figures has already been explained to depend on the position of the middle term in the premisses. The difference of mood depends on the quantity and quality of the propositions

when complete and while completely realized by the mind, may be said to collapse into a judgement whose terms are interfused. *Cf.* p. 311, *infra*.

[1] Cf. *infra*, pp. 280–285.

composing the syllogism. This may be the same in different figures, or different in the same figure : e.g. in the syllogisms

All organisms are mortal
Man is an organism
∴ Man is mortal :

and No unlicensed body may sell liquor to strangers
A college is unlicensed
∴ A college may not sell liquor to strangers :

the figure is the same (the first), but the component propositions are in one case of the form *A*, *A*, *A*, and in the other of the form *E*, *A*, *E*. If the second syllogism be now compared with the following

No good comrade avoids pleasure
All ascetics avoid pleasure
∴ No ascetic is a good comrade :

it will be seen that the component propositions are of the same form in both, *E*, *A*, *E* : but the figure is different.

The different moods have received distinct names in the various figures wherein they occur ; and hence what are called the ' mood-names ' of the various forms of syllogism indicate both figure and mood. What moods are possible in what figures—i.e. what combinations of premisses, as determined by their quantity and quality, will yield what form of conclusion (*A*, *E*, *I*, and *O*) with each position of the middle term—is the general problem to which the formal part of the theory of syllogism has to find an answer. We are now familiar with the technical terms employed in solving the problem. We must next consider the solution.

B. The only method of originally determining what combinations of premisses will yield what conclusion is to try them all, with each position of the middle term, and see. This is what Aristotle did, in the *Prior Analytics*. But when it has been done, it is possible to review the result, and there recognize the nature of the faults committed in those which are invalid, and the rules which therefore must be observed (whether in all syllogisms, or in those of a particular figure) in order to validity. These rules may then be placed in the forefront of our exposition ; it may be shown, by the help of an example, that the breach of them brings invalidity ; and in each figure, out of the whole number of ways in which it is

mathematically possible to combine two premisses, when either may have any one of four forms, we can show which are conformable to the rules that we have found necessary to be observed in that figure.

The syllogism is now generally taught in the latter manner, which is the more formal and systematic. But the other is the more natural, and we shall therefore begin, for the first figure, with that. Indeed the 'rules of syllogism' could not have been known first, and then the valid moods determined thence; their formulation is the result of an investigation of the valid moods conducted without them.

A valid mood of syllogism is immediately seen to be valid by any one who considers it in a particular example, and though the example is particular, the form of inference is seen to be valid universally. The best way, on the other hand, to show that a mood is invalid, is to produce examples in which the premisses and conclusion are of the quality and quantity which that mood requires, and show by them that while the premisses are true, the conclusion may be indifferently true or false. For if you cannot rely on a form of argument to produce a true conclusion from true premisses, it certainly is not a valid form.

Now in the **first figure** the middle term is subject of the major premiss and predicate of the minor. Let us take the possibilities in order.

1. *Both premisses universal.*

a. both affirmative; the mood is valid, and the conclusion *A*:

All organisms are mortal	All *M* is *P*
Man is an organism	All *S* is *M*
∴ Man is mortal [1]	∴ All *S* is *P*

b. both negative; no conclusion follows:

Sounds have no scent	No *M* is *P*
Colours are not sounds	No *S* is *M*
∴ Colours have no scent	∴

Sounds are not visible
Colours are not sounds
∴ Colours are not visible [1]

[1] With actual terms, an universal proposition is often more naturally expressed without the use of the mark of quantity, *All* men or *No* colours. Where this is so, and the content makes it plain that the proposition is

c. *one affirmative and the other negative :*

i. *the major negative* ; the mood is valid, and the conclusion *E* :

No Protestant acknowledges the Pope	No *M* is *P*
Lutherans are Protestants	All *S* is *M*
∴ No Lutheran acknowledges the Pope	∴ No *S* is *P*

ii. *the minor negative* ; no conclusion follows :

Lutherans are Protestants	All *M* is *P*
Calvinists are not Lutherans	No *S* is *M*
∴ Calvinists are not Protestants	∴

Lutherans are Protestants
Romanists are not Lutherans
∴ Romanists are not Protestants

2. *One premiss universal, and one particular.*

a. *both affirmative :*

i. *major universal, minor particular* ; the mood is valid and the conclusion *I* :

What raises prices injures the consumer	All *M* is *P*
Some import-duties raise prices	Some *S* is *M*
∴ Some import-duties injure the consumer	∴ Some *S* is *P*

ii. *major particular, minor universal* ; no conclusion follows :

Some taxes are levied at death	Some *M* is *P*
Excise-duties (*or* Legacy-duties) are taxes	All *S* is *M*
∴ Excise-duties (*or* Legacy-duties) are levied at death	∴

b. *both negative :*

i. *major universal, minor particular* ; no conclusion follows :

Starches contain no nitrogen	No *M* is *P*
Some foods (*or* flesh-foods) are not starches [1]	Some *S* is not *M*
∴ Some foods (*or* flesh-foods) contain no nitrogen	∴

universal, it has not been thought necessary to mark the quantity in that way. But with symbols, because there is then no content to guide us, this is necessary.

[1] It is true that no flesh-foods are starches. But if with premisses true and of the above form the conclusion is to be false, it is impossible to find an example where it would not be equally true to enunciate the minor premiss universally. For suppose that only some *S* is not *M* : then some *S* is *M*, and with the help of the major premiss, no *M* is *P*, it will follow that some *S* is not *P*. But this conclusion was to be false ; therefore no *S* can be *M*.

ii. *major particular, minor universal*; no conclusion follows:

Some quadrilaterals contain no right angles — Some M is not P
The triangle in a semicircle (*or* The pentagon) is not a quadrilateral — No S is M
$\therefore$ The triangle in a semicircle (*or* The pentagon) contains no right angle — $\therefore$

c. one affirmative, and the other negative:

i. *major affirmative and universal, minor negative and particular*; no conclusion follows:

All living things change (*or* contain carbon) — All M is P
Some compounds are not living — Some S is not M
$\therefore$ Some compounds do not change (*or* do not contain carbon) — $\therefore$

ii. *major negative and universal, minor affirmative and particular*; the mood is valid, and the conclusion O:

No political offence is extraditable — No M is P
Some murders are political offences — Some S is M
$\therefore$ Some murders are not extraditable — $\therefore$ Some S is not P

iii. *major affirmative and particular, minor negative and universal*; no conclusion follows:

Some traders are freeholders (*or* are members of Parliament) — Some M is P
No parson trades — No S is M
$\therefore$ No parson is a freeholder (*or* is a member of Parliament) — $\therefore$

iv. *major negative and particular, minor affirmative and universal*; no conclusion follows:

Some plants are not edible — Some M is not P
Beans (*or* Monkshoods) are plants — All S is M
$\therefore$ Beans (*or* Monkshoods) are not edible — $\therefore$

3. *Both premisses particular.*

a. both affirmative; no conclusion follows:

Some Germans are Protestants — Some M is P
Some Calvinists (*or* Romanists) are Germans — Some S is M
$\therefore$ Some Calvinists (*or* Romanists) are Protestants — $\therefore$

b. both negative ; no conclusion follows :

Some things profitable are not pleasant	Some M is not P
Some things popular (*or* pleasant) are not profitable	Some S is not M
∴ Some things popular (*or* pleasant) are not pleasant	∴

c. major affirmative, minor negative :

Some luxuries are taxed	Some M is P
Brandy (*or* A cart) for some purposes is not a luxury	Some S is not M
∴ Brandy (*or* A cart) for some purposes is not taxed	∴ Some S is not P

d. major negative, minor affirmative :

Some men of science do not study philosophy	Some M is not P
Some rich men (*or* philosophers) are men of science	Some S is M
∴ Some rich men (*or* philosophers) do not study philosophy	∴ Some S is not P

This exhausts the possible varieties in form of premisses, so far as the first figure is concerned ; and we have found only four which give any conclusion, namely (to represent them by the accepted symbols, and add the symbol for the conclusion) *AAA* *AII* *EAE* *EIO*

Since the thirteenth century, logicians have given to each of these moods, as well as to those in the remaining figures, a separate name, in which the vowels in order indicate the quality and quantity of the major and minor premisses and the conclusion. The names of these moods of the first figure are Barbara, Celarent, Darii Ferio : and syllogisms of those types are called syllogisms in Barbara, Celarent, &c.[1]

[1] The earliest known work in which these mood-names are found is by William Shyreswood (born in Durham, student in Oxford, taught at Paris, died as Chancellor of Lincoln, 1249 ; *v.* Prantl, iii. 10, Absch. xvii. Anm. 29) : 'Modi autem et eorum reductiones retinentur his versibus—Barbara, &c.' (ib. Anm. 52). They passed into general currency through the *Summulae Logicales* of Petrus Hispanus, afterwards Pope John XXI, who was long believed to be the author of them (*c.* 1226–1277), until Prantl found them in the unpublished MS. of William Shyreswood in the Library of Paris (vol. ii. p. 264). A somewhat similar *memoria technica*, but less ingenious, because it embodies only the form of the moods, and not the rules for the

But an addition has to be made. If the minor premiss is an universal negative proposition, and the major is affirmative, whether universal or particular, then though no conclusion can be drawn in which the major term is denied (or affirmed) of the minor, it is possible to draw a particular conclusion in which the minor term is denied of the major. Thus in 1. *c.* ii. from the premisses

Lutherans are Protestants
Calvinists (*or* Romanists) are not Lutherans

it was impossible to infer whether Calvinists or Romanists were Protestants: the former in fact being so, and not the latter. But it is possible to infer that some Protestants are not Calvinists (*or* Romanists). And in 2. *c.* iii. from the premisses

Some traders are { freeholders / members of Parliament
No parson trades

it was impossible to infer whether any parson was a freeholder, or a member of Parliament: none of them, in fact, being eligible to Parliament, while a rector or vicar is legally a freeholder. But it is possible again to infer that

Some { freeholders / members of Parliament } are not parsons.

Doubtless no member of Parliament is a parson, as no Romanist is

reduction of the moods in the second and third figures to the first (*v.* next chapter) is found in the margin of the treatise attributed to Michael Psellus (1018–? 1079), Σύνοψις εἰς τὴν Ἀριστοτέλους λογικὴν ἐπιστήμην (*Synopsis of Aristotle's Logic*) (according to Prantl, in the same hand as the text, ii. 275. Absch. xv. Anm. 46). Prantl believes the work of William Shyreswood to be borrowed from, and that of Petrus Hispanus to be a mere translation of, the *Synopsis* of Psellus. In an article, however, by R. Stapper (*Die* Summulae Logicales *des* Petrus Hispanus *und ihr Verhältniss zu* Michael Psellus, published in the *Festschrift zum elfhundertjährigen Jubiläum des deutschen Campo Santo in Rom*, Freiburg im Breisgau, 1897, pp. 130 *sq.*; cf. also his *Papst Johannes XXI.* pp. 16-19, Munster i. W., 1898), reason is shown for thinking that the ascription of the *Synopsis* to Michael Psellus is erroneous, and that it is really a translation of the *Summulae*: the Augsburg MS. of the *Synopsis* in which the ascription occurs contains also chapters lacking in the *Summulae*, and partly identical with other works of Psellus; these may have led to his name being placed in the title, which Stapper conceives to be in a hand fifty years later than the bulk of the MS. No other MS. of the *Synopsis* ascribes it to Psellus; all the rest profess to be translations from the Latin; seven give the name of Petrus Hispanus as author, and four that of Georgius Scholarius (Gennadius) as translator. Cf. also Sir William Hamilton's *Discussions*, 2nd ed., pp. 128, 671 sq.: who, however, wrote before Prantl's work appeared.

a Protestant; and those who know this would not trouble to enunciate the subaltern, or particular, propositions; but our premisses do not inform us of the universal; what they do tell us is the truth, even if not the whole truth.

We have thus two further and *indirect* moods, i.e. moods in which the minor term is concluded of the major, not the major of the minor, viz.

AEO
IEO

All / Some } *M* is *P*
No *S* is *M*
∴ Some *P* is not *S*

And there are other indirect moods also. For in Barbara, Celarent, and Darii, it is possible, instead of drawing the direct and natural conclusion, to draw the converse, wherein the major term will be subject and the minor predicate. Thus in 1. *a.* we might have concluded 'Some mortals are men', in 1. *c.* i. 'No one who acknowledges the Pope is a Lutheran', in 2. *a.* i. 'Some things that injure the consumer are import-duties'. There are thus five indirect moods in all: and the whole nine are given in the first two lines of the following hexameters (it is to be noted that the extra syllables after the third, in the fifth and ninth names, are inserted *metri gratia*, and have no significance):—

Barbara Celarent Darii Ferio, Baralipton
Celantes Dabitis Fapesmo Frisesomorum [1]:
Cesare Camestres Festino Baroco: Darapti
Felapton Disamis Datisi Bocardo Ferison.

The first four names in the third line belong to the valid moods in the second figure: the remainder to those in the third. It would be possible to show what moods are valid in these figures by experimenting with all the combinations of premiss possible in respect of quality and quantity when the middle term was respectively predicate or subject in each premiss. But any one who has followed the process for the first figure can work it out for himself in the others; and we may proceed now to the enunciation of the rules of syllogism, and the briefer deduction of the valid moods from them.

[1] The indirect moods of the first are the same as the moods of the fourth figure: cf. pp. 280–285, *infra*.

C. The **Syllogistic Rules** are eight in number, viz.

1. **A syllogism must contain three, and only three terms.** The necessity of this rule is manifest; for we have seen that a syllogism is an argument in which a relation (in the way of subject and predicate) is established between two terms, in virtue of their common relation (in that respect) to a third term. Hence, without a third term there is no syllogism: and if the terms of the conclusion were not related to the *same* third term, there would be no relation established between themselves, and so again, no syllogism.

For example, we can draw no conclusion barely from the premisses *What breathes needs oxygen* and *Fish have gills*. Any one who knew that what has gills breathes might infer that fish need oxygen: but the inference requires the premiss *What has gills breathes* no less than the other two; and falls really into two syllogisms, each containing three terms: though four terms occur in the whole argument, viz.:

(i) What breathes needs oxygen
What has gills breathes
∴ What has gills needs oxygen

(ii) What has gills needs oxygen
Fish have gills
∴ Fish need oxygen

If the middle term is used equivocally—i.e. in different senses in the two premisses—there will in reality be four terms, and no conclusion is possible; e.g. it is true that *no vegetable has a heart*: it is also true that *a good lettuce has a heart*: but to have a heart means something different in these two propositions, and it would be fallacious to conclude that *a good lettuce is not a vegetable*.[1]

A breach of this first rule is technically known as the fallacy of *Quaternio Terminorum* or of *Four Terms*; and where it arises through the equivocal use of the middle term, as the fallacy of *ambiguous middle*.

2. **The middle term must be distributed in one premiss at least.** It will be remembered that a term is distributed, when used in reference to its whole extension; and undistributed, when not so used. Thus in the proposition *All jealous men are suspicious*, the term *jealous man* is distributed (for I expressly refer to all that falls

[1] Conversely, the middle term may be really the same, though verbally different, in the two premisses; and then there is a syllogism, e.g. *Branchiates need oxygen*, and *fish have gills* ∴ *Fish need oxygen*.

within the range of it); but the term *suspicious* is undistributed, for I consider it only as characterizing the jealous, and it may very well have a wider range than that. If again I say that *Some jealous men have killed their wives*, in this proposition neither term is distributed.

Now when the middle term is undistributed in both premisses, it may refer in each to a different part of its extension; and then the major and minor terms are not brought into relation with the *same* term in the premisses at all: hence no conclusion can be drawn.[1]

Examples from the three figures will make plain what is perhaps hard at first to grasp in an abstract statement. If *a Presbyterian is a Christian*, and *some Christians think that the order of bishops was instituted by Christ*, it does not follow that a Presbyterian thinks this. *Christian* is a term that includes more than *Presbyterian*; if all Christians thought that the order of bishops was instituted by Christ, then it would follow that Presbyterians thought so; but if only some Christians think it, how am I to tell that the Presbyterians are among these? Again, in the second figure, from the premisses *Birds fly* and *Eagles fly*, I cannot infer that *an eagle is a bird*; for though birds fly, many creatures may fly which are not birds, and an eagle might be one of these. If in either premiss the middle term were used with reference to its whole extension: if nothing flew but birds, or nothing flew but eagles, and if my premiss informed me of this: then I could conclude that all eagles were birds, or that all birds were eagles; but as it is, I can make no inference. Inference is as obviously impossible in the third figure, with the middle term undistributed. Granted that *some working-men are Tories*, and *some working-men are tailors*: I cannot hence determine whether or not *some tailors are Tories*: for the working-men that are tailors may not be the same working-men as are Tories, and then the inference would be false. But if in either premiss the middle

[1] This is sometimes expressed as follows: though the expression is apt to be misleading (cf. pp. 272, 273). It is said that the premisses assert agreement (or disagreement, if negative) between the major or minor, and the middle, terms; that if the middle term be undistributed in both premisses, the major and minor may respectively agree (or agree and disagree) with a different part of its extension; and therefore we cannot tell that they agree (or disagree) with one another. The vogue of such language is perhaps to be traced to Locke: cf. e.g. *Essay*, IV. xvii. 4: 'It is by virtue of the perceived agreement of the intermediate idea with the extremes, that the extremes are concluded to agree'; cf. also Bacon, *Nov. Org., Distrib. Operis*, 'tametsi enim nemini dubium esse possit quin, quae in medio termino conveniunt, ea et inter se conveniant,' &c.

term were distributed: if *working-men* were referred to in the whole extension of the term, and *all working-men* were spoken of: then a conclusion would follow. For whether all working-men were tailors, and some Tories, or vice versa, in either case the *some* of whom the one term was predicable would be included among the *all* of whom the other term was predicable, and then these two terms (*tailor* and *Tory*) would be predicable—not universally, but in part—one of the other.

A breach of this rule is technically known as the fallacy of *undistributed middle*.

[It is in the third figure, where the middle term is subject in both premisses, that the necessity of distributing it once at least is most obvious. Plainly, there, to say that it is used with reference to a part of its extension only is to say that only part of what it denotes is spoken of; and if this is a different part in the two premisses, there is not really any middle term. Some animals fly, and some are rodents: but they are not the same animals; swallows e.g. fly, and rats are rodents; and it is obvious that our premisses do not justify the inference that the same thing flies and is a rodent. But where the middle term is not subject, there is a certain awkwardness in talking of its distribution. This has already been noticed in discussing the 'quantification of the predicate'.[1] It was then shown that the predicate of a proposition is never really thought of in extension. And yet in explaining the present rule of syllogism, one is tempted to speak as if it were so thought of. A general demonstration of the rule is wanted, applicable equally to any figure; and it is easy to say that if the middle term is undistributed in both premisses, the major and minor may be brought into relation only with different parts of its extension, and therefore not with the same term at all. Or if we speak of agreement between them and the middle term, we have a more seductive formula: we can illustrate with circles, thus:

FIG. 1.

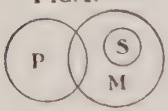


FIG. 2.

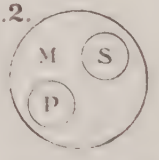
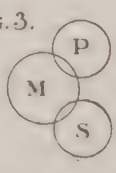


FIG. 3.



The inclusion of one area, wholly or partially, within another symbolizes an affirmative judgement, universal or particular: it is plain that the area *S* may fall wholly within *M*, and *M* partially

[1] Cf. c. ix. pp. 222 sq., *supra*.

[within P, and yet S may lie wholly outside P. This is supposed to show for Fig. 1, that with an undistributed middle we can draw no conclusion; and the other diagrams are as readily interpreted.

Yet a syllogism does not really compare the extension of three terms, and Euler's diagrams put us upon a wrong train of thought. It is true, that unless the middle term be distributed once at least, there is no point of identity in the premisses; and all mediate inference proceeds in some way by help of an identity. It is not true that the point of identity need consist in the same subjects being denoted—in the reference to the same part of the extension of the middle term in both premisses (for which referring to the whole extension in one of them would be an obvious security). In the third figure the inference may no doubt hinge on this; but not in the second, or the first. On the contrary, the inconclusiveness of an argument in the second figure with undistributed middle is best expressed by saying that it does not follow, because the same predicate attaches to two subjects, that these can be predicated one of the other: and in the first figure, that unless P is connected necessarily and universally with M, it is clear that what is M need not be P.[1]

If this discussion of the Undistributed Middle should seem too lengthy, it must be remembered (1) that for working purposes, in order to determine the correctness of a syllogism, the main thing to look to is the distribution of terms: and hence (2) that it is of great importance, in the theory of syllogistic inference, not to misunderstand this reference to distribution. In a later chapter (c. xiv) it will be necessary to consider whether the different figures of syllogism are really different types of reasoning, or the same; and the present discussion will throw light on that enquiry.]

3. **From two negative premisses nothing can be inferred.** A negative proposition denies between its terms the relation of subject and predicate. It is clear that if the major and minor terms are both denied to stand in that relation to the middle term, we cannot tell whether or not they are related as subject and predicate to one another. *Ruminant* may not be predicable of *rodent*, or vice versa: neither *carnivorous* of *ruminant*, or vice versa: we cannot from this infer anything as to the relation of *carnivorous* and *rodent*.

4. **If either premiss is negative, the conclusion must be negative.** The same kind of reflection will justify this rule, as the last. Two terms stand in the relation of subject and predicate; between

[1] The fourth figure has not been considered in this note, but in this matter it raises no question that is different from those that arise on the other figures.

one of them and a third term the same relation is denied; if any inference is possible,[1] it can only be to deny the relation also between the other and the third term.

5. **The conclusion cannot be negative, unless one premiss is negative.** This rule is the converse of the last, and equally obvious. If both premisses are affirmative, and if they justify a conclusion at all, they must establish and not refute our right to predicate the major of the minor.

6. **No term may be distributed in the conclusion, which was not distributed in its premiss.** For if a term is undistributed in the premisses, it is there not used with reference to its whole extension; and this does not justify us in a conclusion which uses it with reference to its whole extension.

A breach of this rule is called an *illicit process of the major*, or *minor, term*, as the case may be.

[With an illicit process of the minor term, if (as in the first and second figures) the minor term is subject in its own premiss, it is obvious that we are treating information about a part of the extension of the term as if it were information about the whole. If all M is P, and some S is M, we can only infer that some S, and not all S, is P. Where the minor term is predicate in its own premiss, or with an illicit process of the major term, the matter requires a little more reflection. The predicate of a judgement (and the major term is always predicate in the conclusion, unless the mood is indirect) not being thought in extension, there is some danger here again lest we should misunderstand a reference to its distribution. Take the following example of illicit process of the minor term, where the minor term is predicate in the minor premiss:

To make a corner in wheat produces great misery
To make a corner in wheat is gambling
$\therefore$ All gambling produces great misery.

My premisses do not primarily give me information about gambling; nevertheless, if there were no gambling except a corner in wheat, the minor term would be commensurate with the middle, and what

[1] It may happen, where the premisses justify no inference, that an affirmative conclusion would in fact be true; e.g. if some M is not P, and all S is M, it may be true that all S is P. Here of course the middle term is undistributed, and therefore there is no real point of identity in the argument. However, it is worth while noticing that the proof of this rule also is difficult to express in a quite abstract way. The notion of *agreement* is employed here again, but merits the same protest as before: if one term agrees with a second, and that disagrees with a third, the first will disagree with the third; but the relation between subject and predicate is too loosely described as one of agreement or disagreement.

[is predicated universally of the latter could be predicated universally of the former. As it is, however, for all the information that is given me, the minor term may be (and in fact it is) of wider extension than the middle; for there are many other modes of gambling besides making a corner in wheat. It is used therefore with reference to a part of its extension only, in the minor premiss; and it is that part which I am told in the major produces great misery; I have no right to extend that information to the whole extension of the term, and say that all gambling produces great misery; my only proper conclusion is that some gambling does so. Again, with regard to the major term: if I argue that productive expenditure benefits the country, and expenditure on art is not productive; and that consequently expenditure on art is of no benefit to the country: I am guilty of an illicit process of the major term. It may not at first sight appear that I have treated information given me about a part of what benefits the country as if it were information about everything that does so. And indeed expenditure which benefits the country is not directly the subject of my thought. Yet it is plain that though productive expenditure may benefit the country, it need not be the only form of expenditure to do so; and hence expenditure on art, though not productive, may be of benefit to the country for some other reason. Yet my conclusion would only be justified if I knew every reason why expenditure could benefit the country, and knew that none of them applied to expenditure on art: whereas my major premiss mentions one ground, and not the sole ground, on which expenditure is beneficial. It is therefore true in effect to say that in the conclusion I treat as referring to its whole extension information which was confined to a part of the extension of the major term; though none the less the extension of the major term is not the proper subject of my thought.[1]]

There remain two rules which are corollaries of those already given, viz.

7. **From two particular premisses nothing can be inferred,** and

8. **If either premiss is particular, the conclusion must be particular.**

The truth of these rules is not evident at first sight; and they can only be established generally—i. e. without reference to mood and figure—by considering what combinations of premisses there are,

[1] Beginners imagine sometimes that the fallacy of illicit process is committed, if a term which is distributed in the premiss is undistributed in the conclusion. This is not so. I must not presume on more information than is given me, but there is no reason why I should not use less.

It will be noticed, therefore, that no particular conclusion can be vitiated by an illicit process of the minor term: and no affirmative conclusion by an illicit process of the major.

both of which, or one of them, is particular; and it will then be seen either that there are not enough terms distributed in these premisses to warrant a conclusion at all; or not enough to warrant an universal conclusion, i.e. one that distributes the minor term.

If both premisses are particular, they must either be both affirmative (*I* and *I*), or both negative (*O* and *O*), or one affirmative and the other negative (*I* and *O*). But in a particular affirmative proposition neither subject nor predicate is distributed; so that the combination of premisses *II* contains no distributed term, and therefore—since the middle term must be distributed if any inference is to be drawn—will yield no conclusion. From *OO*, two negative propositions, a conclusion is impossible. From *I* and *O*, if there were any conclusion, it would be negative; but as the predicate of a negative proposition is distributed, the major term (the predicate of the conclusion) would be distributed in the conclusion; therefore the major term should be distributed in its premiss; and since the middle term must be distributed in the premisses also, we require premisses with two terms distributed in them, to obtain a conclusion; now the combination of a particular affirmative with a particular negative provides only one distributed term, viz. the predicate of the latter (*O*); and therefore from them also a conclusion is impossible.

A similiar line of reasoning will establish rule 8; no combination of premisses, whereof one is particular, contains enough distributed terms to allow of an universal conclusion. For again, either both are affirmative (*A* and *I*), or both negative (*E* and *O*), or one affirmative and the other negative (*A* and *O*: *E* and *I*). The two negative premisses may be struck out as before. The combination of *A* with *I* contains only one distributed term, the subject of the universal affirmative (*A*); and as the middle term must be distributed if the reasoning is to be valid, the subject of *A* must be the middle term; hence the minor term will be one of those that are undistributed in the premisses, and therefore also in the conclusion (of which it is the subject) it must be undistributed—i. e. the conclusion must be particular. The combinations *A* and *O*, *E* and *I* both contain two distributed terms; viz. in the former the subject of the universal affirmative and the predicate of the particular negative, in the latter the subject and predicate of the universal negative; but both of them require negative conclusions, in which the major term is distributed; in both therefore the terms distri-

buted in the premisses must be the major and middle, and the minor term be one of those that are undistributed, so that the conclusion again will be particular.

The above rules are all contained in four rude hexameter lines :

Distribuas medium, nec quartus terminus adsit ;
Utraque nec praemissa negans, nec particularis ;
Sectetur partem conclusio deteriorem ;
Et non distribuat, nisi cum praemissa, negetve.

The third line (that the conclusion must conform to the inferior part of the premisses) covers both the fourth and eighth rules ; a negative being considered inferior to an affirmative, and a particular to an universal judgement. The fourth line (that the conclusion must not distribute any term, unless the premiss does so, nor be negative unless a premiss is so) gives the sixth rule, and the fifth.

D. **Determination of the moods valid in the several figures.**

We have seen that syllogisms are distinguished in mood according to the quantity and quality of the propositions composing them ; and in figure according to the position of the middle term in the premisses. The validity of a syllogism, and the character of the conclusion that can be drawn, depend very largely on the distribution of the several terms—middle, major, and minor—in the premisses ; and this again on the question whether the middle term is subject, and one of the others predicate, in a premiss, or vice versa. Hence a combination of premisses which yields a conclusion in one figure, may yield none in another : e.g. *All M is P, All S is M* yields the conclusion *All S is P* ; but *All P is M, All S is M* yields no conclusion, though the quantity and quality of the premisses are unchanged. We shall therefore have to take the possible combinations of premisses in each figure in turn, strike out those which yield no conclusion in that figure, and ask what kind of conclusion—i. e. whether universal or particular[1]—the others yield in it.

Now as there are four kinds of proposition, so far as quantity

[1] For this depends on the distribution of terms in the premisses, which varies according to the figure : whether the conclusion is affirmative or negative depends on whether both premisses are affirmative or not, a point which can be determined without asking where the middle term stands, i. e. what the figure is.

and quality are concerned—*A*, *E*, *I*, and *O*—and our premisses must be two in number, there are sixteen combinations of premisses mathematically possible.

These combinations are as follows, the premisses being indicated by the conventional vowels, and the major premiss in all cases by the vowel which stands first.

AA	*EA*	*IA*	*OA*
AE	*EE*	*IE*	*OE*
AI	*EI*	*II*	*OI*
AO	*EO*	*IO*	*OO*

It is not however necessary to try the validity of all these sixteen combinations in each figure in turn ; for four can be seen to yield no conclusion on a ground holding in all figures alike, without reference to the position of the middle term, viz. those wherein both premisses are negative, *EE*, *EO*, *OE*, *OO*. Four more are excluded by the rules just given, viz. (i) *II*, *IO*, *OI* (as well as *OO* again) on the ground that both premisses are particular, and (ii) *IE*, on the ground that it involves illicit process of the major term ; for since one premiss is negative, the conclusion would be negative, and so distribute the major term, while the major premiss, being a particular affirmative, would not distribute that term, whether it were subject in the premiss or predicate. But the inconclusiveness of these four combinations cannot be rightly understood, as that of combinations of two negative premisses can be, without taking examples in the several figures : the rules from whose truth it follows being themselves a generalization of what we discover in so doing.

There remain eight combinations of premisses, not excluded by any general rule, on whose validity we cannot pronounce without reference to the figure and the position of the middle term, viz.

AA *AE* *AI* *AO* *EA* *EI* *IA* *OA*

It will be found that four of them are valid in the first figure, four in the second, and six in the third ; there are also five indirect moods of the first, or moods of the fourth, figure : making in all nineteen moods.

In the **first figure**, the middle term is subject of the major premiss and predicate of the minor : hence in this figure *M P*

1. *The minor premiss must be affirmative* : for if it were *S M*
negative, the conclusion would be negative, and so distri- *S P*

bute the major term P; the major term must therefore be distributed in the major premiss; but as it is there predicate, it cannot be distributed unless the major premiss is also negative (since no affirmative proposition distributes its predicate): we should thus have two negative premisses, or else an illicit process of the major term.

2. *The major premiss must be universal*: for since the minor is affirmative, its predicate M, the middle term, will be undistributed; therefore M must be distributed in the major premiss; and for this purpose the major premiss, of which it is the subject, must be universal. These rules however do not hold for the *indirect moods*.

In this figure, therefore, the premisses AE, AO are invalid, by rule 1: IA, OA by rule 2[1]; AA, EA, AI, EI are valid. The conclusions which they yield will be respectively A (universal affirmative), E (universal negative), I (particular affirmative), and O (particular negative); and the *moods*—in which the quantity and quality of the conclusion are indicated, as well as of the premisses—are AAA, EAE, AII, EIO. Their names are Barbara, Celarent, Darii, Ferio. But in the first three of these moods, as we have seen, the converse conclusions can also be drawn; and with the premisses AE, IE, a particular conclusion follows denying S of P; and so we get also the indirect moods AAI, EAE, AII, AEO, IEO, whose names are Baralipton, Celantes, Dabitis, Fapesmo, Frisesomorum.

In the **second figure** the middle term is predicate in both premisses: hence in it

$$\begin{matrix} P\ M \\ S\ M \\ S\ P \end{matrix}$$

1. *One premiss must be negative*, for otherwise the middle term would be undistributed.

2. *The major premiss must be universal*: for since one premiss is negative, the conclusion will be negative, and so distribute the major term P: P must therefore be distributed in the major premiss; i. e. as it is here the subject thereof, the major premiss must be universal.

Hence the premisses AA, AI, IA are invalid, by rule 1: the

[1] e. g. from the premisses *Contemporary evidence is of great historical value*, *Tradition is not* (or *Some inscriptions are not*) *contemporary evidence*, it cannot be inferred that *Tradition is not* (or *Some inscriptions are not*) *of great historical value* (*AE*, *AO*): from the premisses *Some pointed arches are* (or *are not*) *four-centred*, *All Gothic arches are pointed*, it cannot be inferred that *All Gothic arches are* (or *are not*) *four-centred* (*IA*, *OA*).

premisses *OA* (and *IA* again) by rule 2[1]; *EA*, *AE*, *EI*, *AO* are valid. The *moods* are therefore *EAE*, *AEE*, *EIO*, *AOO*; their mood-names are Cesare, Camestres, Festino, and Baroco.

In the **third figure** the middle term is subject in both premisses: hence in it

M P
M S
S P

1. *The minor premiss must be affirmative*, for the same reason as in Fig. 1 (the major term, in both figures, being similarly placed in its premiss).

This rule excludes the premisses *AE*, *AO*[2]: the remaining combinations, *AA*, *AI*, *EA*, *EI*, *IA*, *OA*, are valid. But because the minor term in this figure is predicate of the minor premiss, and the latter is affirmative, the minor term will not be distributed in it; hence it must not be distributed in the conclusion; and therefore in all cases

2. *The conclusion will be particular.*

The *moods* are consequently *AAI*, *AII*, *EAO*, *EIO*, *IAI*, *OAO*: their mood-names are Darapti, Datisi, Felapton, Ferison, Disamis, Bocardo.

[It is impossible at this point to pass over the so-called **fourth figure.** We have seen above (pp. 268–269, 279) that in the first figure, besides the four direct moods, there are five 'indirect' moods, i. e. moods in which the conclusion affirms or denies the minor term of the major. In so describing these moods, we base the distinction of major and minor terms on their meaning: the major term is the more comprehensive; it signifies the general nature of, or some element in, the being of that real subject which the minor term stands for. If we consider 'terms of thought' we may say that the minor is characterized by the major, not vice versa. This relation is natural between two terms, when (as in Fig. 1) we can find a middle term predicable of one of them, and the other of it. But two terms of which the same predicate may be respectively affirmed and denied (as in Fig. 2), or which may be affirmed, or respectively affirmed and denied, of the same subject (as in Fig. 3), need stand in no such relation. And if this relation is ignored, and being major or minor term is made to consist barely in being predicate or subject of the conclusion, then we cannot describe any mood as affirming or

[1] e.g. from *Some* (or *All*) *daisies have a great number of flowers within a single calyx*, *All* (or *Some*) *compositae have a great number of flowers within a single calyx* it cannot be inferred that *Some*, or *All*, *compositae are daisies* (*AA*, *AI*, *IA*): nor from *Some annuals are not* (or *are*) *hardy*, *All poppies are hardy*, that *Some poppies are not* (or *are*) *annuals* (*OA*, *IA*).

[2] e.g. from the premisses *All ostriches have wings*, *No ostriches can* (or *Some ostriches cannot*) *fly*, it cannot be inferred that *No creatures that can fly have wings* or that *Some creatures that can fly have no wings* (*AE*, *OA*).

[denying in its conclusion the minor of the major. Instead therefore of the scheme [1]

(1) *M P* What is sensible is in the mind
S M Material things are sensible
∴ *P S* ∴ Some things in the mind are material things

we must have the scheme

(2) *P M* Material things are sensible
M S What is sensible is in the mind
∴ *S P* ∴ Some things in the mind are material things.

Aristotle, as already remarked, did not recognize a fourth figure, but he recognized the possibility of concluding indirectly in the first figure, though not as a thing peculiar to the first. In one place he says [2]: 'It is clear that in all the figures, when there is no [direct] syllogism, if both premisses are affirmative or both negative nothing at all necessarily follows, but if one is affirmative and one negative, and the negative is universal, a syllogism always arises with the minor as predicate to the major: e. g. if all or some *B* is *A*, and no *C* is *B*: for, the premisses being converted, it is necessary that some *A* is not *C*. And similarly in the other figures; for by means of conversion a syllogism always arises.' This covers the moods Fapesmo and Frisesomorum in Fig. 1. Elsewhere he points out that 'whereas some syllogisms are universal [in their conclusion] and some particular, those which are universal always have more conclusions than one; of those which are particular, the affirmative have more conclusions than one, but the negative have only the [direct] conclusion. For the other propositions convert, but the particular [negative] does not' [3]. He means that any syllogism concluding to *E*, No *S* is *P*, gives also, by conversion of that,

[1] It will be noted that the real terms indicated by *S* and *P* respectively in (1) are indicated by *P* and *S* in (2), because in (1) *S* and *P* symbolize minor and major in the sense of less and more comprehensive, in (2) minor and major in the sense of subject and predicate of the conclusion, and what is minor in the former sense is major in the latter, and vice versa.

[2] *Anal. Pri. a.* vii. 29ᵃ 19 Δῆλον δὲ καὶ ὅτι ἐν ἅπασι τοῖς σχήμασιν, ὅταν μὴ γίνηται συλλογισμός, κατηγορικῶν μὲν ἢ στερητικῶν ἀμφοτέρων ὄντων τῶν ὅρων οὐδὲν ὅλως γίνεται ἀναγκαῖον, κατηγορικοῦ δὲ καὶ στερητικοῦ, καθόλου ληφθέντος τοῦ στερητικοῦ ἀεὶ γίνεται συλλογισμὸς τοῦ ἐλάττονος ἄκρου πρὸς τὸ μεῖζον, οἷον εἰ τὸ μὲν Α παντὶ τῷ Β ἢ τινί, τὸ δὲ Β μηδενὶ τῷ Γ· ἀντιστρεφομένων γὰρ τῶν προτάσεων ἀνάγκη τὸ Γ τινὶ τῷ Α μὴ ὑπάρχειν. ὁμοίως δὲ κἀπὶ τῶν ἑτέρων σχημάτων· ἀεὶ γὰρ γίνεται διὰ τῆς ἀντιστροφῆς συλλογισμός. It is plain that ὅταν μὴ γίνηται συλλογισμός means 'when there is no direct syllogism'.

[3] *Anal. Pri. β.* i. 53ᵃ 3 ἐπεὶ δ' οἱ μὲν καθόλου τῶν συλλογισμῶν εἰσὶν οἱ δὲ κατὰ μέρος, οἱ μὲν καθόλου πάντες ἀεὶ πλείω συλλογίζονται, τῶν δ' ἐν μέρει οἱ μὲν κατηγορικοὶ πλείω, οἱ δ' ἀποφατικοὶ τὸ συμπέρασμα μόνον. αἱ μὲν γὰρ ἄλλαι προτάσεις ἀντιστρέφουσιν, ἡ δὲ στερητικὴ οὐκ ἀντιστρέφει. What Aristotle says here would cover the Subaltern Moods (cf. p. 285, *infra*); but he had not got them in his mind; he would not have regarded them as drawing a different, but part of the same, conclusion.

[the conclusion No P is S, and any concluding to A or I, All S is P or Some S is P, gives also the conclusion Some P is S. We have therefore here a recognition of the possibility of the other three indirect moods of Fig. 1, Baralipton, Celantes, and Dabitis: whose conclusions are merely the converse of those which follow directly from the same premisses in Barbara, Celarent, and Darii.

These observations are applied to all three figures, because Aristotle thought that in the second and third also the major and minor terms, though not distinguishable, as in the first, by having different positions in the premisses, could yet be distinguished by their meaning, so that we could tell whether a syllogism concluded directly or indirectly, and distinguish, e. g., between concluding directly in Camestres and indirectly in Cesare, or directly and indirectly in Darapti. And sometimes this may be done. To be rock is of the being of all granite, to be granite is not of the being of all rock; hence 'rock' and 'granite' are by their meaning relatively major and minor. Now from the premisses

Some rocks are sedimentary
Granites are not sedimentary

I cannot conclude that Granites are not rocks; but I can conclude, indirectly, that Some rocks are not granites. Again, in gases lighter than air, to be lighter than air is a character of the gas, not vice versa; hence from the premisses

Steam and hydrogen are lighter than air
Steam and hydrogen are gases

the natural conclusion is that Some gases are lighter than air, though the converse, that Some things lighter than air are gases, also follows. But often enough in the second and third figures there is nothing in their meaning to make us regard one of two terms as major and the other as minor, rather than vice versa, and then their position in the conclusion must be taken to decide it. And if we make that decide it always, there is still no syllogism in Figs. 2 and 3 which cannot formally be referred to one of the direct moods. The above example in Fig. 2 may be treated as in Festino, thus:

Granites are not sedimentary
Some rocks are sedimentary
$\therefore$ Some rocks are not granite

and with premisses A and E in Fig. 2, according as the subject of A or of E is predicate in the conclusion, the syllogism may be referred to Camestres or to Cesare; for each mood yields the converse of the conclusion of the other. From the premisses

Spiders have eight legs
Insects have not eight legs

[it is more natural to conclude in Cesare that Spiders are not insects than in Camestres that Insects are not spiders, because 'insect' is a more generic term than 'spider'. But whichever conclusion is drawn, the syllogism can be referred to a mood whose form covers it. So with Fig. 3, which has no universal conclusions; an indirect syllogism with premisses *AA* still has the form of Darapti, an indirect syllogism with *AI* or *IA* the form of Datisi or Disamis. But in Fig. 1 it is otherwise. Here, if we draw the indirect, or converse of the direct, conclusion in Barbara, Celarent, or Darii, we cannot by transposing the premisses make the premiss containing the predicate of the indirect conclusion the major premiss, and yet preserve the scheme of the figure, because in this figure the middle term has not the same position in both premisses, and their transposition alters the position of it, and so, if the figure of a syllogism is determined by the position of the middle term, alters the figure also. And if from the premisses *AE* or *IE*, which have no direct conclusion in this figure, we draw the indirect conclusion, and therefore treat *E* as major premiss because containing the predicate of the indirect conclusion, again the middle term becomes predicate and the minor subject of its premiss, and the scheme of the figure is altered. Thus it is easy to see why the indirect moods of Fig. 1 came to be regarded as belonging not to Fig. 1 at all, but to a separate fourth figure. For in many syllogisms of Figs. 2 and 3 there is nothing to settle which shall be called the major and which the minor term but their position as predicate or subject of the conclusion. And settling it thus, we do not need to admit indirect moods in these figures. Only those syllogisms then remain outstanding which in Fig. 1 from premisses that admit of no direct conclusion draw indirect conclusions, or from premisses that yield direct conclusions draw the converse of those. These syllogisms, if the distinction of major and minor terms is still made to depend on their position in the conclusion, do not belong to the scheme of Fig. 1, and a fourth figure is therefore instituted, in which the middle term is predicate in the major premiss and subject in the minor.

That what Aristotle notices about the three figures generally, in the passages quoted above, works out rather differently in the first and in the other two was very early noticed; and an explicit recognition of the five indirect moods as supplementary moods of Fig. 1 is attributed to his pupil and successor in the Lyceum, Theophrastus.[1] If Averroes is right in saying that Galen was the first to regard them as belonging to a distinct figure,[2] the view of Theophrastus held the field for some five centuries. Averroes himself

[1] *v.* Prantl, i. 365, Abschn. v. Anm. 46, where the passages from Alexander, who ascribes the addition of these moods to Theophrastus, are quoted.

[2] Prantl, i. 570–574.

[disagreed with Galen, and in this he was followed by Zabarella,[1] one of the best of the scholastic commentators on Aristotle, whose *De Quarta Figura Syllogismi Liber* is still worth reading on the subject; though in the reason he gives for not regarding the Galenian as really a fourth and independent figure he relies in part upon the questionable analysis which regards all syllogism as an application of the principle called the *Dictum de omni et nullo* (cf. *infra*, p. 296). The real objection to Galen's view is that it implies a defective insight into the character of the thinking in these forms of argument, and treats the syllogism too much as a matter of verbal manipulation. In the fourteenth chapter an endeavour is made to explain the grounds on which this verdict rests. The external and mechanical way of regarding syllogism, which underlies the reference of these moods to a fourth and separate figure, finds what is hardly more than its logical issue in some of the later scholastic writers, who erect separate moods on no better ground than the order in which the premisses are enunciated, without there being any actual difference in the premisses or conclusion.[2]

Granted, however, that we are to acknowledge a fourth figure, the following will be the special rules of it: it must be remembered that as referred to this figure we call that premiss the major which as referred to the first figure we should call the minor, and vice versa.

1. *If either premiss is negative, the major must be universal:* for if either premiss is negative, the conclusion must be negative, and will distribute the major term; which in this figure is subject of the major premiss; and if it is to be distributed there, the premiss must be universal (cf. Fig. 2).

2. *If the major premiss is affirmative, the minor must be universal:* for the middle term, as predicate of an affirmative proposition, will not be distributed in the major premiss; it must therefore be distributed in the minor premiss, where it is subject; and therefore the minor premiss must be universal.

3. *If the minor premiss is affirmative, the conclusion will be particular*: for the minor term, as predicate of an affirmative proposi-

[1] And by others, e.g. Lambert of Auxerre, thirteenth century *med.*, quoted Prantl, iii. 30, Abschn. xvii. Anm. 121.

[2] e.g. Petrus Mantuanus, quoted Prantl, iv. 178. Petrus, in the edition of his *Logica* dated 1492, gives as an example of a syllogism in Cesare, 'Nullus homo est lapis, omne marmor est lapis, igitur nullum marmor est homo'. If the conclusion drawn from the premisses enunciated in this order is 'Nullus homo est marmor', he calls the mood Cesares; but were they enunciated in the opposite order, and the latter conclusion drawn, he would call it Camestres. By such and other even more questionable methods, Petrus compiles fifteen moods in Fig. 1, sixteen in Fig. 2, eighteen in Fig. 3, and eleven in Fig. 4. Cf. also Crackenthorpe, *Logicae Libri Quinque*, Oxoniae, 1670, p. 197, who appears to treat the moods of Fig. 4 and the indirect moods of Fig. 1 as two different things.

[tion, will not be distributed in the premiss, and so must not be distributed in the conclusion, which will therefore be particular.

Hence the premisses *OA* are invalid by the first rule: *AI* and *AO* by the second[1]; *AA*, *AE*, *EA*, *EI*, *IA* are valid; but *AA* will afford only a particular, instead of an universal, conclusion. The *moods* are thus *AAI*, *AEE*, *EAO*, *EIO*, *IAI*; and their mood-names, as moods of the fourth figure, are Bramantip, Camenes, Fesapo, Fresison, Dimaris.

The complete *memoria technica*, with the fourth figure replacing the indirect moods of the first, is commonly given in English textbooks nowadays as follows[2]:—

> Barbara Celarent Darii Ferioque prioris;
> Cesare Camestres Festino Baroco secundae;
> Tertia Darapti Disamis Datisi Felapton
> Bocardo Ferison habet; quarta insuper addit
> Bramantip Camenes Dimaris Fesapo Fresison.
> Quinque subalterni, totidem generalibus orti,
> Nomen habent nullum, nec, si bene colligis, usum.

The meaning of the last two lines is explained in the next paragraph.]

It will be noticed that in five out of these nineteen moods the conclusion is universal, viz. in Barbara and Celarent in Fig. 1, Cesare and Camestres in Fig. 2, and Celantes in Fig. 1 (= Camenes in Fig. 4). It is, of course, possible *a fortiori* to draw the particular, or subaltern, conclusion in any of these cases; and the syllogism is then said to have a *weakened* conclusion, or to be in a *subaltern mood*. Subaltern moods would be used by no one who was asking what could be inferred from given premisses; for it is as easy to see that the universal conclusion, as that the particular, can be drawn from

[1] e. g. from the premisses *Some change is not motion*, *All motion is change*, it cannot be inferred that *Some change is not change* (*OA*): nor from *All great critics are scholars*, *Some scholars are pedants*, that *Some pedants are great critics* (*AI*): nor from *All members of the Government belong to the party in power*, *Some of the party in power are not in the Cabinet*, that *Some of the Cabinet are not members of the Government* (*AO*).

[2] I have not been able to trace this form of the mnemonic verses any further back than to Aldrich's *Artis Logicae Rudimenta*. A good many writers have tried their ingenuity in devising variations upon the original lines. Watts has a version recognizing only fourteen moods, the indirect moods of Fig. 1 appearing neither in that capacity nor as moods of Fig. 4. Sir William Hamilton (*Discussions*, p. 666) also offers 'an improvement of the many various casts of the common mnemonic verses'. But the reader will probably wish for no more. In various modern textbooks, Baroco and Bocardo are spelt with a *k*, in order that *c* medial may not occur with a different meaning from *c* initial.

them. But in seeking for the proof of some particular proposition, we might very likely find premisses that would really prove the universal; yet, since we are only using them to prove the particular, our reasoning would fall into one of the subaltern moods. Still, we should see that our premisses proved more than we had set out to establish, and substitute at once the wider thesis; the subaltern moods are therefore of little importance, and are not included in the enumeration of valid moods of syllogism.

[It would have been possible to determine what moods are possible in each figure, without enunciating the *special rules* (as they are called) *of the different figures*. It might merely have been pointed out, e. g., that in the first figure *AA* would yield an *A* conclusion, *AE* involve an illicit process of the major term, *AI* yield an *I* conclusion, *AO* again involve an illicit process of the major, *EA* yield an *E*, and *EI* an *O* conclusion, *IA* and *OA* involve an undistributed middle. And if it were asked why the mood *IAI* is invalid in this figure, the proper answer is not because in the first figure the major premiss must be universal (though that is the second rule of this figure), but because such a combination of premisses in it involves an undistributed middle; the rule being directed to avoiding this fallacy, and not the fallacy condemned because it breaks the rule. The rules, however, if the grounds on which they rest are understood, give in a general form the principles which must be observed in each particular figure. Therefore the knowledge of these rules helps us to master the theory of syllogism; but only if their grounds are understood. It is better to know what moods are invalid in each figure, and what fallacy they severally commit, than to know the special rules and apply them in a mechanical manner, without being able to justify them.]

CHAPTER XIII

OF THE REDUCTION OF THE IMPERFECT SYLLOGISTIC FIGURES

ARISTOTLE distinguished between syllogisms which were only valid (ἀτελεῖς) and syllogisms which were perfect (τέλειοι). In the latter, the necessity of the inference appeared sufficiently from the premisses as they stand; in the former, they required to be supplemented, in order that it might be seen. The second and third figures, in his view, were in this plight. Their validity, though real, needed proving, by means of the first figure. By converting one of the premisses in the two imperfect figures, he showed that we might obtain a syllogism in the first or perfect figure, either with the same conclusion or with one from which that could be recovered by conversion; where this direct method of validating an imperfect mood fails, we can still validate it indirectly, by proving, in a syllogism of the first and perfect figure, that the falsity of its conclusion is inconsistent with the truth of its premisses.[1]

The process of exhibiting by the help of the first figure the validity of syllogisms in the other two (or three) is called **Reduction.** A knowledge of the method of reducing the imperfect moods to moods of the first figure belongs to the traditional part of the theory of syllogism. The present chapter will explain this; in the next we must ask whether the process of Reduction, sanctified by the tradition of many centuries, is really necessary, in order to validate the imperfect figures.

Directions for Reduction are concealed in the mood-names of 'Barbara Celarent'. Those who have thoroughly mastered the theory of syllogism will see at a glance how a given imperfect mood may be reduced; but the mood-name enables one to do it, as it were, with a mechanical correctness.

[1] This method of establishing the validity of a syllogism *per impossibile* is applicable to all the imperfect moods; but the direct method was preferred where it is available.

Reduction, as already stated, is either *direct* or *indirect*. **Direct Reduction** of an imperfect mood to the first figure consists in showing that from premisses either the same as in the original syllogism, or inferred immediately by conversion from these, the original conclusion, or one from which it can be immediately inferred, follows in the first figure.

As the figures are distinguished from one another by the position of the middle term in the premisses, it is plain that, to reduce a figure from one of the imperfect figures to the first, we must alter the position of the middle term. In the second and third figures, it occupies the same position in both premisses, being predicate in the second, and subject in the third, whereas in the first figure it is subject of the major premiss and predicate of the minor. We must, therefore, convert one premiss of a syllogism in the second or third, in order to reduce it to the form of the first. In the second we should naturally convert the major, for there it is in the major premiss that the middle term is out of place; in the third, the minor. But it may happen that this would give us a combination of premisses which, in respect of quality and quantity, cannot stand; e.g. in a syllogism in Disamis (Fig. 3), by converting the minor premiss *A*, we should get the combination *II*, which yields no conclusion. We therefore have sometimes to *transpose* the premisses, making our original minor premiss the major, and vice versa, and converting in the second figure that which becomes the major, in the third that which becomes the minor. Where the premisses are transposed to make a syllogism in the first figure, they will give a conclusion in which the terms of the original conclusion have been transposed likewise; and it will be necessary to convert this conclusion in order to recover that of the original 'imperfect' syllogism.

By way of illustration, we may take the following example in Camestres, the form of which, as indicated by the vowels of the mood-name, is

All *P* is *M*
No *S* is *M*
∴ No *S* is *P*

If we were to argue that a spider is not an insect because it has not six legs, our argument would fall quite naturally into the above form:

Insects have six legs
The spider has not six legs
∴ The spider is not an insect.

Now if we want to get the same conclusion in the first figure, we cannot convert the major premiss ; for that would give us a particular major

Some animals with six legs are insects

and no conclusion as to whether a spider is an insect or not would follow.[1] We must therefore convert the minor premiss, which being *E* can be converted without change of quantity : and transposing at the same time, form the syllogism in Celarent :

No animal with six legs is a spider
Insects have six legs
∴ No insect is a spider

From this conclusion we can recover by conversion the original conclusion

The spider is not an insect

Had our argument run slightly differently, to the effect that the spider is not an insect because it has eight legs, it would have fallen into a syllogism in Cesare :

No insect has eight legs	No *P* is *M*
The spider has eight legs	All *S* is *M*
∴ The spider is not an insect	∴ No *S* is *P*

Here the major premiss can be converted simply, being *E* : and transposition is not required. The premisses

No animal with eight legs is an insect
The spider has eight legs

conform to Celarent, and yield at once the original conclusion.

The indirect moods of the first figure (the moods, as others regard them, of the fourth figure) fall into two groups, when we wish to show that their conclusions (or others yielding them by conversion) can be obtained directly in the first figure from the same premisses (or from premisses which these yield by conversion). Three, Baralipton, Celantes, and Dabitis, simply draw the converse of the conclusion which the same premisses yield directly in the first figure ; all we have to do therefore is to draw the direct conclusion and convert it. But Fapesmo and Frisesomorum yield no direct conclusion. From the premisses

Every soldier serves his country
Women are not soldiers

[1] Though it would follow by an 'indirect conclusion' in Frisesomorum that some insects are not spiders.

I cannot infer that Women do not serve their country. The only conclusion is that Some who serve their country are not women. Now if this is to have the form of a direct syllogism in the first figure, *women* must be the major term, *soldiers* the minor: but if 'Women are not soldiers' were the major, and 'Every soldier serves his country' the minor premiss, the terms would occupy the wrong positions in the premisses. To obviate this, I must convert both premisses; then indeed I shall get the syllogism

No soldier is a woman
Some who serve their country are soldiers
∴ Some who serve their country are not women

which does prove my original conclusion in a direct mood of the first figure, Ferio; though whether it is the most natural way of removing any doubts I may have had about the validity of the indirect inference in Fapesmo must be considered in the next chapter.

[If these moods, instead of being regarded as belonging to the first figure, are placed in a fourth, their reduction will be formally a little different. To reduce the first three, we shall simply have to draw the conclusion which naturally follows from the same premisses in the first figure, and then convert it; but this will now be said to involve transposition of the premisses: for what is major regarded as in the fourth figure is minor regarded as in the first, and vice versa: thus

Fig. 4. Bramantip.	Fig. 1. Baralipton.
Men of stout heart are free	The free are happy
The free are happy [1]	Men of stout heart are free

∴ Some who are happy are of stout heart

The premisses in Baralipton are premisses in Barbara; those in Bramantip are not so, till they are transposed.

On the other hand, in the last two moods only conversion and not transposition will now be necessary; for the fourth figure already regards the universal negative premiss in Fesapo or Fresison (= Fapesmo or Frisesomorum) as the major, because it contains the term which is predicate in the conclusion, though it is subject in the premiss; conversion will bring it to the position which the major term should hold in its premiss in the first figure; and so with the minor; and our original conclusion then follows in Ferio.]

Whether, in reducing a syllogism of any imperfect mood, the premisses need transposing; which, if any of them, must be con-

[1] τὸ εὔδαιμον τὸ ἐλεύθερον, τὸ δ' ἐλεύθερον τὸ εὔψυχον κρίναντες, Thuc. ii. 43.

verted ; whether we have to convert the conclusion obtained in the syllogism of reduction, in order to recover the original conclusion ; and in which mood of the first figure the validating syllogism will be—all these matters are indicated by the consonants of the mood-names. The significant consonants [1] are :

1. *the initial*, always the same as that of the mood in Fig. 1 to which the imperfect mood must be reduced.

2. *m* (= *muta*), which indicates that the premisses must be transposed.

3. *s* (= *simpliciter*), which indicates that the premiss, or conclusion [2], signified by the preceding vowel must be converted simply.

4. *p* (= *per accidens*), which indicates that the same must be converted by limitation.

5. *c* (= *conversio syllogismi*), which, occurring medially, indicates that we must employ the process of Indirect Reduction, to be explained immediately.

In order to illustrate the mechanical use of these instructions, it will be enough to work out in symbols the reduction of a single mood, Disamis. That, as the mnemonic tells us, is in Fig. 3 ; the middle term is therefore subject in both premisses. The major, being indicated by *I*, is a particular affirmative, and the minor, being indicated by *A*, an universal affirmative ; the conclusion similarly a particular affirmative. Our syllogism is therefore to be of the type :—

Some *M* is *P*	*I*
All *M* is *S*	*A*
∴ Some *S* is *P*	*I*

In reducing it, the *m* of the mood-name indicates that we must transpose the premisses, and the *s* that we must convert simply the premiss indicated by the vowel after which it stands ; the *D* that we shall so obtain a syllogism in Darii, thus :—

All *M* is *S*
Some *P* is *M*
∴ Some *P* is *S*

[1] Except the initials, these are explained in the old lines—

Simpliciter verti vult *S*, *P* verti per acci,
M vult transponi, *C* per impossibile duci.

If any one is horrified at the doggerel, he may be assured that much worse things could have been quoted in earlier chapters.

[2] i.e. not the conclusion of the *original* syllogism (which has to be obtained again as it stood), but the conclusion of the *validating* syllogism.

The simple conversion of this conclusion, enjoined by the *s* after the third vowel in Disamis, gives us

Some *S* is *P*

This process of Direct Reduction cannot be applied to the two moods, Baroco and Bocardo. The reason is obvious. In order that the middle term may occupy a different position in the two premisses, as the first figure requires, one of the premisses in the second and third figures must be converted. In these moods, the premisses are an universal affirmative and a particular negative proposition. The latter, *O*, cannot be converted either simply or *per accidens*; the converse of *A* is *I*; and so by converting that we should obtain two particular premisses. These syllogisms can, however, be validated by the process of Indirect Reduction.

Indirect Reduction, or **Reduction per impossibile**, consists in showing, by a syllogism in the first figure, against which no objection can be taken, that the falsity of the conclusion in the original syllogism is inconsistent with the truth of its premisses. This is done as follows :—

Baroco is of the form

All *P* is *M*	All negroes have curly hair
Some *S* is not *M*	Some natives of Africa have not curly hair
∴ Some *S* is not *P*	∴ Some natives of Africa are not negroes

Now if this conclusion is false, its contradictory will be true, i. e. that All natives of Africa are negroes. We can then combine this with our original major premiss to form a syllogism in Barbara, thus :—

All *P* is *M*	All negroes have curly hair
All *S* is *P*	All natives of Africa are negroes
∴ All *S* is *M*	∴ All natives of Africa have curly hair

But the conclusion thus obtained contradicts the original minor premiss ; hence if the original premisses are true, the conclusion we drew from them cannot be false, and our original syllogism is therefore valid.

The method of reducing a syllogism in Bocardo is the same : except that here by combining the contradictory of the conclusion with the original minor we reach a result inconsistent with the original major premiss ; while in the former case, by combining

it with the major, we deduced a conclusion contradictory of the minor. The medial *c* in the mood-name directs us to substitute for the premiss *indicated by the vowel after which the c is placed* the contradictory of the conclusion.[1]

[All the imperfect moods could be validated in this indirect manner[2]: take, e.g., Darapti—All M is P, All M is S $\therefore$ Some S is P; if this is false, then No S is P; and All M is S; $\therefore$ No M is P; which is inconsistent with the truth of the original major premiss. The first figure, on the other hand, cannot be appealed to in order to confirm itself; if we suppose its conclusion to be false, and combine the contradictory thereof with one of the premisses, it is only by a syllogism in the second or third figure that we can deduce a conclusion inconsistent with the other premiss; e.g. in Barbara (All M is P, All S is M $\therefore$ All S is P); if the conclusion is false, then Some S is not P; and All M is P; $\therefore$ Some S is not M—which contradicts the original minor; and again, Some S is not P, and All S is M $\therefore$ Some M is not P—which contradicts the original major; but the arguments are in the second and third figures.]

[1] It is possible to validate the moods Baroco and Bocardo by the direct method, if we employ the processes of permutation, and conversion by negation. From Baroco we obtain a syllogism in Ferio, thus: *Baroco*, All P is M, Some S is not M $\therefore$ Some S is not P: *Ferio*, No not-M is P, Some S is not-M $\therefore$ Some S is not P; from Bocardo we obtain a syllogism in Darii: *Bocardo*, Some M is not P, All M is S $\therefore$ Some S is not P: *Darii*, All M is S, Some not-P is M $\therefore$ Some not-P is S $\therefore$ Some S is not P. Names have been given to the two moods in place of Baroco and Bocardo, by logicians who considered these methods of reduction to be preferable, in which the processes to be followed are indicated. These processes have been relegated to a note, and the names suppressed, because there is no purpose in burdening what may be called the mechanical part of the theory of syllogism with any fresh refinements. 'Barbara Celarent' may be retained and explained, on historical grounds; we need not add to it. On the other hand, the question as to whether the imperfect moods need validating, and if so, what is the most proper way of doing it, will be discussed in the next chapter.

[2] Though for Fig. 4 the syllogism which employs the contradictory of the original conclusion as one of its premisses will yield a conclusion contradicting the *converse* of one of the original premisses.

CHAPTER XIV

OF THE PRINCIPLES OF SYLLOGISTIC INFERENCE

When I argue that because $A = B$ and $B = C$, therefore $A = C$, my reasoning proceeds upon the same principle as when I argue that because $X = Y$ and $Y = Z$, therefore $X = Z$. This principle is expressed in the familiar axiom that things which are equal to the same thing are equal to one another. In the particular inference, $A = B$, $B = C$ $\therefore A = C$, I do not deduce any conclusion from that axiom, as from a major premiss. It has indeed sometimes been contended that the argument is really syllogistic; that it should be written

> Things equal to the same thing are equal to one another
> A and C are things equal to the same thing
> $\therefore$ A and C are equal to one another.[1]

But the following considerations will show that this is not the case. *Firstly*, we may appeal to an analogous argument, in which a quantitative relation is established between A and C on the ground of the quantitative relations of both to B, although the quantities are none of them equal. If A is greater than B, and B is greater than C, A is greater than C. Are we to maintain that this inference should properly be written

> Things of which one is greater and the other less than the same thing are greater the one than the other
> A and C are things of which one is greater and the other less than the same thing
> $\therefore$ A and C are greater the one than the other ?

The cumbrousness of this would be no reason for refusing to recognize it, if it were correct; and if the other is correct, this must be. Yet where, as in this case, it requires some violence and ingenuity to give a quantitative inference the appearance of a syllogism, it is

[1] Euclid, for example, wrote under the impression that this is the right way of stating such an argument.

not habitually done ; and since men have been content not to force into the form of syllogism the inference '$A>B$, $B>C$ $\therefore A>C$', it may be surmised that they would not have so dealt with the inference '$A=B$, $B=C$ $\therefore A=C$', if it had not been for the apparent ease of the transformation. But appearances may be deceptive ; and it must be noticed *secondly*, that in the syllogism which is supposed to represent the latter inference, viz.

> Things equal to the same thing are equal to one another
> A and C are things equal to the same thing
> $\therefore$ A and C are equal to one another,

our minor premiss and our minor term are both faulty. The minor premiss is not a correct statement of the grounds of our inference ; these are, that A and C are both equal to B, and therefore the major required is 'Things equal to B are equal to one another'. And the minor term 'A and C' is not really a subject of which we demonstrate an attribute ; it is two subjects, which are shown to stand in a certain relation to each other. *Thirdly*, and chiefly, the so-called major premiss is itself established through the so-called minor and its conclusion. It is because I see that if A and C are both equal to B, they are equal to one another, that I recognize the truth of the general principle or axiom. If I were incapable of recognizing the validity of the inference in the case of the three quantities A, B, and C, or X, Y, and Z, I should not be able to recognize the truth of the axiom. The axiom, therefore, is not one of the premisses from which we reason, when we argue that '$A=B$ and $B=C$ $\therefore A=C$' : it is the principle *in accordance with which* we reason. If it were denied, the validity of any particular inference that conforms to it would be denied also ; its truth is therefore involved in that of the particular inferences. But a man may see the validity of the particular inference, without formulating the axiom. This would not be so, if it were really a suppressed major premiss, and 'A and C' a true minor term. In the argument that 'Silver is a good conductor because it is a metal', every one recognizes that it is implied that 'All metals are good conductors' ; and without this premiss, the grounds of the inference are not apparent. But no one requires any further grounds for inferring '$A=C$', than are contained in the premisses '$A=B$ and $B=C$'.

We may therefore dismiss the attempt to reduce this argument to syllogistic form, and recognize in the axiom not a premiss but

the principle or canon of the argument. But the question then arises, whether there is similarly a principle or canon of syllogistic inference. Let us recall what was shown in Chapter XI, of which what has just been said is only a corollary. We there distinguished between an argument in which a relation of quantity was established between two terms, through their relation in quantity to a common third term: and an argument in which a relation was established between two terms in the way of subject and predicate, through their relation in that respect to a common third term; the latter being syllogism. Now the axiom 'Things that are equal to the same thing are equal to one another' is a principle of inference in the domain of quantity. It specifies no particular quantities, but states that two quantities will stand in a certain relation (of equality) to one another, if they stand in certain relations (of equality) to a third. May there not be a corresponding principle in syllogistic inference—one which specifies no particular terms, but states that two terms will be related to each other as subject and predicate in a certain way, if they are so related in certain ways to a third term?

Such a principle has been supposed to be furnished in the *Dictum de omni et nullo*; and a consideration of this, and of other canons which have been proposed in its place, will throw a good deal of light on the nature of syllogistic inference, and the difference between its different types or figures.

The phrase 'Dictum de omni et nullo' is really a short title by which to refer to a principle too long to enunciate always in full; just as we refer to statutes or papal bulls by their first word or two. The principle may be expressed thus—*Quod de aliquo omni praedicatur* [*dicitur*, s. *negatur*], *praedicatur* [*dicitur*, s. *negatur*] *etiam de qualibet eius parte*: What is predicated [stated, *or* denied] about any whole is predicated [stated, *or* denied] about any part of that whole.[1]

[1] I have quoted Zabarella's formulation of the Dictum de Omni, *de Quarta Figura Syllogismi Liber*, *Opera Logica*, Coloniae, 1597, p. 115 A. The words in square brackets are not his. There are numerous variants of no particular importance. Crackenthorpe (III. 16, p. 202 in ed. of 1670) gives 'Quidquid affirmatur (*s.* negatur) universaliter de aliquo, idem affirmatur (*s.* negatur) etiam de omni de quo illud praedicatur'. This form seems (as Mansel remarks of Aldrich's) to be more nearly a translation of the passage in the *Categories* than of that in the *Analytics*. The formula 'quod valet de omnibus valet etiam de singulis' (the reference for which I cannot now find) treats the major premiss nakedly as an enumerative judgement; the

If we take syllogisms in the first figure—and it is enough to consider Barbara and Celarent—the meaning of the principle will

same view is implied in speaking of the middle term as a *class*, as e.g. Whately and Bain do.

The passage in Aristotle from which the Dictum de Omni was primarily derived is *Anal. Pri. a.* i. 24^{b} 26–30 *τὸ δὲ ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι θατέρου θάτερον ταὐτόν ἐστιν. λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι, ὅταν μηδὲν ᾖ λαβεῖν τῶν τοῦ ὑποκειμένου, καθ' οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως* ('That one term should be contained in another as in a whole is the same as for one to be predicated of all another. And it is said to be predicated of *all* anything, when no part [= logical part] of the subject can be found, of which the other term [the predicate] will not be true; and to be predicated of *none*, similarly'). Aristotle is here explaining the meaning of expressions which he is about to use in the *Analytics*; if mortal is predicated of animal or man *κατὰ παντός*, it means that there is no animal (e.g. man) or man (e.g. Socrates) who is not mortal. And no doubt that is involved in the truth of the universal proposition; but it does not follow that Aristotle thought of the universal proposition as no more than an enumerative judgement about every species (or individual) of which the subject-term can be predicated. He uses the formula *τὸ μέσον ἐστὶν ἐν ὅλῳ τῷ πρώτῳ* ('the middle is contained in the major as its whole') as well as *τὸ πρῶτον κατηγορεῖται κατὰ παντὸς τοῦ μέσου* ('the major is predicated of all the middle') to indicate the relation of the major to the middle term in Fig. 1 (and similarly with the relation of the middle to the minor); and *ὅλον* means a logical whole or universal, not an aggregate of individuals. Elsewhere he says of that figure, *εἰ γὰρ τὸ Α κατὰ παντὸς τοῦ Β καὶ τὸ Β κατὰ παντὸς τοῦ Γ, ἀνάγκη τὸ Α κατὰ παντὸς τοῦ Γ κατηγορεῖσθαι· πρότερον γὰρ εἴρηται πῶς τὸ κατὰ παντὸς λέγομεν* ('For if *A* is predicated of all *B*, and *B* of all *C*, *A* must be predicated of all *C*: for we have already stated what we mean by predicating of all') (*Anal. Pri. a.* iv. 25^{b} 33–4, 37–40). Doubtless if it is involved in saying 'All *B* is *A*', that every *B* is *A*, and in saying 'All *C* is *B*', that every *C* is *B*, then it is involved that every *C* is *A*; but the universal proposition need still not be viewed as a statement about individuals. Indeed if it were, each particular *C* must be already known to be *A* in making the judgement 'All *C* is *A*', and therefore the *inference* that all *C* is *A* would be unnecessary. Aristotle himself points this out in *Anal. Post. a.* i, and makes it plain that in his view the universal proposition was not an enumerative judgement about known individuals; and he hardly ever uses a singular term to illustrate the minor of a syllogism. And although we must admit that in regarding Fig. 1 as the only perfect figure, and in exhibiting the necessity of the inference in Fig. 1 as he does in the words last quoted, Aristotle lays too much stress on the aspect of extension, and not enough on that of necessary connexion of characters within the subject, yet he largely corrects this himself in his account of demonstration, and he did not think that the essential meaning of the universal proposition, and what constituted the nerve of the reasoning, lay in the fact that it made an assertion about every individual falling under it.

There is another passage sometimes quoted as the source of the Dictum, viz. *Cat.* iii. 1^{b} 10 (e.g. Mansel's *Aldrich*, p. 85, note *a*: Baldwin's *Dictionary of Philosophy and Psychology*, s. voc. *Aristotle's Dictum*). The section runs as follows: *ὅταν ἕτερον καθ' ἑτέρου κατηγορῆται ὡς καθ' ὑποκειμένου, ὅσα κατὰ τοῦ κατηγορουμένου λέγεται, πάντα καὶ κατὰ τοῦ ὑποκειμένου ῥηθήσεται, οἷον ἄνθρωπος κατὰ τοῦ τινὸς ἀνθρώπου κατηγορεῖται, τὸ δὲ ζῷον κατὰ τοῦ ἀνθρώπου· οὐκοῦν καὶ κατὰ τοῦ τινὸς ἀνθρώπου κατηγορηθήσεται τὸ ζῷον· ὁ γάρ τις ἄνθρωπος καὶ ἄνθρωπός ἐστι καὶ ζῷον* ('When one thing is predicated of another as of a subject *de*

be plain. All (or No) *B* is *A*, All *C* is *B* ∴ All (or No) *C* is *A*. Here it matters not for what real terms *A*, *B*, and *C* stand, any more than

quo, all that is asserted of the predicate will be asserted of the subject as well; e.g. man is predicated of a particular man [as subject *de quo*], and animal of man, and therefore animal will be predicated also of the particular man; for the particular man is man and animal'). But its context dispels any presumption that this passage is an enunciation of the Dictum. There is nothing about syllogism in the *Categories* at all. In the previous chapter a distinction was made between different kinds of ὄντα—beings, or entities. Some καθ' ὑποκειμένου τινὸς λέγεται, ἐν ὑποκειμένῳ δὲ οὐδενί ἐστιν: they are said, or predicated, of a subject, but do not inhere in a subject; *man*, e.g., is predicated of Caesar, but not inherent in him. Others ἐν ὑποκειμένῳ ἐστί, καθ' ὑποκειμένου δὲ οὐδενὸς λέγεται, they inhere in a subject, but are not predicated of a subject; as Priscian's grammatical knowledge inhered in the soul of Priscian, but is not predicable of any subject which could be said to be Priscian's grammatical knowledge. Others again καθ' ὑποκειμένου τε λέγεται καὶ ἐν ὑποκειμένῳ ἐστίν, they both are predicated of a subject and inhere in a subject; as knowledge is predicated of Priscian's grammatical knowledge, and inhered in the soul of Priscian. Others, lastly, οὔτ' ἐν ὑποκειμένῳ ἐστιν, οὔτε καθ' ὑποκειμένου λέγεται, they neither inhere in a subject nor are predicated of any; such are concrete individuals, like Caesar and Priscian. Here the opposition between καθ' ὑποκειμένου λέγεσθαι, being predicated of a subject, and ἐν ὑποκειμένῳ εἶναι, inhering in a subject, is parallel to that between essential and accidental predication. If I say of Caesar that he is a man, of grammar that it is a science, of colour that it is a quality, those predicates give the general being of their subjects, the subjects are essentially those, or (as it may be put) are their subjects *de quo*. But if I say of Priscian that he is a grammarian, or of a map that it is coloured, grammar and colour are not what the soul of Priscian, and the map, essentially are; they inhere in them, and the soul of Priscian and the map are their subjects *in quo*. In the language of the *Categories*, that *is in* a subject, which being in anything not as a part of it cannot exist apart from that wherein it is; ἐν ὑποκειμένῳ λέγω, ὃ ἔν τινι μὴ ὡς μέρος ὑπάρχον ἀδύνατον χωρὶς εἶναι τοῦ ἐν ᾧ ἐστίν (ii. 1ª 24–25). Colour cannot exist except in a body, but is not a part of the body: nor grammar except in a soul, but it is not part of a soul.

Upon these distinctions succeeds immediately the sentence quoted at the head of the last paragraph; and it must clearly be interpreted with reference to them. The connexion seems to be as follows. There exist (1) individual substances, like Caesar, which are subjects *de quo*, and subjects *in quo*, but neither predicable of nor inherent in anything else; (2) universal substances, like man, predicable of individual substances, and not inherent in anything; (3) individual attributes (or accidents), like Priscian's grammatical knowledge, which is a subject *de quo*, because the universal attribute is predicable of it, and it of no attribute else, but is not a subject *in quo*, being itself inherent in an individual substance, not that wherein anything inheres; (4) universal attributes, like knowledge, predicable of individual attributes and inherent in individual substances. *Man* then is not an ὑποκείμενον or subject, but is predicated of a subject; nevertheless we find terms predicated of *man*, in such a proposition as 'Man is an animal'. What then is *animal*? for it is not a subject, and apparently not predicated of a subject, for *man* is not really a subject. The answer is, that *animal* is really predicated of the subject whereof *man* is predicated, and therefore, like *man*, falls into the second of the above classes.

If we consider this doctrine on its own account, it is open to considerable

in the axiom of equals it mattered what real quantities were intended. Whatever they are, suppose that *A* can be affirmed or denied of all *B*,

criticism. There is the same failure to distinguish different senses of *ὑποκείμενον, subject*, as in Mill's discussion of connotative terms (cf. quotation *supra*, p. 148). As subject *de quo*, it is the individual, whether substance or attribute, the instance of an universal; as subject *in quo*, it is the individual substance. Thus in relation to knowledge, the grammatical knowledge of Priscian is a subject *de quo*, as in relation to man, Priscian is; and language allows me to say that the former is a knowledge, and the latter is a man. But again in relation to knowledge, Priscian, or the soul of Priscian, is the subject *in quo*; and language allows me to say not that Priscian is a knowledge, but that he is knowing, though both man and knowledge are sometimes said to be 'predicated of' Priscian: clearly not in the same sense. When the subject of a predicate is its subject *de quo*, then if the predicate is in the category of substance, the subject is a substance; if the predicate is in some other category, the subject is in that category; but when the subject of a predicate is its subject *in quo*, the subject is a substance, the predicate in some other category than substance. Now the language of the Greek, when it distinguishes *καθ' ὑποκειμένου λέγεσθαι* and *ἐν ὑποκειμένῳ εἶναι*, being predicated of a subject and inhering in it, does not suggest that the word *ὑποκείμενον, subject*, has any but one sense; the difference is put as if lying in the relation of the predicate to it; but really to be a subject of inherence, or substance, is not the same as to be a subject *de quo*, or individual, though some individuals are individual substances.

There is a further difficulty in the passage. It professes to distinguish kinds of *ὄντα*, things or entities. Now when we say that an attribute inheres in a substance, we mean, according to the teaching of the passage, that an individual attribute inheres in an individual substance; and these are rightly distinguished as things of different kinds. But when we predicate something, whether substance or attribute, of its subject *de quo*, we do not mean that an individual *is* the universal of which it is an instance. 'His grammatical knowledge inheres in Priscian'; here the words *His grammatical knowledge* denote an individual attribute, and *Priscian* denotes an individual substance. But when I say that Priscian is a man, or that attribute of his a knowledge, *man* does not denote an universal substance, nor *knowledge* an universal attribute. Yet these are what are said to be predicated of their subjects *de quo*. We have seen (*supra*, pp. 33-35) that the same abstract term is commonly used as a general name of attributes, and as name of the universal whereof they are instances. But here the general concrete term is treated as the name of the universal whereof individual substances are instances, and *man* is said not to be, but to be predicable of, a subject. The *word* 'man' is doubtless so predicable; but what it denotes is some individual subject. Aristotle however treats general concrete names as names of universals. In *de Interpr.* vii. 17[a] 38–[b]1 we are told that some things are universal, some individual; universal, what can be predicated of more than one, individual, what cannot; man e.g. is an universal, Callias an individual (*ἐπεὶ δ' ἐστὶ τὰ μὲν καθόλου τῶν πραγμάτων τὰ δὲ καθ' ἕκαστον· λέγω δὲ καθόλου μὲν ὃ ἐπὶ πλειόνων πέφυκε κατηγορεῖσθαι, καθ' ἕκαστον δὲ ὃ μή, οἷον ἄνθρωπος μὲν τῶν καθόλου, Καλλίας δὲ τῶν καθ' ἕκαστον· κτλ.*).

But whatever the scruples which the whole passage raises, the words in question are far from enunciating the *Dictum de omni et nullo*. In the syllogism 'All men are animals, Socrates is a man ∴ Socrates is an animal'—if indeed Aristotle would have called that a syllogism (cf. *infra*, p. 321) – *man* is predicated of Socrates *ὡς καθ' ὑποκειμένου*, as its subject *de quo*, and *animal κατ' ἀνθρώπου λέγεται*, not *ἐν ἀνθρώπῳ ἐστίν*, it is predicable of, not

it can be affirmed or denied of each particular subject, *C* or any other, included in *B*. Here, according to a tradition which has been strong, is the fundamental principle of syllogistic inference. In this *Dictum* is nakedly displayed what is the nerve of our reasoning, whenever we syllogize in the concrete. It is the assurance that *A* is true of all *B*, which satisfies us that it is true of this *B*, viz. of *C*; the business of reduction is to bring imperfect syllogisms into a form, in which we can see at once that the principle applies to them; and the title of the first to be the perfect figure lies in its conforming to the formula of the *Dictum de omni et nullo*.

There are several objections urged against the claims of this formula. In the first place, it suggests the 'nominalist' doctrine expressed by Hobbes, when he said that reasoning is but the right ordering of names in our affirmations. It suggests that our ground for affirming or denying that *C* is *A* lies in the fact that *A* is *said* of all, or no, *B*, and *B* is *said* of *C*. Clearly it is because we believe that *B is A*, and *C is B*—not because *B is called A*, and *C is called B*—that we assert the conclusion. However, this nominalist interpretation of the Dictum is not necessary; it is not as thus interpreted that it will be here discussed; and therefore this objection may be dismissed.

It may be said secondly, that if the reduction of the other figures

inherent in *man*, being in the same category, and its general being; *animal* therefore is predicated of Socrates as its subject *de quo*, i.e. Socrates is an animal. The conclusion is justified by the rule in the *Categories*. But to most syllogisms it has no application. 'All organisms are mortal, Man is an organism ∴ Man is mortal.' Here the minor term is not an ὑποκείμενον, or subject *de quo*, in the sense of the passage in the *Categories*, but something predicated of a subject; and though the middle is predicable of the minor, the major is inherent in the middle. Again if Priscian was a grammarian, and a grammarian is scientific, Priscian was scientific; but here though the minor term is an ὑποκείμενον, an individual substance, the middle is predicated of it not as its subject *de quo*, but as its subject *in quo*; it is not therefore a case ὅταν ἕτερον καθ' ἑτέρου κατηγορῆται ὡς καθ' ὑποκειμένου, where one thing is predicated of another as of a subject *de quo*, and so does not fall within the scope of the rule. Once more, if all men are jealous, and Priscian was a man, Priscian was jealous; here the middle is predicated of the minor term as of a subject *de quo*; but as in the proposition 'All organisms are mortal', so it is in this major premiss; jealousy is not something which κατὰ τοῦ κατηγορουμένου λέγεται; man is to jealousy not subject *de quo*, but subject *in quo*; we cannot, according to the language of the context, say that jealousy καθ' ἀνθρώπου λέγεται, is predicated of man, but that it ἐν ἀνθρώπῳ ἐστίν, inheres in a man. There is therefore no justification for finding in this rule a statement of the Dictum. Whether Aristotle would have accepted the Dictum as a correct expression of the principle of syllogistic inference is another question, to which the answer depends very much on how we interpret the Dictum.

to the first is not necessary, i. e. if the true character of our reasoning in them is not more clearly displayed in the first figure, the Dictum is not the principle of all syllogistic inference. In claiming to be that, it denies any essential difference between the different figures; and those who think them essentially different are so far bound to question the analysis of syllogistic inference which the Dictum implies. This is quite true; but we can hardly discuss the relation of the different figures, until we have settled whether the Dictum expresses correctly the nature of our reasoning in the first.

We come therefore to what is the main criticism which has been urged against the Dictum, and against all syllogistic inference, if it be supposed that the Dictum is a true analysis of its nature. It is said that a syllogism would, on this showing, be a *petitio principii*. By *petitio principii*, or *begging the question*, as it is called in English, is meant assuming in one of your premisses what you have to prove. Of course, the premisses must implicitly contain the conclusion; otherwise you would have no right to draw it from them, and could deny it, while admitting them: this much is true of every kind of cogent inference, whether syllogistic or not, though it has been sometimes treated as a peculiarity of syllogism by persons who thought they could find other kinds of inference not obnoxious to it. But you do not beg the conclusion in the premisses, except where the conclusion is necessary to establish one or other of the premisses. For example. I may know that treason is a capital offence; and the law might make it treasonable to publish libels against the sovereign; and in that case, from the premisses, *All treason is a capital offence*, *To libel the sovereign is treason*, I could infer that *To libel the sovereign is a capital offence*. In this argument, there is no *petitio principii*; I can learn the truth of both premisses by consulting the statute-book, and do not need to be aware that it is a capital offence to libel the sovereign, in order to know either of the premisses from which that conclusion is deduced. But the case is different in such a syllogism as that *All ruminants part the hoof*, and *The deer is a ruminant* ∴ *The deer parts the hoof*. I have no means here of ascertaining the truth of the major premiss, except by an inspection of the various species of ruminant animals; and until I know that the deer parts the hoof, I do not know that *all* ruminants do so. My belief in the constancy of structural types in nature may lead me to expect that a rule of that kind, found to hold good in all the species which I have examined, holds good universally; but this presumption, so long

as it rests merely on the examination of instances, is not conclusive; I should not accept the conclusion merely on the strength of the premisses, but should seek to confirm it by an examination of the hoof of the deer; the case of the deer therefore is necessary to establish the rule.

Now it has been alleged that all syllogism is a *petitio principii*[1]; and the allegation has gained colour from the *Dictum de omni et nullo*. 'That which is affirmed or denied of any whole may be affirmed or denied of anything contained within that whole.' What do we mean by a *whole* here? If it is a class or collection, if the major premiss is to be understood in extension, then it can hardly be denied that it presupposes a knowledge of the conclusion. If in the proposition *All B is A*, I mean not that *B as such* is *A*, but that *all the B's* are *A*, I must certainly have examined *C* (if that is one of them) before making the assertion; and therefore the major premiss, *All B is A*, rests (*inter alia*) on the present conclusion, *C is A*. According to this view, the major premiss of a syllogism is (at least in most cases[2]) a statement of fact about the whole of a number of particulars; it is really an enumerative, and not a true universal, judgement.[3] We make it, not because of any insight that we have into the nature of *B* and *A*, and into the necessity of their connexion: but simply because we have examined everything in which *B* is found, and satisfied ourselves that *A* is equally present in all of them.

There is indeed another sense in which the major premiss may be enunciated without our having insight into the necessary connexion of characters in things, and in which it no longer makes a collective assertion about every one of a number of particulars. If I say that all gold is yellow, I need not mean to assert that every piece of metal, which by other qualities I should identify as gold, is also yellow —a statement for which I certainly cannot claim the warrant of direct experience. I may mean that a yellow colour is one of the qualities on the ground of which I *call* a substance gold; or, in

[1] Cf., e.g., Mill, *System of Logic*, II. c. iii. Mill's own way of avoiding the charge is not very successful.

[2] Where general rules are made by men, as in the case of laws, we can of course know them, in advance of any knowledge about the particular acts or events to which they refer. Such syllogisms, therefore, as that about libelling the sovereign, given in the last paragraph, can in no case be alleged to beg the question. If any other authority (such as revelation) acquaints us with general rules, they will serve as major premisses of equally unexceptionable syllogisms. All other general propositions have, by the extremer critics, been interpreted in the way mentioned in the text.

[3] For this distinction, cf. *supra*, p. 177.

Locke's language, that it is included in the *nominal essence* of gold. By a nominal essence, Locke means what J. S. Mill called the connotation of a name—those attributes which, in using a general name, we imply to belong to the subjects called by it. We may collect together in our thought any set of attributes we like and give a name to the assemblage of them ; and then it will, of course, be true to say that anything called by the name, *if rightly called by it*, possesses any of the attributes included in the signification of the name. The general proposition ceases, in that case, to be enumerative ; but it does not become really universal. It becomes a verbal proposition. Gold is yellow, because we do not choose to call anything gold which is not yellow ; but we are not asserting that there is any necessary connexion between the other attributes for which a parcel of matter is judged to be gold, and this of yellowness. Given such and such attributes, we call it gold ; and therefore gold has all these. Let any one of them be wanting, and we should not call it gold ; therefore that is not gold which is not yellow ; but there may be a parcel of matter, for all that we mean to affirm, which has all the other qualities of gold, but is of the colour of silver.[1]

Locke did not suppose that the ordinary man, who says that gold is yellow, means only to assert that yellowness is one of the attributes included by him and others in the nominal essence (or connotation) of the word gold : but rather that he means, that with certain other qualities collected in the ' complex idea ' to which the name *gold* is attached the quality yellow is constantly conjoined. This however, on Locke's view, we cannot *know* ; for knowing is perceiving a necessary agreement or disagreement between our ' ideas ' (it would be better to say, connexion or disconnexion between the characters of things) ; and this in regard to our ' ideas of substances ' we do not perceive.[2] It is not our present business to discuss this ; we have not to ask how many of the general propositions of the sciences state connexions known to be necessary (though, if we did, we should find Locke not very far in that matter from the truth), nor what means there are (if any) of proving universal propositions about such matters of fact. We are concerned

[1] Cf. Locke's *Essay*, III. vi. §§ 6, 19, and also pp. 92 sq., *supra*, on Definition.

[2] Cf. Locke, *Essay*, IV. vi, esp. §§ 8, 9. Miss Augusta Klein has justly objected to me that in the first edition of this book I represented Locke as holding propositions about ' nominal essences ' to be more verbal than he really does.

with the theory of syllogism, and the allegation that it begs the question. We found that if the major premiss be interpreted in extension as an enumerative judgement, the charge is true; and that the *Dictum de omni et nullo* at least lends colour to such an interpretation. We have now seen that there is another interpretation, according to which the major premiss may be known to be true without examination of every individual instance included under the subject of it, but only by becoming a verbal proposition. On this interpretation the syllogism will still be a *petitio principii*, though not in the way which the *Dictum de omni et nullo* suggests. For though the major premiss will no longer presuppose a knowledge of the conclusion, the minor will do so. If nothing is to be called gold unless it is yellow, I cannot tell that a parcel of matter is gold, in which I have found the other qualities which the name implies, unless I have first seen that it is yellow. Of course, colour being the most obvious of the properties of a substance, I am not likely ever to be in the position of inferring the colour of a substance from its name; but the argument is the same if I took some unobvious quality, like solubility in aqua regia. If that is part of the nominal essence of gold, then I cannot tell that a particular parcel of matter with the familiar weight and colour of gold is gold, until I know that it is soluble in aqua regia. I do not therefore infer its solubility from the knowledge that it is gold, but I call it gold because I know it to be thus soluble.[1]

We need not dwell longer on the view that a general proposition is only warranted by agreement as to the meaning of a name, nor on the consequences, fatal enough, which this view would entail on the syllogism. Reasoning is not a mere process of interpreting names; and it is not the principle of syllogistic inference, that whatever a name means may be affirmed of the subjects called by it. In considering the charge that the syllogism is a *petitio principii*, it was necessary to notice the view which makes the *petitio* lie in the minor premiss, as well as that which makes it lie in the major. We must now return to the latter, and to the Dictum which is supposed to countenance it.

We saw that the crucial question here concerned the nature of the major premiss; is it universal, or merely enumerative? is it

[1] It will now be seen why a syllogism was explained to beg the question, if it presupposed the conclusion not in the premisses together, but in either of them singly; all syllogisms in a sense presuppose it in the premisses taken together (though they do not presuppose a knowledge of it).

based on an enumeration of particulars, or on the connexion of universals? If it is enumerative, and rests on a previous review of all the particulars included in the middle term, the charge of *petitio* is sustained. We should then accept the *Dictum de omni et nullo* as the general principle of syllogism, the 'whole' of which it speaks being understood as a whole of extension, a collection or class; but we should scarcely be able to speak of syllogistic *inference*.

Now Aristotle, who thought syllogism to be the type of all *demonstration*, could not possibly have understood the major premiss in this way.[1] He thought that, although we might know as a fact that C is A, yet we did not understand it, without seeing that it *must* be so; and to see that it must be so is to see that in it which makes it so—to see that it is A *in virtue of* B. B is a middle term, because it really mediates between C and A; it performs for C the office of making it A, and is *the reason why* C *is* A, not merely *the reason why we know* C *to be* A.

We have already, in discussing the modality of judgements, met with this distinction between the reason for *a thing being* so and so, and the reason for *our knowing it* to be so—between the *ratio essendi* and the *ratio cognoscendi*.[2] When I say that wheat is nourishing, because it contains nitrogen and carbon in certain proportions, I give the reason for its being nourishing: it is this constitution which makes it so. When I say that Mellin's Food is nourishing because Baby grows fat on it, I do not give the reason for its being nourishing, but only the reason for my thinking it to be so: it is not Baby's condition which makes it nourishing, but its nourishing properties which produce Baby's condition. The physical sciences always look for *rationes essendi*, so far as possible; though it may be noted that in what is, in many ways, the most perfect of the sciences, viz. Mathematics, we reason very largely from *rationes cognoscendi*.[2] If $A = B$, and $B = C$, then $A = C$; but it is not *because* A and C are both equal to B, that they are equal to one another, though that is how I may come to know of their equality. The reason why they are equal is that they contain the same number of identical units.[3]

The middle term does not in all syllogisms give the reason why the major belongs to the minor. It does so only in the first figure,

[1] The doctrine of the *Posterior Analytics* must in this respect be taken as overriding the more formal and external treatment of syllogism in the *Prior*.

[2] v. *supra*, pp. 205–206.

[3] But we cannot give this reason for the equality of the units.

and not always there. Because, whenever the middle term really is a *ratio essendi*, the syllogism falls into the first figure, Aristotle called it the scientific figure, *σχῆμα ἐπιστημονικόν*.[1] Why are modest men grateful? Because they think lightly of their own deserts. This implies a syllogism in Barbara. All who think lightly of their own deserts are grateful, and modest men think lightly of their own deserts. But if I try to establish the conclusion by an appeal to instances, pointing out that Simon Lee and Tom Pinch, John Doe and Richard Roe, were modest, and were grateful, I am giving not a reason why the modest are grateful, but reasons which lead me to judge them to be so; and my syllogism falls into the third figure, not the first: These men were grateful, and these men were modest, therefore modest men are (or at least they may be) grateful.

The first figure then is scientific, because a syllogism which makes you know why *C* is *A* falls into that figure; but the middle term in the first figure *need not* be a *ratio essendi*. 'Parallel rays of light proceed from objects at a vast distance; the sun's rays are parallel; therefore they proceed from an object at a vast distance.' Here my syllogism is again in Barbara; but the distance of the sun is not due to its rays (at the earth) being (so far as we can detect) parallel: their being parallel is due to the distance of the sun from the earth. Nevertheless, the syllogisms in which the middle term does account for the conclusion are enough to show that syllogism is not essentially a process of inferring about a particular member of a class what we have found to be true of every member of it. The importance of the scientific, or demonstrative, syllogism in this connexion is, that it most effectually disposes of this analysis of syllogistic inference. It shows that there are syllogisms which cannot possibly be brought under the *Dictum de omni et nullo*, thus interpreted. We shall, however, find that even where the middle term is not the cause of the conclusion, in the sense of being a *ratio essendi*, the Dictum thus interpreted does not give a true account of the nerve of our reasoning.

For syllogism really works through the connexion of concepts, or universals. The major premiss, '*B is A*', is not a collective statement about every *B*, *C* included; if it were, there would certainly be nothing new in the conclusion '*C is A*'. When Jacob lamented 'Me have ye bereaved of my children: Joseph is not, and Simeon

[1] *Anal. Post. a.* xiv. 79ª 17. The rest of the chapter is by no means all of it true. On 'scientific' and 'dialectical' syllogisms cf. *infra*, pp. 398–399.

is not, and ye will take Benjamin away', he added, speaking collectively of the loss of two sons, and the threatened loss of a third, 'all these things are against me'.[1] It would have been no inference to proceed 'Therefore the loss of Simeon is against me', because that was definitely included by the demonstrative *these*. To be 'one of these' is not a common character in each of them, with which a further character is connected; it is not therefore a middle term. Where *B* is a middle term, the major premiss connects *being A* with *being B*.[2] We must not be misled by our use of symbols. In a syllogism of the form '*All B is A, C is B* ∴ *C is A*', we say that *B* is the middle term. The following is a syllogism of that form:

Those who can find things out for themselves are little dependent on education
Men of genius can find things out for themselves
∴ Men of genius are little dependent on education.

Now if we symbolize the major premiss by the formula '*All B is A*', *B* represents the words 'those who can find things out for themselves'. But if we say that *B* is the middle term, *B* really represents the words 'being able to find things out for oneself'. It is that which, in men of genius, is either the ground or the sign of being little dependent on education. The middle term therefore is not the collection of things called by a general name; it is the common nature intended by the name, a ἓν ἐπὶ πολλῶν, something one in many subjects. And the same is of course true of the major term. The minor may indeed be an individual, or a number of individuals, though it need not be so.

The perception that the middle term is not a class but a character, universal and not a sum of particulars, has led to the formulation of a principle intended to express this more satisfactorily than the *Dictum de omni et nullo* does; of which it has already been said that it at least lends itself to an erroneous view of the major premiss, as an enumerative proposition, though it was by no means always so intended. The principle is this—*Nota notae est nota rei ipsius* (and for the negative, *Repugnans notae repugnat rei ipsi*): i. e. what qualifies an attribute qualifies the thing possessing it. Certain objections may be made to this formula also. It suggests that the minor term is always a concrete individual, and that the syllogism

[1] Genesis xlii. 36.

[2] Or, if negative, asserts that *being B* excludes *being A*. It will be remembered that we are discussing the first figure.

refers to this (*res ipsa*) what in the major premiss is stated to characterize its predicates. It speaks also as if one attribute were conceived to qualify another in the same way as an attribute qualifies a concrete subject. And the conception of a mark or *nota* is no improvement on that of attribute.[1] We need not interpret it as a purely external sign, related to what it signifies as a word to its meaning or a letter to a sound. The 'notes' of a thing are its characteristics, as Cardinal Newman spoke of the notes of the Church; they are not the mere indications by which we judge what thing is present, but themselves contribute to make it the thing that it is. Yet the nature of a thing is no less ill conceived as an assemblage of marks than as a bundle of attributes. The notes of the Church would not exhaust the notion of the Church; the marks of a disease, though elements and features of it, would not give a complete conception of what the disease is. There are predicates of a thing which include too much of its nature to be called marks of it. Nevertheless this formula has the great advantage that it does prevent our regarding the middle term as a class which includes the minor in its extension.[2]

But a better formula may be found. Kant said of the syllogism that it subsumed a cognition (i.e. a subject of knowledge) under the condition of a rule, and thus determined it by the predicate of the rule.[3] The rule is given in the major premiss, which connects

[1] Cf. Hegel's *Logic*, § 165, E. T. (Wallace), p. 296: 'There is no more striking mark of the formalism and decay of Logic than the favourite category of the "mark".'

[2] J. S. Mill (*System of Logic*, II. ii. 4 and note) strangely misinterprets the maxim *Nota notae est nota rei ipsius*. He understands by *res ipsa* the major term, and by *nota* the minor; so that the whole, instead of meaning that what qualifies an attribute qualifies the subject of it, comes to mean that what indicates the presence of an attribute indicates what the latter indicates. He naturally gets into great difficulties where the minor term is singular. We may treat the attributes of man as a mark or indication of mortality (though this is rather like saying that a Bank of England note is a mark of the presence of the chief cashier's signature); but we cannot treat Socrates as a mark or indication of the attributes of man. Therefore in the syllogisms *All men are mortal, All kings are men* (or *Socrates is a man*) ∴ *All kings are* (or *Socrates is*) *mortal*, while the minor premiss of the former is paraphrased *The attributes of a king are a mark of the attributes of man*, that of the latter runs *Socrates has the attributes of man*. This is a rather desperate shift. But *res ipsa* never meant the major term, the most general or abstract term in the syllogism; and the whole interpretation, which necessitates a measure so violent, is impossible. The formula is really an abridged equivalent of the passage in Ar. *Cat.* 1^{b} 10–12, quoted p. 297, n., *supra*.

[3] *Krit. d. r. Vern.*, Transcendental Dialectic, Introd. II. B. (p. 215, Meiklejohn's Translation).

a predicate (the major) with a condition (the middle term): the minor premiss asserts the fulfilment of this condition in its subject; and in the conclusion we determine the subject by the predicate which the rule, in the major premiss, connected with this condition. This analysis brings out the essential nature of the major premiss, as a rule connecting a predicate with a condition universally, not an assertion that the predicate is found in every member of a class. It also applies equally where the middle term is, and where it is not, the *ratio essendi* of the major. And it is free from the objections just urged against *Nota notae*.[1] If we were to frame from it a 'canon' parallel to this and to the *Dictum de omni et nullo*, it would run somewhat thus: *Whatever satisfies the condition of a rule falls under the rule*. In the rule 'Whatever is B, is A', being B is the condition, the fulfilment of which involves being A; and to a given subject C fulfilling the condition the rule will apply, and it will be A. We may perhaps accept this as a statement of the nature of the reasoning employed in syllogisms of the first figure. We need not deny that the *Dictum de omni et nullo*, if rightly interpreted, is free from the offences charged against it. If the *omne* be understood as an unity present in many instances—a logical whole or whole of intension, not an aggregate of individuals—then the principle will serve. But the other puts more clearly the nerve of the inference. And it applies to all syllogisms in the first figure, whatever the nature of the middle term: whether it be a mere sign of the major term, as if we said that 'All men with large hands and small eyes are choleric' —where the connexion of the predicate with its condition, though accepted *de facto*, is one for which we can see no necessity: or whether it give, wholly or in part, the reason and the explanation of the major, e.g. in such premisses as that 'All trees fertilized by the wind blossom before their leaves are out', or that, 'Men successful in a work that gives full play to all their faculties are happy'. Whatever our particular syllogism is, we shall find it true to say of it, that it brings a subject under a rule, on the ground that it satisfies the condition of that rule: that it affirms (or denies) a predicate of a subject, on the ground that this subject fulfils the

[1] Kant himself applied this analysis to hypothetical and disjunctive arguments also. In a later chapter, these are more strongly distinguished from 'categorical' syllogisms than he allows. But this need not prevent the acceptance of his analysis. A statement may correctly express the nature of syllogistic inference, even when some arguments, which are not strictly syllogistic, are also alleged to fall under it.

condition with which the predicate (or its absence) is universally conjoined or connected.

This canon is exemplified even where the major premiss rests on an examination of all the instances included under the middle term; so that there is inference there, though not proof of the conclusion. The major premiss is indeed in such a case a sort of memorandum, as Mill says of it,[1] to which we subsequently refer in order to save the trouble of repeating our observations; but a memorandum in general terms requires inference to make use of it. Suppose a man intending to dispose of part of his library; he might look through his books and put a mark in all those which were not worth keeping; if he then forgot what a certain book contained, but finding his mark in it said that it was not worth keeping, he would be syllogizing. He would argue 'Books thus and thus marked are not worth keeping, this book is thus and thus marked, ∴ it is not worth keeping'. There is no real proof here that it is not worth keeping; that could only be determined by reading the book; and his mental note that no book thus and thus marked is worth keeping requires that he have read this book and ascertained that it was not worth keeping before marking it. But he may have forgotten all about it; and he now asserts that it is not worth keeping because by containing the mark it satisfies the condition on which the ascription of that predicate rests. In applying his rule he trusts of course to his past care in reading and marking; and so he may be said to take the major premiss on trust. But that is common enough. Even when an universal proposition is capable of proof, many reason from it syllogistically who never knew the proof, or if they knew it, have forgotten it. We may go further. Subsumption, or bringing a subject under the condition of a rule, which is the nature of syllogism in the first figure, always implies that in a measure the rule is taken on trust. To this extent the major premiss is always a sort of memorandum. For if we understood at the moment the necessary connexion between the middle term and the major, we should appeal to no rule, but in considering C, the minor term, itself, pass from the knowledge that it is B to the further knowledge that it is A. We should indeed realize herein that the connexion of the character A with the character B was not limited to the subject C. But

[1] *System of Logic*, II. iii. 4. Mill's mistake lies not in saying that the major premiss of a syllogism is a memorandum, but in making it a false memorandum, which records that all B is A when we have only observed that X, Y and Z (which are B) are A.

we should rather extract the general rule from consideration of the subject before us than bring it in independently and as it were *ab extra* to the consideration of that subject. And that would not be syllogism. Syllogism does not belong to the level of complete insight into the connexion of facts. In geometry we never syllogize except when we rely upon the results of a previous demonstration whose steps we do not realize in the case before us. 'The triangle in a semicircle has the square on its hypotenuse equal to the squares on the other two sides, because it is right-angled'; that is a syllogism; but if we realized at once the construction of Euclid i. 47 for the figure in iii. 31, the proposition that in a right-angled triangle the square on the hypotenuse is equal to the squares on the other two sides would appear rather as generalized from what we saw to be true in the triangle in a semicircle than as a rule applied to that case. The subsumption in syllogism belongs therefore to thinking which has not complete insight into the necessity of all the facts in its premisses at once. When Aristotle taught that syllogism is the form of demonstrative thinking, he failed to realize this. Because *C's* being *A* is seen to be involved in its being *B*, he thought we used a major premiss, '*All B is A*'. He was nearer the truth when he said that in demonstration our terms are connected *per se*. The putting together of, or the appeal to, premisses already known is not necessary to demonstration. Supposing I already understood that to be an organism involved being mortal, yet if I discovered some thing of strange kind to be an organism, I should know that it was mortal, in virtue of my now understanding the connexion, not in virtue of having understood it before. But because we have constantly to appeal to the conclusion of a previous process of demonstration or other reasoning without re-thinking that process at the time, we are constantly syllogizing; and where the premisses are such of which we remember to have previously satisfied ourselves by reflection or demonstration or inductive argument, or (if they concern facts established by authority) by reference to authority, there syllogism may deserve the name of proof. It is otherwise where, in order to establish a conclusion, we appeal to a premiss which itself needs the help of the conclusion to establish it; this is not proof; yet if the premiss has been so established, and is now appealed to as a record, there is syllogistic inference. Our argument is one whose general form is given in the canon of syllogism.

That canon, like the axiom of equals, is not itself a premiss but

a principle of reasoning. It is easy to see this. Any one denying it would as readily deny the validity of any particular syllogistic argument; but a man may admit the validity of the inference, in a particular case, without needing to consider this general principle. And, as no one could see that *Things equal to the same thing are equal to one another*, who was incapable of seeing the truth of that principle in a given case, so no one could see the truth of the principle that *What satisfies the condition of a rule falls under the rule*, who failed to recognize that if all organisms are mortal, and man is an organism, man must be mortal. What then is the use of the principle, if it is not a premiss of inference? It might be used to stop the mouth of a disputant who denied the conclusion which followed from the premisses he had admitted. We might ask such a disputant, whether he denied the truth of this principle, and unless he was prepared to do that, require him to admit the validity of the syllogism he was disputing. It is true that in consistency he might decline. A man who denies the validity of a given syllogism in Barbara may with equal reason deny the argument which attempts to prove its validity. For that argument will itself take the form of another syllogism in Barbara:

All inferences upon this principle (that what satisfies the condition of a rule falls under the rule) are valid
The syllogism in question is an inference upon this principle
∴ It is valid

Why should a man admit this reasoning, if he will not admit that since

All organisms are mortal, and
Man is an organism
∴ Man is mortal?

The two are of the same form, and this shows that you cannot make the principle of syllogistic inference into the premiss of a particular syllogism, without begging the question.[1] Yet a man who disputes

[1] Cf. an article on 'What the Tortoise said to Achilles', by 'Lewis Carroll', in *Mind*, N. S. iv. 278 (April, 1895). It is obvious that the validity of the latter of these two syllogisms cannot require to be deduced from the principle which stands as major premiss in the former. For if until that is done its validity is doubtful, then the principle by which we are to establish its validity is equally doubtful. Besides, what proves the validity of the former, or validating, syllogism? The validity of a syllogism cannot be deduced from *its own* major premiss; else the fact that all organisms are mortal would show that the syllogism, of which that is the major premiss, is valid.

in a particular case the conclusion that follows from his premisses may hesitate to maintain his attitude, if the principle of reasoning involved is put nakedly before him, and shown to be one which he daily proceeds upon, and cannot disallow without invalidating his commonest inferences. For this reason it may cut wrangling short, if we can confront a man with the principle of the inference he questions. Show him, for example, that the inference ascribes to a subject, in which certain conditions are fulfilled, a predicate connected universally with those conditions, and he cannot longer refuse his assent. For to do what it does *is to be a syllogism*[1]: and therefore valid.

And there have been writers[2] who thought that the only object of knowing the theory of syllogism was to cut short wrangling. But there is another object, connected with a side of logic which the same writers for the most part ignore. Logic is not an art. Its business is to know and understand the processes of thought, and not least the true nature of our processes of inference. To this business belongs the question, what is the principle of a certain inference which we make, and recognize to be valid? To find and formulate that principle—to extricate it from its concrete setting in the matter of a particular argument, and set it out in abstract,—this is the logician's task. Now men may misinterpret the character of syllogism, and formulate wrongly the principle involved; yet if their misinterpretation is generally received for true, the wrong principle will serve in practice to stop dispute as well as the right principle would have done. Those who are agreed that syllogism is conclusive, however they define a syllogism, will accept an argument if it can be shown to accord with their definition; and the same misinterpretation which appears in their account of the general nature of syllogism will appear in their view of particular syllogisms, from which that account is of course derived. Therefore, though it be said that a syllogism is an argument which applies to some one member of a class what is true of every member, yet even this analysis of it, however faulty, will serve to 'stop wrangling'

If it be said that the validating syllogism needs no proof of its validity, the same can be said of the syllogism which it validates. But if it needs a proof, the syllogism which validates *it* will need validating by another, and so *ad infinitum*. No form of inference can have its validity guaranteed by another inference of the same form with itself; for we should be involved at once in an infinite process.

[1] Cf. Ar. *Post. An.* β. vi. 92ᵃ 11–16.

[2] e.g. Locke, *Essay*, IV. xvii. 4.

among persons who accept it. For let a particular argument be exhibited as doing this, and it will be accepted as valid. But the theoretical objections to this analysis of syllogistic inference are in no way lessened by its being practically as useful as any other that men could be brought to accept. The paramount question is, whether it is true: not whether for any purposes it is useful. And the present chapter has been quite disinterested; it has aimed at throwing light on the question, What is a syllogism? i. e. What is the principle of inference which a syllogism exemplifies?

We have ignored of late the imperfect figures, in seeking an answer to this question. They furnished a possible objection to the claims of the *Dictum de omni et nullo*[1]; for if their reduction to the first figure is unnecessary, then the Dictum, which only contemplates the first figure, cannot be the principle of all syllogistic inference. But this objection was deferred, until the Dictum had been examined on its own ground. We must now return to the subject of the imperfect figures.

It may make things clearer, if the view to be taken in the following pages is given summarily at the outset. There are difficulties in any view of the matter; because the same verbal form may be used where the thought in the speaker's mind is different. The true character of an argument depends not on the verbal form, but on the thought behind it. And therefore sometimes the movement of a man's thought, though he expresses himself, e. g., in the second figure, would be more adequately exhibited in the first.[2] In such a case direct reduction may be defensible, though still unnecessary; and yet it may be true that, speaking generally, the direct reduction of the imperfect figures distorts them, and purchases a show of conformity with the first figure at the expense of concealing the genuine movement of thought in them.

It would seem then that syllogisms in the second and third figures do not as a rule merely present under a disguise the reasoning of the first; they are independent types. Their validity is confirmed,

[1] Cf. *supra*, p. 301.

[2] e. g. in this syllogism in Festino, 'No fragrant flowers are scarlet, Some geraniums are scarlet ∴ Some geraniums are not fragrant', I think a man would probably substitute in thought for the major its converse, 'No scarlet flowers are fragrant', and argue to himself in Ferio. With such a premiss, where there is no priority as between the two accidents, fragrant and scarlet, that is the more natural way to argue. But this does not show that all syllogisms in Festino ought to be thus treated.

in the second figure, by the *reductio ad absurdum*,[1] and in the third, by the method which Aristotle called ἔκθεσις, or *exposition*. The fourth figure (or indirect conclusion in the first) is not an independent type ; its first three moods are merely moods of the first figure, with the conclusion converted, as the process of reducing them assumes ; its last two moods draw conclusions which are shown to be valid most naturally by reduction to the third figure.

Let us begin with the second figure. Take the syllogism : *All true roses bloom in summer : A Christmas rose does not bloom in summer ∴ It is not a true rose.* Surely, if a man hesitated for a moment about the necessity of this consequence, he would reassure himself, not by transposing the premisses, and converting the present minor into the statement that *No rose which blooms in summer is a Christmas rose* : but by considering, that a Christmas rose, if it were a true rose, would bloom in summer, whereas it does not. The same remarks will obviously apply to a syllogism in Baroco. Nor is it otherwise with the remaining moods. If *No fish has lungs*, and *Whales* (or *Some aquatic animals*) *have lungs*, then *Whales* (or *Some aquatic animals*) *are not fish.* A man sees at once that if they were, they would not have lungs : whereas they have.

It might be said that the last conclusion could be as naturally reached in the first figure ; that if a man, confronted with the conclusion that *Whales are not fish*, and not feeling that he was clear about its cogency, were to ask himself ' Why not ? ', he would answer ' Because they have lungs ' ; and that this implies a syllogism in Celarent, with the major premiss *What has lungs is not a fish.* Whether this gives the reason why a whale is not a fish (in which case Celarent would be a better way of proving it) we need not dispute ; but there certainly are cases where, in what a subject is, we can find a reason for its not being something else. *Notes that produce beats are not harmonious : The fourth and fifth produce beats ; Therefore they are not harmonious.* This argument might be set forth in the second figure : *Harmonious notes do not produce beats : The fourth and fifth produce beats ; Therefore they are not harmonious* : but here undoubtedly the syllogism in Celarent is better than the syllogism in Cesare ; and any one who knew that concord was dependent on regular coincidence in vibrations and discord on the clashing of them, would extricate from the major premiss of the latter syllogism the major of the former, and think

[1] Called by Aristotle ἀπαγωγὴ εἰς τὸ ἀδύνατον.

in Celarent. Nevertheless it is only this knowledge which makes him do so; and without it he might perfectly well validate to himself his conclusion by considering that if those notes were harmonious, they would not produce the beats they do. If the middle term gives a *ratio essendi*, we naturally put our reasoning into the first figure.[1] The Chinese are not admitted into the United States, for fear lest they should lower the white labourer's standard of living. The likelihood of their doing this is the cause of their exclusion. It would be unnatural to express this in Cesare—

None admitted into the United States are likely to lower the white labourer's standard of living
The Chinese are likely to lower it
∴ The Chinese are not admitted into the United States.

But we are not concerned to prove that no arguments expressed in the second figure are better expressed in the first; only that there are arguments which are more naturally expressed in the second, and which we should not, if challenged, attempt to validate by reduction to the first. Thus I may argue that *Notes which produce beats are not harmonious*, and *A note and its octave are harmonious*, ∴ *They do not produce beats*; and it is as much a distortion to put this into the first figure by conversion of the major premiss as to put the previous example which used that major premiss into the second figure by the same means. Again, if I give, as a reason why whales are not fish, that they have not the characteristics of fish, such as breathing through gills, laying eggs, &c., my syllogism may very well be in Camestres—*All fish breathe through gills*, and *Whales do not* ∴ *A whale is not a fish*; if I still ask myself why not, I should probably answer, 'Because if it were a fish, it would breathe through gills, which it does not do'. The conclusion states a fact of difference between two things, which the premisses prove but do not account for; and the proof in the second figure may be said to be here the primary form.[2] Moreover, if I were to recur to the first figure in order to establish this inference, it would naturally be by contraposing the major premiss

[1] It must not be forgotten that most reasoning which explains facts through their causes is not syllogistic at all; but if it is syllogistic, it will be in the first figure. Cf. *supra*, p. 305.

[2] Hence the statement, frequently quoted from Lambert (*Neues Organon*, vol. ii. p. 139; *Dianoiologie*, iv. § 229. Leipzig, 1764), that the second figure points us to the differences between things: 'Die zweite Figur führt auf den Unterschied der Dinge, und hebt die Verwirrung in den Begriffen auf.'

What does not breathe through gills is not a fish
Whales do not breathe through gills
∴ Whales are not fish

for the absence of a feature essential to any fish may be treated as explaining why a thing is not a fish. But the syllogism to which Camestres is supposed to be reduced is not the above; it is the following—

What breathes through gills is not a whale
A fish breathes through gills
∴ A fish is not a whale

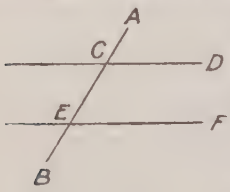
from which the original conclusion that a whale is not a fish is recovered by conversion. Now this argument, instead of relying on something in whales (viz. the absence of gills) to show that they are not fish, relies on something in fish (viz. the presence of gills) to show that they are not whales; whereas whales are really the subject of my thought. The same line of reflection may be applied to the argument, *Matter containing active bacilli putrefies: Frozen meat does not putrefy ∴ It contains no active bacilli*; where no one could maintain that non-putrefaction was really the cause of matter containing no active bacilli.

Thus the second figure is really different in type from the first; although reasonings which would naturally fall into the first *may* be thrown into the second. And the difference is this, that the second is fundamentally indirect, the first direct. In the second, we see the validity of the conclusion through the contradiction that would be involved in denying it; in the first (though, of course, it would be equally self-contradictory to admit the premisses and deny the conclusion) the perception of this is not a 'moment' in our thought. It may fairly be said that the first figure is prior to the second, in the sense that it is involved in the perception of the contradiction which would result from admitting the premisses and denying the conclusion in the second. But that does not justify us in reducing the second to the first. For it is an essential part of our thought in the second figure, to see that the conclusion must follow on pain of contradiction; and not merely to see the validity of the first-figure syllogism, by help of which the contradiction is developed. There is therefore a movement of thought in the second figure which is absent from the first. This is what makes a new type of it; and this is why its direct reduction, representing second-figure

syllogisms as only first-figure syllogisms in disguise, is wrong, and therefore superfluous.

It may be asked, is even indirect reduction necessary? Is not the validity of the argument plain, without our being at pains to show that, if it were disputed, we should be involved in a contradiction? Cannot a man appreciate that if No A is B, and C is B, then C is not A, without the necessity of pointing out that C would not otherwise, as it is, be B? The answer is that a man may certainly not require this to be pointed out, inasmuch as he sees it at once to be involved in the premisses. The so-called indirect reduction is really a part of the thought grasped in the syllogism; not something further, by which, when a man has already made his inference, and realized the act of thought involved in making it, he then proceeds to justify his act. It rather brings out what is in the inference, than reduces or resolves it into another. Hence a man may feel it to be unnecessary, but only because it is a repetition, not because, if he did not see it, the syllogism would still be seen to hold without it.

Yet it must not be supposed that a form of argument is valid only because to question it would involve a contradiction. With equal reason it might be said that unless the argument were valid, there would be no contradiction in rejecting it. Hence, in the second figure, the contradiction that would ensue if we denied the conclusion, is not the reason for admitting the conclusion, but the perception of it is involved in realizing its validity. An analogy may help us. If a straight line, falling on two other straight lines, makes the exterior and the interior and opposite angles on the same side of it equal, the two lines must be parallel. Strictly speaking, this cannot be proved by reasoning; we just see, when we try to draw the figure otherwise, that it must be so. But this necessity may be brought out indirectly by the consideration, that if $B\,E\,F$ were to be greater than $B\,C\,D$, $E\,F$ and $C\,D$ would cut $A\,B$ at a different slant, and therefore incline towards one another; and the perception of this is really part of seeing the necessity of the original proposition. Nevertheless it cannot be given as a reason for the truth of that proposition; for unless the lines were parallel when the angles $B\,E\,F$, $B\,C\,D$ are equal, they would not necessarily tend to meet when each cuts $A\,B$ at a different slant. The confirmation, such as it is, is obtained by looking at the



same matter from another side ; and so it is in the second figure of syllogism. The truth of one side cannot really be separated from the truth of the other, and therefore the one is not dependent on the other ; but it is not fully appreciated without it. The development of the contradiction involved in denying the conclusion in the second figure is a development of the system of relations between the terms alleged in the premisses, or of the consequences involved in these. It is not, like a suppressed premiss, something without the consideration of which the argument is altogether broken-backed ; but it is something involved in the full appreciation of the argument.

If then the second figure is not a mere variation of the first, it follows that the principle or canon on which the first proceeds is not that of the second. If the above account of the nature of our reasoning in the second figure is correct, its principle is this, that no subject can possess an attribute which either excludes what it possesses or carries what it excludes.

Of the third figure we must give a different account. Its two most noticeable features are that the middle term is subject in both premisses, and the conclusion always particular. For this reason it has been well called the *inductive* figure ; for induction (whatever else besides may be involved in it) is the attempt to establish a conclusion upon the evidence of instances. The *terms of the conclusion* are always general. The conclusion declares two characters to be conjoined, or (if negative) disjoined : *Sailors are handy, The larger carnivora do not breed in captivity.* In the premisses we allege instances of which both characters can be affirmed ; or of which one can be affirmed and the other denied ; and these instances are our evidence for the conclusion. But *the conclusion* is not general ; we are never justified, by a mere citation of instances, in drawing a really universal conclusion. If All B is A, and All B is C, we cannot say that All C is A ; in traditional phraseology, C is undistributed in the minor premiss, and therefore must not be distributed in the conclusion ; and the thing is obvious, without any such technicalities, in an example ; if all men have two arms, and all men have two legs, it does not follow that all animals with two legs have two arms ; for birds have two legs, besides men, and have not arms at all, but wings. Yet, though our instances will never *justify* a really universal conclusion, they may *suggest* one ; and they will at any rate overthrow one. The instances of Queen Elizabeth or Queen Victoria,

of Catherine of Russia or Christina of Sweden, will disprove the proposition that *No woman can be a statesman*; and truth is often advanced by establishing the contradictory of some universal proposition, no less than by establishing universal propositions themselves.

Now what is the true nerve of our reasoning in such arguments? It is the instance, or instances. We prove that some C is A, or some C is not A, because we can point to a subject which is at once C and A, or C and not A. Unless we are sure that the same subject is referred to in both premisses, there can be no inference: *Some animals are quadrupeds*, and *Some animals are vertebrates*; but they might be different animals, and then there would be no instance of a vertebrate that had four legs. But if either premiss is universal—if e.g., with *mammal* as our middle term, we take the premisses *Some mammals are quadrupeds*, and *All mammals are vertebrates*—then it follows that *Some vertebrates are quadrupeds*; for the 'some' mammals of the major premiss are included among the 'all' of the minor, and therefore we could pick out, from among the latter, instances of animals that were both vertebrate and quadruped. The instances, however, instead of being vaguely indicated as 'some' of a whole class or kind, may be specified by name; and then the nature of our reasoning is unambiguous; we are manifestly arguing through instances. In order to show that *A woman may be a statesman*, we can appeal to the four queens mentioned above; these were statesmen, and these were women; and therefore some women have been (or women may be) statesmen. But whether the instances in which C and A are united, or C is present without A, be cited by name, or only indicated as 'some' of a whole class, in both cases alike it is on them that the reasoning hinges, and it is by producing them that a sceptic could be confuted, who refused to admit the conclusion.

Aristotle called this production of the instance by the name ἔκθεσις, or Exposition. He conceived that the proper mode of validating a syllogism in the third figure was by direct reduction,[1] but added that it was possible to validate it *per impossibile* or by 'exposition': 'if all S is both P and R, we may take some particular S, say N; this will be both P and R, so that there will be

[1] Except, of course, where the major premiss is a particular negative and the minor an universal affirmative proposition (Bocardo), in which case we can only proceed *per impossibile* or by exposition. *Anal. Pri.* a. vi. 28^b 15–21.

some R which is P'[1]; and what is possible where both premisses are universal and affirmative is equally possible in any other mood. This seems to exhibit the real movement of thought in the third figure better than the artificial process of direct reduction. For, in the first place, if the middle is a singular term, as in this figure it often is (though Aristotle took little note of such cases), the conversion of a premiss is forced and unnatural. In words I may say that since Queen Elizabeth and Queen Victoria were statesmen, and some women were Queen Elizabeth and Queen Victoria, therefore women may be statesmen; but in thought, Queen Elizabeth and Queen Victoria will still be subject in the minor premiss. And secondly, even where the middle is a general term, direct reduction often conceals, rather than expresses, our thought. *No ostrich can fly, All ostriches have wings ∴ Some winged animals cannot fly*: here, though it is possible to substitute for the minor premiss *Some winged animals are ostriches*, the other is the form in which we naturally think; the more concrete term stands naturally as the subject of our thought.

It may be admitted that there are cases where direct reduction is unobjectionable. *No clergyman may sit in Parliament*, and *Some clergymen are electors to Parliament ∴ Some electors to Parliament may not sit in it*. Here it would be as natural to say that *Some electors to Parliament are clergymen*; for the franchise, and the clerical office, are each an 'accident' of a man, and either, *elector to Parliament* or *clergyman*, can equally well be subject in the proposition, and the other predicate. But the character of the argument seems changed by this alteration. *Clergymen* are no longer the instance which shows that a man may be entitled to vote without being entitled to sit; the middle term is now a status in virtue of which certain voters cannot sit. The point contended for is not that there may not be syllogisms in the third figure, whose conclusion could be equally well, or even better, obtained with the same middle term in the first: but that the movement of thought characteristic of the third figure is not, and cannot be reduced to, that of the first; and that reduction, as a general principle, is thereforesuperfluous and misleading: the true confirmation of the validity of the syllogism lying in the perception that, if the premisses are true, there actually are instances of the fact alleged in the conclusion.

One objection to this view of the third figure needs consideration.

[1] *Anal Pri.* a. vi. 28^{a} 24–26.

It may be said that the production of a particular instance in support of the conclusion does not do full justice to the grounds on which we accept it, in cases where the middle term is general and both premisses universal. All horned animals ruminate, and they all part the hoof ; this, it may be urged, is better ground for concluding that cloven-footed animals may be ruminants, than if I merely called attention to the cow in my paddock. To settle this, let us look for a moment at the two meanings, which (as we saw before) may be intended by a particular proposition.[1] If I say that *Some C is A*, I may either mean to refer to certain unspecified but definite members of the class *C*, and predicate *A* of them ; or without any special thought of any particular case, I may mean to declare the compatibility of the two characters, *C* and *A*, in one subject. In the latter case, I can also express my meaning by the problematic judgement *C may be A* ; which contains no doubt the thought of unknown conditions under which it *will* be so. Now supposing I understand the proposition in the latter sense, the cow in my paddock is as good a middle term as horned animals generally ; supposing I understand it in the former sense, then my conclusion, that *Some cloven-footed animals ruminate*, undoubtedly has more to rest on, when the premisses speak of *all horned animals*, than when for middle term I refer only to a cow or two in a neighbouring paddock. But it is also really a different conclusion ; the ' some ' intended are a larger number of unspecified animals in the one case than in the other ; and it is only by the production, or ' exposition ', of all the instances to which our ' some ' refers, that the reference to them all, in the conclusion, may be justified.

It may fairly be said that the argument, in this view of it, does not really amount to a syllogism : it comes to this, that if all horned animals ruminate, and all part the hoof, then *all cloven-footed animals that are horned* ruminate. If the exact sphere of the conclusion is thus borne in mind when we say that *some cloven-footed animals* ruminate, and we *mean* by ' some ' *all that are horned*, there is not really and in thought that elimination of the middle term in the conclusion which is characteristic of syllogism. It would not be reckoned a syllogism if we argued that since Wolsey was a cardinal and Wolsey was chancellor, he was both chancellor and a cardinal[2] ; neither is it a syllogism (though it is inference) to argue, from the

[1] Cf. *supra*, pp. 178–180, 199.
[2] Cf. Bain's *Logic, Deduction*, p. 159 (ed. 1870).

premisses above, that all horned animals are both ruminant and cloven-footed : from which it follows that all cloven-footed animals that are horned are ruminant.

We may admit the view of the last paragraph to be the right one. Supposing that when we conclude, in the third figure, that Some C is (or is not) A, we refer in thought, though not in words, just to those particular instances, and no others, which in the premisses were stated to be both B and A (or not A), then we have not got a proper syllogism. Still our conclusion rests entirely on the production of those instances, few or many, beyond which our thought refuses to travel. The true and characteristic syllogism in the third figure, however, intends its conclusion in the other sense : as a statement of the proved compatibility or separability of two attributes. And to establish this too it relies on the production of an instance ; nor are many instances really more sufficient than one, to establish mere compatibility, except as minimizing the risk of mal-observation. The appeal need not indeed be to an individual it may be to a kind. If we want to prove that an evergreen may have conspicuous flowers, we can cite the rhododendron ; and we may mean by that not any particular specimen, but any of the species generally.[1] But very often, and mostly where one premiss is particular,[2] and of course always where the premisses are singular, it is on determinate individual instances that we rely ; and one instance or one species is enough. Therefore it is by exposition —by a production, not of course in bodily form, but in thought, of one instance or species—that we justify the inference to ourselves ; we actually make this appeal in our minds, if we realize the ground of our conclusion. Persons familiar with a type of reasoning may draw conclusions from premisses as it were by precedent, and without realizing the evidence on which they act ; but whenever we are fully conscious of what we are about, there is, in the third figure,

[1] It may be urged that the appeal is really to specimens, not species : for the species does not blossom. The question raised is not peculiar to the third figure. If I argue that the rhododendron is popular because it flowers brilliantly, it may equally be said that rhododendrons do so, not 'the rhododendron'. The relation of an universal truth to its instances is involved in the question, which is an important one. But it need not complicate the present discussion.

[2] Not always, even there ; I may argue that all breeds of dog are domesticated, and some are savage, and therefore some domesticated breeds of animal are savage (Disamis). Here I am speaking, and thinking, throughout not of individual animals but of their kinds.

the recognition that the conclusion is proved by its exemplification in a case cited, or included in what we cite.

Of course there is a way in which the number of instances makes a real difference to the conclusion which we are inclined to draw. The case of Prince Bladud is alone enough to show that a man who washes in the waters of Bath may recover of a disease. The two events, however, may be accidental and unconnected. But if cases were multiplied, we should begin to suspect there was a connexion between the use of these waters and the cure of certain ailments; or if the ailments which disappeared after taking the waters were of all sorts, we might begin to look on Bath waters as a panacea. For establishing a *connexion* between two attributes the number and variety of instances are matters of great importance; but for establishing *compatibility* one instance is enough. Now the third figure does not prove more than a compatibility; and never can prove a connexion, however many the instances are; and though the number of instances may make a connexion highly probable, yet we are influenced in reaching such a conclusion by other considerations besides the mere number of the instances. For example, a man who observed in several cows the combination of the cloven foot with the ruminating stomach would be much less inclined to suppose that there was any general connexion between these characters in nature, than if he had observed the same thing in an equal number of beasts belonging to as many different species. For we are accustomed to find characters constant throughout one species, and failing when we go beyond it; so that the accumulation of instances would be discounted by the fact that they all belonged to the same kind. Again, we might meet a Privy Councillor in a light suit, and yet not be led to regard the next man we met in a light suit as a Privy Councillor; but if we met a Guardsman in a breastplate, we should very likely suppose the next man in a breastplate to be a Guardsman. The readiness with which we infer connexion is controlled by our general knowledge of the kinds of attributes that are connected; such considerations do not appear in our premisses, but greatly influence our thought. Hence it is, that those who are thoroughly familiar with the facts of a science, or of some historical period, can make inferences from isolated facts which to persons ignorant of the field of investigation, and the controlling principles applicable to it, appear foolhardy. But all this belongs to rather a different department of logical theory, the Logic of Induction. It

remains true that so far as we bring no extraneous considerations to bear, and are guided only by the facts contained in our premisses, we can infer no more than the compatibility of two characters (or their separability) from any number of instances ; and we can infer thus much from a single instance.

It should be noticed, before leaving the consideration of the third figure, that it always argues from a *ratio cognoscendi*. It is not because the rhododendron has brilliant flowers, that this attribute can be combined with evergreen foliage ; if it were not that there is no incompatibility between them, the rhododendron could not exhibit both. Our instance merely teaches us that the two are compatible ; it is the ground of our assertion, not the ground of the fact asserted. And this in itself is enough to show that there is a real difference between the nature of our reasoning in the third figure, and in the first—at least when our syllogisms in the first figure are scientific ; and that the attempt to reduce all syllogisms to one typical form imposes an unreal appearance of conformity upon arguments which are essentially disparate.

[The fourth figure of syllogism remains for consideration.[1] It has this peculiarity, that its premisses as they stand, if we transpose them, present the arrangement of terms required by the first figure. And three of its moods (Bramantip, Camenes, and Dimaris), when thus regarded as being in the first figure (= Baralipton, Celantes, Dabitis), afford conclusions of which those drawn in the fourth figure are merely the converse ; but the other two moods (Fesapo and Fresison) yield no conclusion in the first figure, from which the conclusion in the fourth might be obtained. Are we therefore to regard this figure as presenting a separate type of inference from the first, or was Aristotle right in disregarding it ?

Let us look first at the moods which are reduced to the first figure by a mere transposition, and without any alteration, of the premisses. In the premisses *All nitrogenous foods are flesh-forming*, *All grains are nitrogenous*, if we treat *flesh-forming* as the major term, we have a syllogism in Barbara ; but if we treat *grains* as major term, our syllogism is in Bramantip, and the conclusion is that *Some flesh-forming foods are grains*. It is surely true that the cogency of this inference, as compared with the other, is peculiarly unobvious. The conclusion is not what we should naturally

[1] This note may, of course, be equally well regarded as a discussion of the indirect moods of the first figure. But if a new type of inference were involved in them, the erection of a fourth figure would be justified. As that is the question under discussion, it seems fairer to call them moods of the fourth figure at the outset.

[draw from the premisses; and we need to look a little closer, in order to convince ourselves that it necessarily follows. And this conviction comes to us when we realize either that from the given premisses it follows that *All grains are flesh-forming*, and our other conclusion follows by conversion from that: or else that if no flesh-forming foods were grains, no nitrogenous foods would be grains; and that in that case grains could not all, or any, of them be nitrogenous. The same remarks would apply *mutatis mutandis* to syllogisms in Camenes or Dimaris; and we may therefore conclude that these moods are not evidently cogent without a further act of thought than their formulation in the fourth figure displays. Are we therefore to treat them as belonging to the first figure? The reason for doing this is, that the simplest and directest way of justifying the inference which they contain is by drawing a conclusion in the first figure from their premisses, and converting it.

The two remaining moods, Fesapo and Fresison, are less easily disposed of. As the same considerations apply to both, it will suffice to take an example of the former. *No animals indigenous to Australia are mammals, All mammals are vertebrate ∴ Some vertebrates are not indigenous to Australia*; if we transpose these premisses, no direct conclusion follows; we cannot tell from them whether any of the animals indigenous to Australia are vertebrate, or not; so that if our argument requires validating, we must validate it either by direct or indirect reduction, or by exposition. That it does need validating seems to follow from the fact, that in its present form it is no more obvious than the three preceding moods of the fourth figure; no one ever argues in the fourth figure, and that shows that it does not adequately exhibit the movement of thought in inference. Aristotle exhibited the validity of this mood[1] by converting both premisses (i.e. by direct reduction): *No mammal is indigenous to Australia*, and *Some vertebrates are mammals*; and this is a more natural way of putting the argument. But there are cases in which conversion would substitute a less natural mode of expression in the premisses; e.g. from the premisses *No mineral waters are alcoholic* and *All alcohol is taxed*,[2] we can infer that *Some things taxed are not mineral waters*: it would be less natural, although it would yield the same conclusion, and that in the first figure, to say that *Nothing alcoholic is a mineral water*, and *Some things taxed are alcoholic*. Again we may proceed by indirect reduction; we may argue that if all vertebrates were indigenous to Australia, then since no animals indigenous there are mammals, no vertebrate would be a mammal; we thus reach a

[1] i.e. of Fapesmo and also Fresison=Frisesomorum: v. *Anal. Pri. a.* vii. 29ᵃ 24–27.

[2] It would complicate the illustration too much to make the exception required by methylated spirits.

[conclusion inconsistent with the premiss *All mammals are vertebrate*, and that shows that our original argument cannot be disputed ; but we should more naturally say that No mammals are vertebrate than that No vertebrates are mammals ; and the former contradicts more directly the premiss that *All mammals are vertebrate*. Still more do we feel this, if we apply indirect reduction to our other example ; there, if Everything that is taxed were a mineral water, then since No mineral waters are alcoholic, Nothing taxed is alcoholic ; it is clearly more natural to say that No alcohol is taxed, and that exhibits better the contradiction with our premiss. If we employ the method of ἔκθεσις or exposition, we must convert the premiss *No animals indigenous to Australia are mammals* ; then we have it given that mammals, in any instance that we like to take, are not indigenous to Australia, and are vertebrate ; from which it follows that an animal is sometimes vertebrate, and not indigenous to Australia. Similarly we may convert *No mineral waters are alcoholic*.

Thus we have in this mood an argument undoubtedly valid, yet lacking something to be obvious ; it is possible to validate it in several ways, either bringing it into the first figure by conversion of both premisses, or into the third by conversion of one, or leaving the premisses and showing, as in the second figure, that the falsity of the conclusion is inconsistent with their truth. Which of these methods is preferable ? and to what figure should the mood be referred ? or is it really of a fourth sort ? That it is not of a fourth sort is shown by the fact that without one of these methods of validation its conclusiveness is not apparent, that conversion of both premisses reduces it to the first figure, and that if reduction *ad impossibile* or exposition is to be used, it may as well, and better (as will be explained two pages below), be regarded as a syllogism belonging, by the nature of its premisses, to the first figure, but needing validation by this means. Perhaps the first of the above questions will be best answered, if we ask in what way, by the use of the same middle term, the conclusion of the given syllogism could most naturally be reached. How are we to prove that *Some vertebrates are not indigenous to Australia*, using *mammals* as our middle term ? or that *Some things taxed are not mineral waters*, using *alcohol* as middle term ? In both cases we should appeal to an instance in point ; the mammals may be cited to show the former, and alcohol to show the latter. It would seem therefore that exposition is the natural way of validating the argument ; or in other words, that we realize its cogency most readily if we realize that in the major premiss there is involved a converse, from which the conclusion follows at once in the third figure.

Are we then to reckon the mood to the third figure, and not (with Aristotle) to the first ? Aristotle would, of course, have said that since the third figure itself needed validating through the first,

[we had stopped half-way in reducing it to the third; but if, as has been held above, the third figure is really a different type of inference, our question cannot be settled thus. Let us recall the meaning of the distinction between major and minor terms. The distinction is not purely formal and external. A term is not really the major term because it is made the predicate, and minor because it is made the subject, in a conclusion. It is the meaning of the terms themselves which determines which *ought to be* subject, and which predicate, and therefore which is major and which minor. Otherwise, Aristotle would have recognized the fourth as a separate figure. We may take a syllogism in Darii, and by transposition of the premisses produce one in Dimaris; e.g. the premisses *White is conspicuous at night*, *Some flowers are white*, whose natural conclusion is that *Some flowers are conspicuous at night*, furnish instead, if we transpose the premisses, the conclusion that *Some things conspicuous at night are flowers*. But this is an obvious inversion, for it is the flower which is conspicuous, and not the conspicuous, as such, which is a flower. It is true that there are cases where either conclusion is equally natural, as there are propositions which may be converted without contortion. *Those who are friendless are unhappy*, *Some rich men are friendless* ∴ *Some rich men are unhappy*; or, in Dimaris, *Some unhappy men are rich*. Here the conclusion in Darii is the natural conclusion to draw, because the premisses give the reason why a rich man is sometimes unhappy, but not why an unhappy man is sometimes rich; yet, considered apart from the premisses, either conclusion is an equally natural form of judgement. But the reason is, that the concrete subject *men* is retained throughout; in the conversion, the attributes *rich* and *unhappy* change places, but the subject of which they are attributes is retained in its place. Now these are merely coincident attributes, and neither is properly the subject of the other; we feel this in making the judgement; and instinctively convert *Some rich men are unhappy* not into *Some unhappy are rich men* (where the concrete term 'rich *men*' could not be predicated of 'unhappy' as such) but into *Some unhappy men are rich*. When, however, this is not the case—when the subject-concept contains the ground of the predicate-concept, or is the concrete whole in which the latter inheres as one feature—then the former is properly the minor and the latter the major term, and no verbal artifice which inverts them can alter what the fact is for our thought.

Hence in the first three moods of the fourth figure, reduction to the first does no more than recognize in outward form as major and as minor terms what we must acknowledge to be so in our thought. But in Fesapo and Fresison, the conclusion is the *same* as what we should draw in Ferio after their reduction, and not its converse; we have therefore no ground so far for giving a preference to the expression of the argument in the first figure. But the same con-

[siderations which make it not an arbitrary matter, which term is major and which is minor in the conclusion, will help us to determine the right position of the middle term in the premisses. If then the premisses of a syllogism in Fesapo or Fresison were both of them inversions of what would naturally be expressed in the converse form, we should instinctively think them back into the form required by the first figure, in drawing the conclusion. This can hardly occur with Fesapo; for bad logic, as well as verbal contortion, is required in order to express a particular affirmative by an universal converse; and therefore the minor premiss *A* cannot be an inverted way of stating *I*: the original of Fesapo cannot be Ferio. With Fresison it is more possible; that is to say, a syllogism in Fresison may be reached by converting both premisses of one in Ferio (or Celarent); and then it is possible that our thought may validate the conclusion by converting them back again. *Gold does not tarnish, Some ancient ornaments are of gold*: we may, however, say, if we like, that *What tarnishes is not gold*, and *Some things of gold are ancient ornaments*, and from these premisses draw the same conclusion as from the others, that *Some ancient ornaments do not tarnish*; yet our thought, justifying to itself an inference made by outward rule, may fly to the other forms of premiss. If so, it is hard to say that we are not really arguing in the first figure, and in such a case the syllogism which wears externally the garb of the fourth belongs really, and is rightly forced by direct reduction to show that it belongs, to the first. It is however possible even here to convert only the minor premiss in thought, and reach the conclusion in the second figure: by realizing that ancient ornaments, if they tarnished, would not be of gold. But the important cases are not such as these, where the premisses are palpably in an unnatural form, and would be restored to natural form by conversion. They are those in which the position of the middle term, as the predicate of the major premiss and subject of the minor, is the natural position. For here conversion to the first figure produces a result as unnatural as there conversion to the fourth figure produced in the premisses of an argument naturally belonging to the first; *No mineral waters are alcoholic* and *All alcohol is taxed* are propositions put in their natural form; *Nothing alcoholic is a mineral water* and *Some taxed things are alcoholic* are not.

And if that is so, there is only one ground on which we can justify Aristotle in reckoning these moods to the first figure. It is, that what is properly the major term—that is, the most general and comprehensive—does stand as predicate in its premiss, and what is properly the minor term—that is, the most concrete and specific—as subject. Hence looking to the character of the *premisses*, we may fairly say that our syllogism is of the first figure. And it follows that Aristotle is right when he says that we prove the minor, not universally but partially, of the major; for major and minor,

[as we have seen, are such intrinsically, and not barely in virtue of their position in the conclusion; so that where the two criteria lead to opposite results, it is right to base our nomenclature on the former. It was through overlooking this, and taking a purely formal and external view of the distinction of major and minor terms, that some of his successors were led to add a fourth figure to the three of Aristotle. But if we recognize these moods as of the first figure, we must no less recognize that they need validating; and the most natural way of realizing their validity is by the process of exposition which we found to be the characteristic method for the third figure. We need not on this account say that the syllogism belongs to the third figure. The occurrence of a syllogism of the first figure in the reduction *per impossibile* by which we validate the second did not lead us to resolve the second figure into the first. Exposition too, though the most natural, is not the only way in which we can realize to ourselves the validity of these arguments; so that the third figure could not receive them unchallenged. We must be guided, therefore, by the character of the premisses, and assign them to the first: but admit that the conclusion is not really drawn without a further act of inference than appears upon the face of them.]

We may now sum up the results of our enquiry. There are three figures, each with a distinctive character, and the 'imperfect' figures are misrepresented by reduction to the first. The first is the chief, because the demonstrative, but not because the only figure. Arguments in it need not be demonstrative, but when they are, our thought is moving on a higher level of intelligence, though not of cogency, than in the other figures. In realizing the validity of the second figure, the inconsistency involved in denying the conclusion is a more prominent 'moment' in our thought than the necessity of admitting it. The third figure appeals not to relations of concepts, but to experience of the conjunction of attributes (or their disjunction) in the same subject, and from that argues the general possibility, under conditions unspecified, of what is exhibited in a given case. There is no fourth figure; but in the first three moods of the first figure we may also argue to the converse of their conclusions; and two moods may be added, with an universal negative minor premiss, in which, while the major term cannot be denied of the minor without fallacy, the minor can be denied of the major; though such a conclusion is only particular, and realized by the help of conversion or of exposition or of reduction *per impossibile*. It must always be remembered that the character of an argument is determined not by the form into which it is thrown in words, but by that which it assumes in our thought. This is our justification for

recognizing the figures as distinct types. In particular cases, a syllogism may not belong to the figure into which it has been verbally compelled; in others, it may be possible with the same terms to construct syllogisms in more than one figure; but then there must be a real movement of thought in the process of conversion by which the change is effected. The theory of syllogism ought not to be regarded as a lesson in the manipulation of symbols and the application of the formulae. What we have to look to is the character of the thinking involved in it, and to that end we need to *realize* our symbols and see how the varying character of our terms, and of the relations between them in judgement, affects the inference. If our enquiry has done anything to bring this lesson home, its length and intricacy will not have been altogether vain.

[One further question about syllogism must be considered. It was said above (p. 311) of syllogism in the first figure that its use belongs to the stage of incomplete insight into the nature of facts. Yet inference depends on seeing the connexion of facts.[1] How then can we infer syllogistically? The same problem may be reached another way. We may see that a syllogism is valid, without knowing whether its premisses are true, or even knowing them to be false; or we may follow out syllogistic arguments with symbols, not knowing what they stand for.[2] Now to see the validity of an argument is a process of inference. How then can inference depend on seeing connexions of fact? Again, it is well known that, although a false conclusion cannot be validly drawn from true premisses, a true conclusion may be validly drawn from premisses one or both of which are false; here then we reach the truth by inference, yet clearly not by tracing out the connexions of fact.

The problem cannot be solved by distinguishing between the

[1] Cf. *supra*, p. 240.

[2] Neither this fact, nor the fact that the validity of an argument may be considered independently of the truth of the premisses, is confined to syllogism. Indeed all symbolic logic is an investigation of validity. Mr. Hugh MacColl (*Symbolic Logic*, § 52) makes the strange statement that 'it is a demonstrable fact that *not one syllogism* of the traditional logic—neither Darapti, nor Barbara, nor any other—is valid in the form in which it is usually presented in our text-books, and in which, I believe, it has always been presented since the time of Aristotle'. The reason he gives is that it asserts the premisses and the conclusion, as well as the implication of the latter in the former; and he thinks Barbara, invalid as commonly formulated, is valid in the form '*If* every *A* is *B*, and every *B* is *C*, *then* every *A* is *C*'. But that is exactly what is meant by saying that the traditional form is valid. To call an argument valid is not to call either its premisses or its conclusion true, but to say that if the premisses are true, the conclusion is true. The difficulty is, how we can know this, without knowing whether the premisses or conclusion are true.

[logic of consistency and the logic of truth. Doubtless in the theory of syllogism we have no more than an account of what conclusions we must admit, if we are to be consistent, when we have admitted certain premisses. That indeed is all we have in any attempt to formulate by the help of symbols types of argument that are found recurring with various real terms. But what is meant by the condition 'if we are to be consistent'? Consistency is not a matter of arbitrary convention; it is determined by what is possible in the nature of things. Inconsistency may be a disregard of the 'laws of thought'; but these, as we know[1], are laws of things. That consistency requires us to admit a certain conclusion if we have admitted certain premisses means then that the nature of things requires it. Yet if the premisses and conclusion are false, and if we are working with symbols, how have we the nature of things before us?[2]

The problem is partly one that arises in regard to all hypothetical thinking. In the inductive sciences, for example, we are constantly forming hypotheses, whose consequences we proceed to deduce, only to reject the hypotheses if their consequences differ from observed facts. Now here the premisses and the conclusion are both false, yet the inference from those to this is or should be sound, and clearly rests on perceiving connexions of fact. What is seen to involve certain consequences is something in the nature of the facts supposed.[3] Thus in support of the view that so-called acquired characters are not inherited it has been argued[4] that, if the conditions of town-life which injure the growth and health of the individual disposed the individual to produce correspondingly feebler offspring, and again this offspring to produce a yet feebler, and so continuously, the later generations of town-dwellers would be less resistent to the injurious influences of towns than the earlier, the stock, so far as not replenished from country districts, would die out, and infants with a long town ancestry, transferred to healthy surroundings and reared there, would grow up markedly feebler than infants of country ancestry reared with them. But these

[1] Cf. *supra*, p. 13.

[2] In *Mind*, vol. xix. N. S. 76, pp. 544-546, I drew a distinction between the ἀνάγκη εἶναι, the necessity for certain facts to be thus and thus, which is apprehended in demonstrative thinking, and the ἀνάγκη λέγειν, the necessity to say one thing if we have said another, which was alone considered in the formal treatment of syllogism. Professor J. A. Smith pointed out to me the futility of this distinction. What is meant by my being compelled to *say* anything? As far as talking goes, I can say what I please. But the compulsion here is a logical compulsion, not a moral or physical; and so I have not got away from apprehending connexions of fact. By admitting the premisses I am compelled to admit the conclusion only because, if what the premisses express exists, what the conclusion expresses exists also. But how can I see this connexion unless I am considering existents?

[3] I have learnt from Professor Cook Wilson the importance of this.

[4] By Dr Archdall Reid, *The Principles of Heredity*, pp. 335–337.

[things do not happen; the Jews e.g. have lived mainly in towns for many centuries, and often under worse conditions than most town-dwellers; yet they thrive better, not worse, in towns than men of other stocks. In this argument, it is because, or so far as, we understand what are injury, transmission, identity of relation between every two members in a series of terms, &c., that we can follow the deduction.[1] Whether in the facts of town-life prolonged over a succession of generations we have an example of all these is another question, and is the question that has to be settled. So in syllogism; we so understand what an universal relation is between one character M and another P, as to see that it involves the presence of P in any subject S exhibiting M. The man who sees that if the premisses of a given syllogism are true, the conclusion is true, moves by insight into relations which are displayed in some facts, even if not in the terms of his syllogism. And syllogistic reasoning can be used in 'indirect reduction' just as hypothetical reasoning is used in refuting theories in the inductive sciences.

We may get some further light from our procedure in geometry. When we wish to follow a geometrical demonstration about a triangle or circle, we draw a triangle or circle; but our power to follow the demonstration is not dependent on our figure being really triangular or circular. We are thinking, as Plato says,[2] not about the figure we draw, but about what it represents. So it is when we use symbols in working out syllogisms, and equally when we use premisses with real terms, that we know to be false. 'Whoever knows Greek is a compound householder, and all snapdragons know Greek $\therefore$ All snapdragons are compound householders' is a syllogism whose validity we can only grasp if we think about relations no more exemplified in the terms before us than is equality of radii in a badly drawn circle, but yet understood because exemplified in some real terms. It is still therefore on insight into connexions of fact that reasoning rests.

[1] If any one likes to say that the consequence depends on the fact that the relation of parent to offspring is an asymmetrical transitive relation, we can agree. But if we could think of no example of such relations, we could not think out their implications. Our reasoning therefore rests on our perceiving real connexions. This is so even in non-Euclidean geometries. We do not really know what would happen if straight lines making equal angles with the same straight line could meet; but when it is said that in consequence a triangle might contain more than two right angles, that is because we see that, where x is a positive quantity, $1 + 1 + x > 2$. In other words, we are guided by our insight into real connexions whenever we follow out the implications of a false hypothesis, whether the falsity of the hypothesis is only revealed by the discrepancy between its consequences and observed fact, or is self-evident. From an unintelligible hypothesis we cannot reason. We must reason therefore from what is intelligible in the hypotheses of meta-geometry; and this is relations and analogies exemplified in fact. Implication is connexion of fact, not of mere thought, or statement.

[2] *Rep.* vi. 510 D, E.

[As to drawing a true conclusion from false premisses (e. g., if in the syllogism in the last paragraph we put, for the term *compound householder*, the term *labiate*), if it is simply a question of seeing its validity, what has been said already applies here equally, and the truth of the conclusion is irrelevant. But if a man is led by arguing correctly from false premisses believed true to entertain a true belief, he has not been thereby enabled to understand the real connexion between the terms in his conclusion. All that he understands there at all is the subject-attribute relation, which he rightly supposes to be exemplified between the terms of the conclusion, without seeing the connexion there: as a man might rightly suppose, on others' testimony, that a portrait was like its original, because he knows from other examples what likeness is, though he had never seen the original of this portrait. He does not reach an understanding of the connexion which the conclusion states between its terms. That is impossible apart from the apprehension of the connexion of its terms with the middle term, and therefore impossible if its terms are not really connected with the middle term taken. Consequently in drawing a true conclusion from false premisses, not by way of logical exercise, but in actual life, when we express in premisses and conclusion what we do think, the inference does not really make use of the special (or material) nature of the terms, any more than the inference of the geometer makes any use of the irregularities or length or thickness or other special characters of the lines he draws; and the connexion alleged in the conclusion between just this nature, say being a snapdragon, and just that, say being a labiate, is not made manifest, and so, in strictness, not concluded, but we come to believe it for a reason independent of the nature of those terms. That reason is, that it happened to be by help of those terms and the middle term that we contemplated a certain connexion of relations displayed frequently in what is real, though not in those terms. We believe the conclusion, therefore, as Aristotle said, accidentally (κατὰ συμβεβηκός). Only by studying the structure of snapdragons and other flowers, and detecting in them a common character variously modified, should we understand that they are labiates.]

CHAPTER XV

OF HYPOTHETICAL AND DISJUNCTIVE REASONING

THE form of argument which we have been examining under the name of Syllogism has for its premisses only categorical propositions; but there are forms of argument to which the name has been extended, in which this is not so. In what have been called Hypothetical and Disjunctive Syllogisms, hypothetical and disjunctive propositions figure in the premisses. For reasons, however, to be considered later, it appears better not to call them syllogisms, but to speak rather of hypothetical and disjunctive *arguments*. They are processes of argument that recur with great frequency both in ordinary thought and in the reasonings of science.

In a **hypothetical argument**, one premiss is a hypothetical proposition, connecting a *consequent* with a *condition* or *antecedent*: the other is a categorical proposition,[1] either affirming the antecedent or denying the consequent. From these follows as conclusion a categorical proposition, either affirming the consequent or denying the antecedent. In the former case, the argument is said to be in the **modus ponens** or **constructive**: in the latter case, in the **modus tollens** or **destructive**. Examples will make this clear.

1. The *modus ponens* is of the form :—

If A is B, it is C	If A is B, C is D	If A is C, B is C
A is B	A is B	A is C
$\therefore A$ is C	$\therefore C$ is D	$\therefore B$ is C

e. g

If the soul is uncreated, it is indestructible
The soul is uncreated
$\therefore$ It is indestructible

or

If all men are born equal, slavery is unjust
All men are born equal
$\therefore$ Slavery is unjust

[1] But cf. *infra*, p 337, iii.

or If men have obligations towards their friends, they have them towards their enemies
Men have obligations towards their friends
∴ They have them towards their enemies.

The following points should be noted further :—

i. The subject of the minor premiss may either, as in the foregoing examples, be the same as the subject of the antecedent in the major premiss (if we may retain the name of major for the hypothetical and of minor for the categorical premisses in this form of argument), or it may be a term that we recognize as included therein, falling under it. Thus we may argue that

If a beautiful thing is rare, it is costly
Diamonds are rare
∴ They are costly.

Here it is implied and recognized that diamonds are beautiful things. The argument might of course be expressed

If anything is at once beautiful and rare, it is costly
Diamonds are at once beautiful and rare
∴ They are costly.

But diamonds are still 'subsumed' as a special case under a rule that applies beyond them; the condition in the major premiss does not concern *them in particular*.

ii. We saw in a previous chapter that the distinction of affirmative and negative has no application to hypothetical judgements—for every hypothetical judgement *connects* a consequent with a condition, whether that consequent itself be expressed in the form of an affirmative or of a negative statement: it would be no hypothetical judgement to say that 'If the weather changes at full moon, it does *not* follow that the change will last'.[1] Hence the character of the *modus ponens* is unaltered, whether the antecedent or the consequent (and therefore the conclusion) be affirmative or negative. I may argue

If the North American colonies were unrepresented in Parliament, they ought not to have been taxed by Parliament
They were unrepresented in Parliament
∴ They ought not to have been taxed by Parliament.

[1] This is the denial of a hypothetical judgement, but not itself hypothetical: being equivalent to saying 'It is not true that if', &c.

Here my conclusion is negative; but the argument is still in the *modus ponens*: for by that is meant not the mood which is affirmative in its conclusion, but the mood which *establishes* the consequent set down in the major premiss. The reader will easily see that if the antecedent were of the form 'If A is not B', it would still make no difference to the character of the argument.

iii. It is possible to argue with both premisses and the conclusion hypothetical, in the form :—

If A is C, it is D	If C is D, E is F	If A is D, B is D
If A is B, it is C	If A is B, C is D	If B is D, C is D
$\therefore$ If A is B, it is D	$\therefore$ If A is B, E is F	$\therefore$ If A is D, C is D

e.g. If the price of an imported article rises, those who manufacture the same article at home will charge more for it

If a tax is imposed upon the importation of an article, the price of the imported article rises

$\therefore$ If a tax is imposed upon the importation of an article, those who manufacture the same article at home will charge more for it.

The remarks made in the last paragraph apply, *mutatis mutandis*, to this form of the *modus ponens* also; and the subject of the antecedent may be in one premiss the same with that of the consequent, and in the other different. It is unnecessary to illustrate all these variations.

2. The *modus tollens* is of the form :—

If A is B, it is C	If A is B, C is D	If A is C, B is C
A is not C	C is not D	B is not C
$\therefore$ It is not B	$\therefore$ A is not B	$\therefore$ A is not C

e.g. If matter is indestructible, it is uncreated

Matter is not uncreated

$\therefore$ It is not indestructible

or If the earth did not rotate, the winds that blow from the poles to the equator would not be deflected westward

But they are deflected westward

$\therefore$ The earth does rotate.

or If any one has a natural right to a vote, every one has

Not every one has

$\therefore$ No one has.

It is plain that the observations made above with regard to the

modus ponens are equally applicable, *mutatis mutandis*, to the *modus tollens*.

Thus, given a hypothetical proposition, we can proceed to draw an inference whenever we have a further premiss given us, either *affirming the antecedent* or *denying the consequent*. But from the affirmation of the consequent, or the denial of the antecedent, no conclusion follows. Arguments of the form

If A is B, it is C
A is C
$\therefore$ It is B

or A is not B
$\therefore$ It is not C

are invalid. It is true that if a member of the Commons House of Parliament is declared a bankrupt, he loses his seat; but it is not true that if he loses his seat, it must be because he has been declared a bankrupt, or that if he is not declared a bankrupt, he may not still lose his seat. For the connexion of a consequent with a condition does not preclude the possibility that there are other conditions upon which the same consequent may follow; so that the fact of the consequent having occurred is no proof of this particular condition; nor is the fact that this particular condition is not fulfilled any proof that the consequent has not occurred in virtue of the fulfilment of some other condition with which it is connected. Obvious as these considerations are, yet these are among the commonest errors to occur in men's reasonings. We are all of us apt to conclude, that by disproving the allegations advanced in support of a proposition, we have disproved the proposition itself; or that by showing that facts agree with the consequences of some hypothesis which we have formed, we have established the truth of that hypothesis. We do not realize that it would be necessary to show, not only that the facts agree with the consequences of our hypothesis, but that they do not agree with the consequences of any other. The Teutonic races have during the last three centuries increased and expanded faster than those which speak languages of Latin stock; and some may be inclined to attribute this to the fact that the former in the main embraced, while the latter rejected, the principles of the Reformation. Grant that the facts are consistent with the hypothesis that this difference of growth is due to a difference of religion; yet if there are other ways of explaining it, what

ground has yet been shown for accepting that way? When facts are equally consistent with the truth and with the falsity of our hypothesis, we have so far no reason for believing it true.

It is then fallacious to draw any inference from the affirmation of the consequent, or the denial of the antecedent, in a hypothetical argument. It is sometimes said that to do the former is to commit the fallacy of undistributed middle; and to do the latter, to commit the fallacy of illicit process of the major term: for that the argument

If A is B, it is C
A is C
$\therefore A$ is B.

may be exhibited in the form

$A\ B$ is C
A is C
$\therefore A$ is $A\ B$

and the argument

If A is B, it is C
A is not B
$\therefore A$ is not C

may be exhibited in the form

$A\ B$ is C
A is not $A\ B$
$\therefore A$ is not C

And valid hypothetical arguments, it is said, may be similarly 'reduced' to categorical syllogisms; when it will be found, that the *modus ponens* is really a syllogism in Barbara, and the *modus tollens* one in Camestres.[1]

It seems to be an error thus to identify hypothetical reasoning with syllogism. In syllogism, as we have seen, a relation is established between two terms in the way of subject and predicate, by means of their common relation in the way of subject and predicate to a third or middle term. Hypothetical reasoning rests upon another relation than that of subject and predicate—the relation of condition and consequent; and there is not necessarily any middle term. Where antecedent and consequent, in the hypothetical

[1] A number of modern text-books teach this doctrine. For an older authority cf. Zabarella, *In Lib. Prior. Anal. Tabulae*, p. 158, 'syllogismus hypotheticus an valeat necne cognoscitur per eius reductionem ad categoricum.'—*Opera Logica*, Coloniae, 1597.

premiss, have the same subject—where that proposition is of the form 'If *A* is *B*, it is *C*'—a middle term may at times be found, and the reduction effected; but where that is not so—where it is of the form 'If *A* is *B*, *C* is *D*' or 'If *A* is *C*, *B* is *C*'—there a middle term is wanting, and the violent nature of this process of reduction becomes manifest.

'If the value of gold is affected by the amount of labour needed to obtain it, improvements in mining machinery must raise prices. The value of gold is affected by the amount of labour needed to obtain it. Therefore improvements in mining machinery raise prices.' We are not concerned here with the truth of this hypothetical proposition. So many circumstances, many of them varying independently of one another, combine at any time to affect the course of prices, that it would be hard to rest on observation the effect which it is here asserted that improvements in mining machinery ought to have. Our concern, however, is with the character of the argument; it is clearly difficult to reduce it to a syllogism. There is nothing asserted of improvements in mining machinery, which in turn is asserted universally to raise prices; the connexion between the value of gold and the amount of labour needed to obtain it is not a predicate of improvements in mining machinery, nor is raising prices a predicate of that connexion. It is a consequence of it; but that is another matter. Attempts have indeed been made to get round this difficulty. It is said that the major premiss may be expressed in the form 'The case[1] of the value of gold being affected by the amount of labour needed to obtain it is the case of improvements in mining machinery raising prices. The existing case is the case of the value of gold being affected by the amount of labour needed to obtain it. Therefore the existing case is the case of improvements in mining machinery raising prices.' But such linguistic *tours de force* do not alter the nature of the argument which they conceal. What does that major premiss mean? Interpreted literally, it is undoubtedly false. Modification in the value of gold, because gold has become easier or harder to obtain, is not a rise in prices due to improvements in mining machinery. The one fact may be dependent on the other, but the one *is not* the other. It is not therefore until we mentally substitute for this premiss the hypothetical proposition it attempts to

[1] Had I written, for *the case*, *all cases*, the proposition would have been still more absurd. But the contention should be examined in its strongest form.

supersede, that we assent to it at all; the 'reduction' is purely verbal; our *meaning* remains unchanged, and cannot be put into the categorical form. Nor does the minor premiss stand criticism any better. *What* case is 'the case of the value of gold being affected by the amount of labour needed to obtain it'? To say 'the existing case' is useless, unless we are told what the existing case is a case of. If it is a case of the value of gold being affected by the amount of labour needed to obtain it, the proposition becomes tautological, and the conclusion will only repeat the major premiss [1]: if it is a case of something else, we ought in the first place to have that something stated, in order that we may know what the proposition means; and in the second place, when it was stated, we should find the proposition had become false, in the same way as the major premiss, literally interpreted, was false. It is clear then that this syllogism is far from exhibiting more correctly the true character of the hypothetical argument in question; on the contrary, the hypothetical form exhibits the true nature of the argument thus violently forced into a syllogism.

Had we indeed taken an example in which the subject of the antecedent was the same with the subject of the consequent in the major premiss—in which, to put it otherwise, the major premiss was of the form 'If A is B, it is C': then the process of reduction to syllogism would not have appeared to be so difficult or violent. For then the condition on which it depends that A is C is a condition *fulfilled in A*. 'If the moon rotates in the same period as it revolves, it must present always the same face to the earth. It does rotate in the same period as it revolves. Therefore it does present always the same face to the earth.' 'If Christian nations had the spirit of Christ they would avoid war. They do not avoid war. Therefore they have not the spirit of Christ.' There is little change made, if we substitute for these arguments the following syllogisms:

A body rotating in the same period wherein it revolves round another body presents always the same face to the other
The moon is a body rotating in the same period wherein it revolves round the earth [2]
∴ The moon presents always the same face to the earth

[1] The case of A is the case of B: the existing case of A is the case of A: therefore the existing case of A is the case of B.

[2] It will be seen that in this minor premiss not only is the *moon* 'subsumed' under the more general notion of a *body* rotating, &c.: but the *earth* is also

and Those who have the spirit of Christ avoid war
Christian nations do not avoid war
∴ Christian nations have not the spirit of Christ.

Indeed, if it be granted that the hypothetical premiss is unaltered, otherwise than in verbal form, by reduction to the form of a categorical proposition, we must grant that the argument is unaltered by reduction. And there are logicians who have contended that all universal judgements are really hypothetical[1]; from which it would follow that there is no real difference between a syllogism in Barbara or Camestres, when it has a genuinely universal (i. e. not a merely enumerative) major premiss, and a hypothetical argument in the *modus ponens* or the *modus tollens*—though the former rather than the latter would demand reduction. Yet there do seem to be some judgements which, in their context, intend to affirm the existence of the subject about which assertion is made, and not merely to assert that something would be true about it if it existed. To say that, if Christian nations had the spirit of Christ, they would avoid war, leaves it an open question whether any have that spirit; to say that those who have the spirit of Christ avoid it, naturally implies that there are such. The reduction of a hypothetical argument to a syllogism is no merely verbal change, if it substitutes one of these forms of statement for the other.

Attention ought to be called to one other change incidental to this reduction in the last two examples. Our hypothetical major concerned the moon and the earth, or Christian nations: in the syllogism, the major concerned any two bodies in which certain conditions are fulfilled, or any in whom the spirit of Christ is found. Thus in the syllogism, a principle is stated in more general form than in the hypothetical proposition. Here again, more than a merely formal change is involved. It is true that no one could assent to the proposition, that if the moon rotates in the same period wherein it revolves, it must present always the same face to the earth, without seeing that its truth has nothing to do with the fact that the bodies

subsumed under the more general notion of the *other body*. Hence it is difficult to express the argument completely in symbols. Suppose that we write 'Any X is Y, the moon is X ∴ the moon is Y': now here, in the major premiss, X = 'body rotating in the same period wherein it revolves round another body'; in the minor premiss, X = 'body rotating in the same period wherein it revolves round the earth'; and similarly with Y. The argument is none the less a syllogism; the difficulty is linguistic; but we are really bringing the *moon in its relation to the earth* under the condition of a rule. Aristotle recognizes this: cf. *Post. An.* β. xi. 94ᵃ 36–ᵇ7.

[1] Cf. pp. 183, 185, n. 1, *supra*.

in question are the moon and the earth, but holds equally for any two bodies; so that the more general form of the universal categorical proposition given above is obviously justified. Yet it is not the mere form of the hypothetical judgement which enables us to see this; and it might be contended in the other case that the more general form of the categorical judgement is not justified, and that we ought not to have said more than that 'Nations which have the spirit of Christ avoid war'. It might be said that if a Christian *nation* had the spirit of Christ, it would avoid war; but that an *individual* may be morally bound to take part in warfare, though he has that spirit, when the nation to which he belongs has it not. Now there is, doubtless, in every true hypothetical judgement of the form 'If A is B, it is C', some general principle involved: we may express this as '$\alpha\ \beta$ is γ'. But if A is some determinate individual, or case of a particular kind, and if the condition B is similarly determinate, we may know that if A is B, it is C, without knowing generally what conditions β, occurring in what kind of subject α, will involve the predicate γ. Where this is the case the hypothetical form is more natural to the expression of our argument than the syllogistic.[1]

We find, then, that even when antecedent and consequent have the same subject in a hypothetical major, reduction of the hypothetical argument to syllogism may mean a real change in the nature of the argument used; and that where they have different subjects, such reduction can only be effected to outward appearance, and by violent means: for here the condition on which it depends that C is D is not a condition asserted to be realized in the nature of C itself; in other words, there is no middle term.[2] No doubt

[1] If the subject of the antecedent in the hypothetical premiss be a singular term, and we know of no general term under which it falls which can be substituted as subject in its stead, the impossibility of reducing a hypothetical argument to syllogism is specially obvious; for we cannot replace such a hypothetical proposition by any categorical proposition. 'If he marries her, he will be happy; he will marry her ∴ He will be happy'—is an example in point.

[2] The inference in a hypothetical argument might hence be called *immediate*; but such an expression would readily give rise to misunderstanding. It is immediate in the sense of having no true middle term: and in this it differs from syllogism; it is also immediate in the sense, that given the premisses, nothing more is needed in order that we may see the necessity of the conclusion: and in this sense, syllogism, and indeed every step of valid argument when fully stated, is immediate. But it was in yet another sense that the processes of conversion, &c., were called immediate, and distinguished from syllogism: viz. that in them we passed from a *single*

there is an unity embracing both condition and consequent; they belong to a system, of which it might be said that, when affected by the condition, *it* exhibits the consequence. Sometimes this admits of ready expression. 'If the rainfall is deficient, the hay-crop is light': we may express this by saying that 'Grass which is insufficiently supplied with moisture makes only a small growth that can be used for hay'. In other cases, the interconnexion of facts within a whole does not admit of being stated except in hypothetical form. And anyhow, it must be contended that hypothetical reasoning is not identical in character with syllogism, and that we ought not to pretend to validate it by reducing it to syllogism, nor to identify the fallacies involved in argument from the denial of the antecedent or the affirmation of the consequent with the syllogistic fallacies of illicit process of the major term or undistributed middle.

In a **disjunctive argument,** one premiss is a disjunctive proposition; the other is a categorical proposition, affirming or denying one of the alternatives in the former. From these follows as conclusion a categorical proposition, denying or affirming the other alternative. In the former case, the argument is said to be in the

proposition to another inferred therefrom, without anything further being required as a means of reaching the conclusion. (Cf. *supra*, p. 232, n. 3). Hypothetical arguments are not immediate in this sense. Given that 'If A is B, it is C', I cannot conclude that A is C, unless I also know that A is B: nor could I conclude that A is C, from the fact that A is B, without the hypothetical premiss. I can, however, conclude from 'If A is B, it is C' to 'If A is not C, it is not B', without any further knowledge: and to this we saw that some forms of so-called immediate inference amounted.

The conditions of valid hypothetical reasoning are of course recognized by Aristotle (cf. e. g. *Top.* β. iv. 111^b 17–23 *et al.*); but he does not speak of hypothetical syllogisms. The term συλλογισμὸς ἐξ ὑποθέσεως has a different meaning—viz. a syllogism proving the antecedent of a hypothetical proposition, and therefore, *by virtue of the acceptance of that hypothesis*, proving the conclusion. Let it be granted that if A is B, C is D: then any syllogism which proves that A is B will by virtue of this agreement establish also that C is D: but without such agreement, it would not have been shown at all that C is D: that is therefore said to be proved only *ex hypothesi*. In a case between University College, Oxford, and the City of Oxford (v. *Times* of July 5, 1904) arising out of a claim by the College to put a bridge between two blocks of buildings on either side of a narrow street called Logic Lane without payment of any acknowledgement to the City, it was agreed that if the soil of Logic Lane were vested in the College, the College was entitled to do this (subject to any building regulations which the City had power to make); the arguments advanced on behalf of the College (which established its case) were directed to show that it was owner of the soil; but, ἐξ ὑποθέσεως, the College showed by the same arguments that it was entitled to erect the bridge without acknowledgement.

modus ponendo tollens: in the latter case, in the **modus tollendo ponens.** Examples and observations follow.

1. The *modus ponendo tollens* is of the form

A is either *B* or *C*	Either *A* is *B* or *C* is *D*	Either *A* or *B* is *C*
A is *B* or	*A* is *B* or	*A* is *C*
∴ It is not *C*	∴ *C* is not *D*	∴ *B* is not *C*

e.g. 'Possession by devils' is either a form of mental derangement, or supernatural
It is a form of mental derangement
∴ It is not supernatural

or Either the interests of religion require the maintenance of the Temporal Power, or the Popes are actuated by worldly motives in continuing to claim it
The interests of religion do require its maintenance
∴ The Popes are not actuated by worldly motives in continuing to claim it

or Either Newton or Leibniz invented the calculus
Newton invented it
∴ Leibniz did not

2. The *modus tollendo ponens* is of the form

A is either *B* or *C*	Either *A* is *B* or *C* is *D*	Either *A* or *B* is *C*
A is not *B* or	*A* is not *B* or	*A* is not *C*
∴ It is *C*	∴ *C* is *D*	∴ *B* is *C*

e.g. The belief in a golden age rests either on history or on hope
It does not rest on history
∴ It rests on hope

or Either God is unjust, or no man is eternally punished
God is not unjust
∴ No man is eternally punished

or Either Aristotle or Eudemus wrote Bks. v, vi, vii of the *Nicomachean Ethics*
Eudemus did not write them
∴ Aristotle did write them.

The following points should be noted :—

i. It is sometimes contended that the *modus ponendo tollens* is

invalid : that the affirmation of one alternative does not justify the denial of the other. This will depend on the interpretation given to the disjunctive proposition. If the alternatives therein stated are mutually exclusive, the argument is valid : if otherwise, it is not. Whether they are so intended can only be determined in a given case by reference to the context and the matter of the judgement ; but mutually exclusive alternatives may exist, and therefore a valid argument in this mood is possible. Of the examples given above, the third is clearly the most open to objection ; for Newton and Leibniz may well have invented the calculus independently, as they are now believed to have done. In the first, it is implied that if we can otherwise account for the phenomena of demoniacal possession, we shall not attribute them to supernatural agency ; and the argument may be considered valid, provided that we are justified in that view.[1] The second is more doubtful ; men may do from bad motives what ought anyhow to be done, and the motives of the Popes in maintaining their claim to temporal power might be worldly, even though their possession of it were required in the interests of religion. The premisses do not really prove the unworldliness of their motives ; but they show that we need not assume the contrary, in default of further evidence. The validity of the present mood of disjunctive argument will, in fact, depend on what hypotheticals are implied in its disjunctive premiss ; for we have seen (p. 187, *supra*) that the disjunctive judgement '*A* is either *B* or *C* ' may imply, though it is not reducible to, the hypothetical judgements ' If *A* is *B*, it is not *C* ', ' If *A* is *C*, it is not *B* ', ' If *A* is not *B*, it is *C* ', and ' If *A* is not *C*, it is *B* '. If the alternatives are mutually exclusive, all four will be implied, and the *modus ponendo tollens* will be valid. If not, we cannot get, out of the proposition ' *A* is either *B* or *C* ', the propositions ' If *A* is *B*, it is not *C* '—' If *A* is *C*, it is not *B* '. To say that ' Either the interests of religion require the maintenance of the Temporal Power, or the Popes are actuated by worldly motives in continuing to claim it ' will mean that if the interests of religion do not require it, they must be so actuated ; but not that if the interests of religion do require it, they cannot be so actuated ; and therefore to argue from the premiss that the interests of religion do require it is to argue from the denial of the antecedent in a hypothetical argument.

[1] The *argument* may be valid even though the *conclusion* be false : the truth of the conclusion further presupposes that of the minor premiss.

Here we might leave this matter, with this as our result—that the validity of the *modus ponendo tollens* depends on the alternatives in the disjunctive premiss being mutually exclusive, and that there is no way of determining on merely formal considerations whether they are so [1]; that the form of argument is not universally invalid, because they may be so; but not universally valid, because they may not. It is, however, worth noticing that quite independently of this doubt about the validity of the *modus ponendo tollens* in any given case, the *modus tollendo ponens* is of more importance on other grounds. We are more often interested in proving one alternative by disproof of others, than vice versa. A prisoner indicted on a charge of murder may indeed be content to show that, whoever committed the crime, he did not; and his ends may be satisfied by proving an *alibi*. But the ends of justice are not satisfied except by discovering the murderer. And so it is with disjunctive argument generally; its use lies more in what it can establish than in what it can overthrow.

ii. As in hypothetical, so also in disjunctive argument, the major premiss may make a more general assertion, which in the conclusion is applied to some special case. Thus a man might argue

> Every man at forty is either a fool or a physician
> My son at forty is not a physician
> ∴ He is a fool

or from the premiss 'Either God is unjust, or no man is eternally punished', I might have concluded that I shall not be eternally punished.[2]

iii. The mood of a disjunctive argument is not affected, any more than the mood of a hypothetical argument, by the quality—

[1] It might be said that we could give an unambiguous form to the argument by writing it thus: '*A* is either *B* only, or *C* only, or both *B* and *C*: it is *B* only ∴ it is neither *C* only, nor both *B* and *C*.' But here there seems to be no inference; for if we already know that it is *B only*, we must already know that it is not *C*. The inference rests upon the knowledge that *A* is *B*, and that *B* and *C* are mutually exclusive: if we are doubtful of the latter point, and only know that *A* is *B*, we cannot tell whether it is *C* or not: and this information is all that we have; we must not substitute for the minor premiss '*A* is *B*' a different one, '*A* is *B* only'.

[2] The subsumption involved may be expressed if we like in a separate and syllogistic argument: thus

> Every man at forty is either a fool or a physician
> I am forty
> ∴ I am either a fool or a physician: but I am not a physician, &c.

affirmative or negative—of the minor premiss or the conclusion. Arguments of the type

A is either B or C
A is not B
$\therefore$ It is C

are in the same mood as those of the type

A is either not B or not C
A is B
$\therefore$ It is not C

I establish one alternative by way of rejecting the other, equally whether from the premisses

A diplomatist must either be insincere or fail
Bismarck did not fail

I conclude that he was insincere, or whether I conclude that he was not honest from the premisses

A diplomatist is either not honest, or not successful
Bismarck was successful

Attempts have been made to reduce disjunctive arguments also to syllogistic form. We have seen that a disjunctive proposition implies two or perhaps four hypotheticals; and every disjunctive argument can be exhibited as a hypothetical argument using for major premiss one of these. But as hypothetical argument is not syllogism, we do not thereby make disjunctive argument into syllogism; nor do we really identify it with hypothetical argument; for the hypothetical major premiss expresses only a part of the meaning of the disjunctive proposition, from a perception of the relations involved in which a disjunctive argument proceeds to draw its conclusion.[1]

and having reached the conclusion 'No man is eternally punished', I can with the minor premiss 'I am a man' draw the conclusion that I shall not be eternally punished. This act of subsumption is a different act of inference from the disjunctive argument.

[1] The term *hypothetical* was long used (following Boetius) *sensu latiore*, to cover both what have in this chapter been called hypothetical and what have been called disjunctive arguments; and for hypothetical, in the narrower sense employed above, the term *conjunctive*. *Conditional*—originally equivalent to *hypothetical* in the wider sense—has by some who retained the wider sense for the latter been used as equivalent to *conjunctive* (cf. Sir W. Hamilton's *Discussions*, p. 150). A few points may be noted here which did not seem worth a place in the text.

1. The order in which the alternatives in the disjunction are mentioned

being irrelevant, it makes no difference to the nature of the argument whether we proceed from the affirmation of the first to the denial of the second, or from the affirmation of the second to the denial of the first.

2. A disjunction may contain more than two members: e. g. it may be of the form *A* is either *B* or *C* or *D*. In this case, if the minor is categorical, the conclusion will be disjunctive; and in the *modus ponendo tollens*, a disjunctive minor will give a categorical conclusion—*A* is either *B* or *C* ∴ it is not *D*. But the minor '*A* is neither *B* nor *C*', which is needed in order to get a categorical conclusion in the *modus tollendo ponens*, is not a disjunctive proposition. But such details involve no fresh principle of reasoning, and need not be pursued, any more than it is necessary to work out all the variations that are possible according as the disjunction is between two predicates of the same subject, or two subjects of the same predicate, or two assertions differing both in subject and predicate, when either or both assertions in each of these cases are affirmative or negative.

3. An argument of the form '*A* is either *B* or *C*: *C* is either *D* or *E* ∴ *A* is either *B* or *D* or *E*' is not a disjunctive argument, but the application of syllogism to one limb of a disjunctive proposition.

CHAPTER XVI

OF ENTHYMEME, SORITES, AND DILEMMA

This chapter deals with certain forms or modes of stating an argument which introduce no new principle of reasoning beyond those now already discussed, but for one reason or another deserve a special name and mention.

An **enthymeme** indeed is not a particular form of argument, but a particular way of stating an argument. The name is given to a syllogism with one premiss—or, it may be, the conclusion—suppressed.[1] Nearly all syllogisms are, as a matter of fact, stated

[1] By Aristotle the term *ἐνθύμημα* is defined as *συλλογισμὸς ἐξ εἰκότων ἢ σημείων*, *Anal. Pri. β.* xxvii. 70ª 10. Its nature is discussed in that chapter and in various passages of the *Rhetoric*. Roughly speaking, *εἰκός* is a general proposition true only for the most part, such as that *Raw foods are unwholesome*; in applying this to prove the unwholesomeness of some particular article of diet, we are open to the objection that the article in question may form an exception to the rule; but in practice we are often compelled to argue from such probable premisses. A *σημεῖον* is *either* a particular fact, to which one can appeal in support of a general proposition, because if the proposition were true, the fact would follow as a consequence of it: thus we may argue that 'The wise are just, for Socrates was wise and just': where Socrates is the *σημεῖον* (*Rhet. a.* i. 1357ᵇ 11); *or* it is a particular fact appealed to as evidence of another particular fact, because the existence of one such fact implies the previous or subsequent or concurrent existence of the other: thus 'Pittacus is liberal, because ambitious men are liberal, and Pittacus is ambitious': here his ambition is the *σημεῖον* of his liberality (*Anal. Pri. β.* xxvii. 70ª 26). In this case, the appeal to a *σημεῖον* implies a general principle which, if it is irrefragable, gives to the *σημεῖον* the nature of an evidence, or *τεκμήριον* (*Rhet. a.* ii. 1357ᵇ 3); to argue from a *τεκμήριον* is not, however, to argue from the true cause of the effect; for this would be scientific syllogism, and not *ἐνθύμημα*. It may be added that, where the general principle implied is not irrefragable, but true for the most part, it is hard to distinguish the *συλλογισμὸς ἐκ σημείου* from a *συλλογισμὸς ἐξ εἰκότος*. It should be noted that Aristotle includes under *σημεῖον* that which, as a consequence of something else, is assumed, where it exists or occurs, to presuppose it, whether it could exist or occur without the existence or occurrence of that other thing or not; where it could not, we have a *τεκμήριον*; and of this character are what doctors call the symptoms of a disease (and such reasoning from effect to cause is not 'scientific'); where it could, the argument—as Aristotle recognizes—is not really valid; it may be true that persons in a fever breathe rapidly, but I cannot safely infer that a person who breathes rapidly has fever (*ib.* 1357ᵇ 19); there are, of course, symptoms of disease that are of doubtful interpretation. The *ἐνθύμημα* is said to be a rhetorical demonstration, or rhetorical syllogism (*Rhet. a.* i. 1355ª 6, ii.

as enthymemes, except in the examples of a logical treatise, or the conduct of a formal disputation. It must not be supposed, however, that we are the less arguing in syllogism, because we use one member of the argument without its being explicitly stated. Syllogism is an act of thought, and if, in order to perform this act, we need to recognize in thought all three propositions, we are arguing syllogistically, whether we enunciate the whole syllogism or not. That we do recognize a suppressed premiss may be shown by the fact that, if any one were to deny it, we should feel that he was attacking our argument, though we had not expressly asserted it.

The suppressed member may be the major premiss, or the minor, or—less frequently—the conclusion. Medea, in Ovid's play of that

1356^{b} 4), because public speakers make use of the appeal to such probable premisses or signs, and do not expect or provide more strictly demonstrative or scientific arguments. But they also commonly present their thought without enunciating all three propositions of a syllogism, whereas in a set debate one endeavours to get both premisses explicitly admitted, and so establish a conclusion. And Aristotle, in distinguishing the arguments of the platform from those of such debate, probably had in mind both the sort of premisses to which orators appeal, and the mode in which they present their arguments. The chapter (*Rhet. β.* xxi) on γνῶμαι, *sententiae*, or apophthegms, describes a γνώμη as part of an enthymeme, because, when it is justified by another proposition, we have an enthymeme; e. g. 'There is no man free' is a γνώμη, but when we add 'For each is a slave to money or to fortune', there is an enthymeme (1394^{b} 4–6). So a γνώμη which includes the ground of its own statement is called enthymematic, e. g. *ἀθάνατον ὀργὴν μὴ φύλαττε θνητὸς ὤν* ('Nurse not immortal anger, being mortal'). Both the character of the premisses and its incomplete statement seem then to distinguish the enthymeme from other syllogisms, according to Aristotle. That, in spite of the definition quoted from *Anal. Pri. β.* xxvii, above, the latter feature has come to determine the use of the term, may perhaps be due to a later passage in the same chapter, 70^{a} 24–28 *ἐὰν μὲν οὖν ἡ μία λεχθῇ πρότασις, σημείων γίνεται μόνον, ἐὰν δὲ καὶ ἡ ἑτέρα προσληφθῇ, συλλογισμός, οἷον ὅτι Πιττακὸς ἐλευθέριος· οἱ γὰρ φιλότιμοι ἐλευθέριοι, Πιττακὸς δὲ φιλότιμος* ('If the one premiss be stated, there is a sign only, but if the other be taken also, a syllogism: e. g. Pittacus is generous; for the ambitious are generous, and Pittacus is ambitious'). This, however, seems merely to mean that, if I say 'Pittacus is generous, because he is ambitious', I only state the sign; whereas, if I add that the ambitious are generous, I make a syllogism; but this syllogism was implied all along, and is an *ἐνθύμημα*, whether I state it in full or not, because of the character of the premisses. A demonstrative syllogism could not be so called by Aristotle, even though incompletely expressed: e. g. 'The moon is liable to eclipse, because it may be hidden by the earth from the sun'. Cf. on the question Cope, *Introduction to Aristotle's Rhetoric*, p. 103, n. The term enthymeme has more commonly been applied to a syllogism omitting one of the premisses, than to one omitting the conclusion. Sir W. Hamilton (*Discussions on Philosophy, &c.*, pp. 153–158) traces the antiquity of the non-Aristotelian use of the term. It goes back to the oldest of the commentators.

name, asks Jason—*Servare potui, perdere an possim rogas?* here the major premiss *Qui servare possunt, perdere possunt*, is understood: Medea supplies only the minor, and—in the form of a rhetorical question—the conclusion.[1] If I argue that 'those cultivate the land best who have a personal interest in its improvement, and therefore peasant proprietors are the best cultivators', I omit—yet I clearly use, for to deny it would destroy the argument—the minor premiss, that 'peasant proprietors have a personal interest in the improvement of the land'.[2] The conclusion may be omitted from motives of delicacy, or sometimes for purposes of effect, as in the Greek couplet

καὶ τόδε Φωκυλίδου· Λέριοι κακοί, οὐχ ὁ μέν, ὃς δ' οὔ,
πάντες, πλὴν Προκλέους· καὶ Προκλέης Λέριος.[3]

It is, of course, possible that an enthymeme may be contained in what *grammatically* is only a single sentence; as in Goneril's address to King Lear:

You, as you are old and reverend, should be wise,

or in Regan's, later in the play:

I pray you, father, being weak, seem so.

A syllogism, whether expressed in full or as an enthymeme, is a *single* act of inference; it may be analysed into premisses and conclusion, but not into parts which are themselves acts of inference. The premisses may, however, be themselves in turn conclusions reached by other acts of inference: and the conclusion may itself serve as premiss to a further act of inference. A syllogism proving one of the premisses of another syllogism is called, in relation to that, a **prosyllogism**: and a syllogism using as a premiss the conclusion of another is called, in relation to it, an **episyllogism**; where the prosyllogism is expressed in the form of an enthymeme, the whole argument is sometimes called an **epicheirema.**[4] The

[1] This example is used in the *Port Royal Logic*. Pt. III. c. xiv.

[2] I am inclined to think it would be found that the major premiss is more frequently suppressed when the conclusion of the enthymeme is put in the forefront, the minor when we begin with a reason. If we begin with a reason, we like to lay down a general principle.

[3] 'This too is from Phocylides: The Lerians are bad men, not this one only and not that, but all of them except Proclees; and he is a Lerian.'

[4] *v.* Mansel's *Aldrich*, p. 97, note t: and Trendelenburg's *Elementa Logices Aristotelicae*, note to § 33, cited by Mansel. The term ἐπιχείρημα was differently defined by Aristotle (who called it, as well as the ἐνθύμημα, a dialectical syllogism, συλλογισμὸν διαλεκτικὸν, *Top. ε.* xi. 162ᵃ 16): it was an assault upon a position maintained in disputation by the respondent.

following argument contains both a prosyllogism and an episyllogism, and as the former is expressed in abbreviated form, it is also an epicheirema. 'Those who have no occupation have nothing to interest themselves in, and therefore are unhappy; for men with nothing in which to interest themselves are always unhappy, since happiness depends on the success with which we advance the objects in which we are interested; and so wealth is no guarantee of happiness.' Here the central syllogism is

All who have nothing in which to interest themselves are unhappy
Those who have no occupation have nothing in which to interest themselves
∴ Those who have no occupation are unhappy.

The major premiss is proved by a prosyllogism to this effect:

Happy men are those who succeed in advancing objects in which they are interested
Men who have nothing in which to interest themselves do not succeed in advancing any object in which they are interested
∴ Men who have nothing in which to interest themselves are not happy.

And an episyllogism is added thus:

Those who have no occupation are unhappy
Rich men may have no occupation
∴ Rich men may be unhappy.[1]

We have in such a case a *train of argument*, of which the several steps are not each set out in full, though the premisses necessary to complete the sequence of thought are readily supplied, as in an enthymeme. Trains of argument may, of course, be of any length, and vary indefinitely in composition, according to the nature of the separate steps into which they can be broken up; and it would be useless as well as impracticable to invent names for every variety. But there is one well-marked variety to which the name of *Sorites* has been given by logicians.

[1] The schoolmen gave the name of *syllogismus crypticus* to a syllogism which lay so concealed in the wording of an argument, that some process like conversion, or other substitution of equivalent propositions, was necessary in order to show clearly the terms of the syllogism, and their relation: as, here, 'rich men may be unhappy' is taken as equivalent to 'wealth is no guarantee of happiness'.

A **Sorites**[1] may perhaps be defined as *a syllogism in the first figure with many middle terms*; or if it be thought that nothing should be called a syllogism that contains more than one act of inference, as *a polysyllogism*[2] *in the first figure with the intermediate conclusions suppressed.* Schematically, it is of the form

$$\begin{array}{l} A \text{ is } B \\ B \text{ is } C \\ C \text{ is } D \\ D \text{ is } E \\ E \text{ is } F \\ \therefore A \text{ is } F \end{array}$$

where it will be observed that we start with the minor premiss, and each subsequent premiss is, in relation to that enunciated before it, a major.[3]

There must be, at least, two steps, and therefore three premisses, in a sorites, else we should have no series or chain of syllogisms: and there may be any number of steps more than two; the premisses will always be more numerous by one than the steps into which the argument can be resolved.[4] Short sorites are of common occurrence. A well-known example occurs in Romans viii. 29, 30, 'For whom he did foreknow, he also did predestinate to be conformed to

[1] The name is derived from σωρός = heap.

[2] A series of syllogisms, one proving a premiss of another, is called a *polysyllogism*: while each single step of syllogistic reasoning is called a *monosyllogism*.

[3] Where the order in which the premisses are enunciated is reversed, starting with the major and proceeding always to one which in relation to the preceding is a minor premiss, the sorites is called a *Goclenian Sorites*, after Rodolphus Goclenius, Professor at Marburg at the end of the sixteenth century, who first called attention to this form of presenting the argument. But though it is important to notice that the order in which the premisses are commonly placed in a sorites is the opposite of that which is customary in a simple syllogism, it must not be supposed that the character of the argument is affected by reversing the order, or that the Goclenian sorites is a thing, as such, of any importance. The Goclenian is known also as a *regressive*, and the other, or 'Aristotelian', as a *progressive* sorites. Aristotle, however, does not discuss the sorites (though clearly believing it to occur in science, cf. *An. Post.* *a.* xiv. 79ª 30. xx–xxiii), so that the progressive is not entitled to be called Aristotelian. Sir W. Hamilton states that he could not trace the term back beyond the *Dialectica* of Laurentius Valla, published in the middle of the fifteenth century. From the sixteenth century onward it found a regular place in logical treatises. Cf. his *Lectures on Logic*, xix. p. 377.

[4] 'Sorites est syllogismus multiplex . . . Est enim sorites progressio enthymematica, syllogismos continens propositionibus [=praemissis] uno tantum pauciores.' Downam's *Commentarii in Petri Rami Dialecticam*, 1610, p. 653.

the image of his Son. . . Moreover whom he did predestinate, them he also called: and whom he called, them he also justified: and whom he justified, them he also glorified.'

But long specimens are less common, not because long trains of reasoning are rare, but because the successive steps do not generally continue for long together to be of the same form. Leibniz, in the second part of his *Confessio Naturae contra Atheistas*, written in 1668 (and containing doctrines as to the nature of matter which he subsequently abandoned), offers a proof of the immortality of the human soul in the form of a continuous sorites; but even so, many of the propositions are supported by reasons that do not enter into the series of premisses constituting his sorites.[1] In the following transcription the premisses that do not belong to the sorites are placed out of line to the right; and some of them are omitted.

The human soul is a thing whose activity is thinking.

A thing whose activity is thinking is one whose activity is immediately apprehended, and without any representation of parts therein.

A thing whose activity is apprehended immediately without any representation of parts therein is a thing whose activity does not contain parts.

A thing whose activity does not contain parts is one whose activity is not motion: — for all motion is divisible into parts.

A thing whose activity is not motion is not a body: — for the activity of a body is always a motion.

What is not a body is not in space: — for the definition of body is to be extended.

What is not in space is insusceptible of motion.

What is insusceptible of motion is indissoluble: — for dissolution is a movement of parts.

[1] *v.* Erdmann's ed., p. 47.

What is indissoluble is incorruptible : for corruption is dissolution of the inmost parts.
What is incorruptible is immortal.
∴ The human soul is immortal.

We may pass from examples to a consideration of the form of the argument, and the rules of its validity. It will be observed that the predicate of each premiss is the subject of the next, while the subject and predicate of the first and last premiss are the subject and predicate respectively of the conclusion. For each premiss is minor to that which follows, and major to that which precedes it ; and as we start from the minor premiss of the whole argument, each middle term is predicate of one premiss and subject of the next. It follows, that (i) no premiss except the first may be particular, and (ii) none except the last negative ; for in the first figure, the major premiss must be universal, and the minor affirmative ; now each premiss except the last is a minor, in relation to a premiss following it, and must therefore be affirmative ; and each premiss except the first is a major, in relation to one preceding it, and therefore must be universal. This will be easily seen if we resolve the sorites into its constituent syllogisms :

1. beginning from the minor

A is B
B is C
C is D
D is E
E is F
∴ A is F

A is B (i)
B is C (ii)
∴ A is C
C is D (iii)
∴ A is D
D is E (iv)
∴ A is E
E is F (v)
∴ A is F

It is clear that if the first premiss were particular, the conclusion of the first syllogism would be particular ; this stands as minor to the third premiss in the second syllogism, whose conclusion would therefore again be particular, and so ultimately would the conclusion of the whole sorites be ; but if any other premiss were particular, there would be an undistributed middle in the syllogism into which it entered.

2. beginning from the major

E is F (v)
D is E (iv)
$\therefore$ D is F
C is D (iii)
$\therefore$ C is F
B is C (ii)
$\therefore$ B is F
A is B (i)
$\therefore$ A is F

Here, if the last premiss (E is F) were negative, the conclusion of the syllogism in which it stands as major would be negative: this as major to the premiss C is D would make the next conclusion negative, and so ultimately the conclusion of the whole sorites; but if any other premiss were negative, there would be an illicit process of the major term in the syllogism into which it entered. The rules of a sorites are thus nothing but the special rules of the first figure.[1]

A sorites is distinguished from other chains of syllogistic reasoning by the fact that not only is one of the premisses suppressed, at every step of the argument except one, but the intermediate *conclusions*, by which the final conclusion is reached, are all suppressed; for the conclusion of one argument *is* the suppressed premiss of the next. This is, perhaps, what has led logicians to give special attention to it.[2]

The **Dilemma** combines into one argument hypothetical and disjunctive reasoning. Generally it is an argument in which one premiss is a disjunctive proposition, and the other consists of hypothetical propositions connecting with either alternative in the

[1] Either an E or an I proposition may be converted simply. With an I premiss for the first, if it be converted, the sorites *may* be broken up into a series of syllogisms in the third figure; with an E premiss for the last, if it be converted, the sorites *may* be broken up into a series of syllogisms in the second figure. Yet, except for the premiss thus converted, the middle terms stand throughout in the premisses as in the first figure. A series of premisses in the second or in the third figure will not form a sorites: because there would be no series of middle terms, but only one middle term throughout; hence as soon as we come to combine the conclusion of two premisses with the next premiss, we should be involved in *quaternio terminorum*. The sorites is therefore essentially confined to the first figure, though its resolution may involve the second or third.

[2] It is however only one example of what mathematical logicians like Mr. Bertrand Russell call a system of asymmetrical transitive relations.

disjunction an unpalatable conclusion. In one form, however, of the simple destructive dilemma[1] the disjunction may be in the consequent of the hypothetical premiss, and the other premiss be categorical, denying both alternatives in the disjunction.[2] We may therefore define a dilemma, to cover this case, as a *hypothetical argument offering alternatives and proving something against an opponent in either case.* The conclusion may be either the same, whichever alternative is accepted, or different ; in the former case the dilemma is called **simple**, in the latter **complex**. It is called **constructive**, if it proceeds from affirmation of antecedent in the hypothetical premiss to affirmation of consequent ; **destructive**, if it proceeds from denial of consequent to denial of antecedent.

1. *Simple Constructive.*

If *A* is *B*, *E* is *F* ; and if *C* is *D*, *E* is *F*
But either *A* is *B* or *C* is *D*
∴ *E* is *F*[3]

Troops with a river behind them have sometimes been placed in a dilemma none the less painful because it is simple. If they stand their ground they die—by the sword of the enemy : if they retreat they die—by the flood ; but they must either stand or retreat ; therefore they must die.

2. *Complex Constructive.*

If *A* is *B*, *E* is *F* ; and if *C* is *D*, *G* is *H*
But either *A* is *B* or *C* is *D*
∴ Either *E* is *F* or *G* is *H*

Thus we might argue—and this too is unfortunately a dilemma from which it is not easy to see an escape :

If there is censorship of the press, abuses which should be exposed will be hushed up ; and if there is no censorship, truth will be sacrificed to sensation
But there must either be censorship or not

[1] See below, pp. 360–361.

[2] The hypothetical premiss is sometimes called the major, in accordance with the nomenclature used also of hypothetical reasoning : and the other premiss the minor.

[3] Antecedent and consequent may, of course, all have the same subject (if *A* is *B*, it is *D* ; and if it is *C*, it is *D*) : or the same subject in one case and different subjects in the other ; and the minor premiss will vary accordingly. It would be tedious to give each time all these varieties, which involve no difference of principle.

∴ Either abuses which should be exposed must be hushed up, or truth be sacrificed to sensation.

3. *Simple Destructive.*

If *A* is *B*, *C* is *D* and *E* is *F*
But either *C* is not *D* or *E* is not *F*
∴ *A* is not *B*

Plato, in the *Republic*,[1] urges that children should not learn the poems of Homer, from which they will derive very false beliefs about the nature of the gods. One of his arguments might be put thus :

If Homer speaks truth about things divine, the heroes were sons of gods, and did many wicked deeds
But either they were not sons of gods, or they did not do wicked deeds
∴ Homer does not speak truth about things divine.

Again, If *A* is *B*, either *C* is *D* or *E* is *F*
But neither is *C D*, nor is *E F*
∴ *A* is not *B*

Of this character was one of the arguments used by Zeno to disprove the possibility (or perhaps we might say, the intelligibility) of motion :

If a body moves, it must either move in the place where it is, or in the place where it is not
But it can neither move in the place where it is, nor in the place where it is not
∴ It cannot move.

4. *Complex Destructive.*

If *A* is *B*, *E* is *F* ; and if *C* is *D*, *G* is *H*
But either *E* is not *F*, or *G* is not *H*
∴ Either *A* is not *B*, or *C* is not *D*

A nation having colonies like those of Great Britain might fairly urge :

If we give our colonies self-government, we shall make them powerful ; and if we attempt to control their use of it, we shall make them hostile
But either we ought not to make them powerful, or we ought not to make them hostile

[1] III. 391 C–E.

∴ Either we ought not to give them self-government, or we ought not to attempt to control their use of it.

[It is sometimes said that a destructive dilemma is always complex, and such arguments as those given under (3) above would not be allowed to be dilemmas. Mansel's definition (which follows Whately, and has been adopted by others since) definitely excludes the simple destructive; according to him (*v.* his *Aldrich*, p. 108, n. i) a dilemma is 'a syllogism having a conditional major premiss with more than one antecedent, and a disjunctive minor'; as the destructive dilemma proceeds from denial of consequent to denial of antecedent, if there is more than one antecedent its conclusion must be necessarily complex. A number of writers, however, have admitted the simple destructive dilemma; and it seems very difficult to exclude examples of the first form above given, at any rate. The simple constructive (If *A* is *B*, *E* is *F*; and if *C* is *D*, *E* is *F*) may be written

If *A* is *B* or *C* is *D*, *E* is *F*
But either *A* is *B* or *C* is *D*
∴ *E* is *F*

The simple destructive runs

If *A* is *B*, *C* is *D* and *E* is *F*
But either *C* is not *D* or *E* is not *F*
∴ *A* is not *B*

It may be said that there is a disjunction in the hypothetical premiss of the former, and not of the latter; but this does not seem to constitute an essential difference, such as would render one a dilemma and the other not. In the former, one or other of two alternatives must be affirmed, and whichever be affirmed, the same conclusion follows, because it is logically a consequent of affirming either alternative; in the latter, one or other of two alternatives must be denied, and whichever be denied, the same conclusion follows, because it is logically a consequent of denying either alternative. The essence of the dilemma seems to lie in the fact of confronting a man with alternatives at once ineluctable and unpleasant: cf. the definition quoted by Mansel from Cassiodorus, loc. cit.: *Dilemma, quod fit ex duabus propositionibus pluribusve, ex quibus quidquid electum fuit, contrarium esse non dubium est.* And therefore the other example given above—Zeno's argument about motion—seems also to be fairly called a dilemma.[1] It is true that its second premiss is not disjunctive at all, but denies a disjunctive proposition; it does not assert the truth of one of two alternatives, but the falsity of both. But the whole argument is a combination of the hypothetical and

[1] So Minto takes it, *Logic, Inductive and Deductive*, p. 224.

[the disjunctive, and drives a man into a corner by way of alternatives between which his choice is alleged to be confined. If we are to maintain that a body moves, we have to assert one or other of two propositions which are both self-contradictory ; and that seems a good example of being placed between the devil and the deep sea. The simple constructive dilemma is a hypothetical argument in the *modus ponens* ; its hypothetical premiss has a disjunctive antecedent and a simple consequent, and therefore the other premiss must be disjunctive and the conclusion simple. The simple destructive dilemma of the second form given above is a hypothetical argument in the *modus tollens* ; its hypothetical premiss has a simple antecedent and a disjunctive consequent ; the other premiss must therefore be the denial of a disjunctive proposition, and the conclusion the denial of a simple one. But the denial of a disjunctive proposition is a categorical, whereas the affirmation of it is of course a disjunctive proposition. Hence the difference which has led to refusing the name of dilemma to this form of argument ; yet its parallelism with the simple constructive seems correct and clear. It may be asked why there are two types of simple destructive dilemma, against one type of simple constructive. The answer seems to be this. In the destructive dilemma, I may overthrow the antecedent, either if its truth involves two consequents, one or other of which I can deny, or if its truth involves one or other of two consequents, both of which I can deny ; and each case involves a disjunction. In the constructive dilemma, I can establish the consequent, either if two antecedents involve its truth, both of which I can affirm, or if either of two antecedents involve its truth, one or other of which I can affirm. But here the former case does not constitute a dilemma, because no disjunction is involved anywhere : If A and B are true, C is true ; but A and B are true $\therefore$ C is true. It would appear therefore that so far from there being no such thing as a simple destructive dilemma, there are two forms of it, against only one form of simple constructive dilemma.]

A dilemma is sometimes spoken of as if it were a peculiarly unsound form of argument. It shares with all inference the property that it is of no material value unless its premisses are true ; but formally it is quite sound, and if there is about it any special weakness, it must lie in some special difficulty in getting true premisses for it. Now it is generally difficult, except where one alternative is the bare negation of the other, to get an exhaustive disjunction ; it is here that any one ' in a dilemma ' would look for a way out ; and it is this difficulty which inspires mistrust of the dilemma as a form of argument.

To show that there is some other alternative besides those, on

one or other of which your opponent attempts to drive you, is called *escaping between the horns* of a dilemma : the alternatives being the 'horns' on which you are to be 'impaled'. In reply to Zeno's dilemma to show the impossibility of motion, it is often said that a body need not move either in the place where it is or in the place where it is not ; since it may move *between* these places. It may be questioned whether this is a very satisfactory solution of the paradox ; for those who offer it might find it hard to say *where* the body is when it is between these places ; if it is not in some other place, the continuity of space seems to suffer disruption. But however that may be, we have here an attempt to escape between the horns of Zeno's dilemma.

The other two ways of meeting a dilemma also bear somewhat picturesque names ; we may *rebut* it, or we may *take it by the horns*. To rebut it is to produce another dilemma with a contradictory conclusion. The old story of Protagoras and Euathlus, without which a discussion of Dilemma would hardly be complete, furnishes a good example of rebutting. Protagoras had agreed with Euathlus to teach him rhetoric for a fee, of which half was to be paid at the conclusion of the instruction, and the remainder when Euathlus won his first suit in court. Observing that the latter delayed to practise, Protagoras thought he was endeavouring to evade payment, and therefore himself brought a suit for the recovery of the second half of his fee. He then argued with the jury that Euathlus ought to pay him, in the following way :

> If, he said, he loses this case, he ought to pay, by the judgement of the court ; and if he wins it, he ought to pay, by his own agreement
>
> But he must either lose it or win it
>
> ∴ He ought to pay.

Euathlus, however, rebutted this dilemma with the following :

> If I win this case, I ought not to pay, by the judgement of the court ; and if I lose it, I ought not to pay, by my own agreement
>
> But I must either win it or lose it
>
> ∴ I ought not to pay.

It will be seen that the rebutting dilemma is produced in this case by connecting in the hypothetical premiss, with either ante-

cedent, the contradictory of the consequent originally connected with the other. With a destructive dilemma the parallel procedure would be to connect with the contradictory of either antecedent the consequent originally connected with the other. But this is not the only way of rebutting; you rebut whenever you produce a dilemma with contradictory conclusion, even though you do it with quite different premisses. Nor can every dilemma be rebutted in this way or in any other way: not in this, for the alternative conditions are not always such with which you can connect the contradictory of each other's consequents. And if a dilemma can be rebutted, it must be for one of three reasons. Either (1) there must (as in the last example) be some impossible assumption in the supposed situation; and some of the ancients spent much ingenuity in imagining situations of this kind, in which our reason was entangled by finding that two contradictory solutions of a problem could apparently be maintained with equal force.[1] Or (2) the premisses are unsound, and premisses equally or more plausible can be found for another dilemma proving a contradictory conclusion; in this case, it would be possible to attack the original dilemma directly, either by showing that you can escape between the horns of it, if the disjunction is not complete, or in the third of the ways mentioned above, by 'taking it by the horns'. Or else (3) as happens, unless there is an impossible assumption in the situation supposed, when we rebut by transposing and denying the consequents or the antecedents, the conclusions of the two dilemmas are perfectly consistent, and the second merely shows that you will escape one or other of the alternatives, of which the first showed that one or other would be incurred. In a complex dilemma whose alternatives are mutually exclusive this is obviously necessary; but it is

[1] Of this nature are the well-known sophisms of the 'Liar' and the 'Crocodile'; Epimenides the Cretan said that all Cretans were liars; if they were, was he lying, or was he speaking the truth?—a crocodile had stolen a child, and promised the mother he would restore it, if she could guess rightly whether he intended to do so or not; if she said he would not restore it, she could not claim the child by his promise, because her taking it would make her guess wrong; if she said he would restore it, she could not claim it, for she guessed wrongly; what was she to say? (cf. Lucian, *Vit. Auct.* § 22, cited Mansel's *Aldrich*, p. 151). The solution of the first is easy unless we suppose that no Cretan ever spoke the truth; in which case the truth of the statement attributed to Epimenides is incompatible with his making it. It may be said generally of both these sophisms, and of the story of Protagoras and Euathlus, that the difficulty arises from supposing that a statement or agreement about certain matters can itself be within the scope of such statement or agreement.

small consolation to any one on the horns of a dilemma, to point it out. When Henry VIII desired to force upon Sir Thomas More the oath of supremacy, More was asked whether he thought the statute 'giving to the King the title of Supreme Head of the Church under Christ' had been 'lawfully made or not. He replied that the act was like a two-edged sword, for "if he said that it were good, he would imperil his soul; and if he said contrary to the statute, it were death to the body".'[1] If a man is threatened with death or damnation, the threat is not proved empty by showing that he will escape damnation or death. Sir Thomas More indeed 'declined to swear at all'. But that also was death to the body.

To *take a dilemma by the horns* (or by one of them) is to accept an alternative offered you, but to deny that the consequence, which the opponent attaches to its acceptance, follows. Perhaps the following will serve for an example. It is held by many naturalists, that species are modified in the course of descent only by the accumulation of many slight variations, and not *per saltum*: variations not being directly adaptive, but being distributed, in respect of frequency and degree, in proportions that follow the well-known 'curve of error', on either side of the standard represented in the parents. Against this it has been argued, that though the cumulative effect of many slight variations might be useful, it will often happen that in the incipient stages, while the distance traversed in the direction of some new peculiarity is still very slight, the variation would be valueless, and therefore not tend to be perpetuated; so that the basis for accumulation would not exist. This line of objection has been applied to the particular case of protective colouring in insects in the following argument.[2] If, it is said, the slight variations, with which the process of mimicry in insects must, as alleged, begin, are of no use in leading birds to mistake the individuals exhibiting them for members of some protected species, then they will not be preserved by natural selection, and no accumulation can take place; while if they are of use, any further and more exact resemblance to the protected species is unnecessary, and could not, if it occurred, be preserved by natural selection. Now against this dilemma we may answer that it does not follow that, because a slight degree of resemblance is useful, any further degree

[1] *Political History of England*, vol. v, by H. A. L. Fisher, p. 350.

[2] See an article on *The Age of the Inhabited Earth*, by Sir Edward Fry, in the *Monthly Review* for January, 1903.

would be superfluous. On a particular occasion a particular insect no doubt needs no greater resemblance than what has actually enabled it to escape ; but with a large number of insects over a long series of occasions, it may well be that the percentage of escapes would be higher with those in whom the resemblance was closer. Thus the dilemma is ' taken by the horns ' ; but that does not settle the important question at issue as to whether variation ever does proceed *per saltum* or not. We saw before that a thesis is not disproved by the refutation of any particular argument brought forward in support of it.

CHAPTER XVII

OF THE FORM AND MATTER OF INFERENCE

So far we have considered and examined some of the commonest forms of inference—syllogism, hypothetical and disjunctive reasoning, and certain complications of these. We have not pretended —what has nevertheless sometimes been maintained—either that the last two can be reduced to syllogism, or that syllogism, even if the term be taken to include the three, is the type to which all valid inference must conform ; though we have maintained, and it will appear more fully in the sequel, that all are forms of great frequency and importance in our thought. Were Logic a purely formal science, the analysis of these forms would be, to those who thought that all reasoning really moved in one or other of them, the end of the task imposed upon that science ; to those who did not think them the only forms in which men's reasoning moves, no other task would be left than to offer a similar analysis of the remainder. But if it is impossible to understand fully the form of thinking without reference to differences in it springing from the nature of that about which we think, then the task of Logic is obviously harder. It will not suffice to work with symbols. We cannot make abstraction of the special character of our terms. Already we have found this to be the case. We saw that what is called demonstrative syllogism in the first figure rests upon a perception of the necessary relation between certain notions, or universals ; while in the third figure such a perception of necessary relation neither need be given in the premisses of a syllogism, nor can be reached in the conclusion. We saw too how hypothetical reasoning, where it differs most from syllogistic, differs because it establishes a connexion between subject and predicate in the conclusion by means of a condition which is apparently extraneous to the nature of the subject ; and yet how our thought recognizes that there must be some wider system to which the subject and that condition both belong, and through which it comes about that the fulfilment of the latter should affect the predicates of the former. None of these things could be explained or understood merely through

symbols : examples were needed not only to show that the arguments symbolized were such as we do actually often use, but because only in suitable examples could those facts of our thought with which we were concerned be realized. The symbols, e.g., are the same, but *do not symbolize the same thing*, when some terms in our syllogism are singular, and stand for individual concrete subjects, whose attributes are set down as we find them, and when they are all general, and signify universal characters of things, between which we perceive connexion.

It will be said that if the form of thought be thus bound up with the matter, and if the matter be different according as we think about different things, an understanding of the form must wait upon a knowledge of these, and the task of Logic will not be complete until we have finished the investigation of what is to be known. In a sense this is true. It may be illustrated by mathematics ; no one can understand the nature of mathematical reasoning except in reflection upon his thinking about number or space or quantity ; it cannot be seen in application to heterogeneous subjects. And it consists with the position which we have taken up from the outset, that Logic is the science which brings to clear consciousness the nature of the processes which our thought performs when we are thinking about other things than Logic. Nevertheless we must bear in mind one or two facts, which may make the task of Logic seem a little less hopeless than it would appear to be, if it had to wait altogether upon the completion of knowledge.

In the first place, the dependence of the form of thought upon the matter is consistent with some degree of independence. It may be impossible to grasp the nature of mathematical proof except in application to mathematical subjects ; but an analysis of one or two examples of geometrical reasoning may serve to show us the nature of geometrical reasoning in general, and after that the form of it will not be any better understood for tracking it through all our reasonings about every figure and space-relation. So also it may be impossible except in examples of the relation of subject and predicate to grasp the distinctive character of syllogistic reasoning ; but we may grasp it there universally, and realize that it will be the same for all terms that stand in those relations. If this were not so, science would be impossible ; for science seeks to reduce a multiplicity of facts to unity of principles. Thus our apprehension of the forms of thought has not to wait upon the completion of our

knowledge so far as that completion means only its extension to fresh subjects of the same kind. If some branch of our knowledge is defective in point of extent—as it would appear, for example, that the science of number must ever continue to be, because the numerical series is by its nature inexhaustible—yet its further extension may involve no change in its character ; and so soon as all the main branches of possible knowledge have been discovered—that is, knowledge about all the main departments of fact—the forms which thought assumes in them can be studied even while our knowledge is incomplete in its extent. The main departments of fact must, of course, be taken to include not merely those which form the subjects of the physical sciences, but equally those of which philosophy treats, and not least the relation of the world to the mind that knows it. It would be rash to assert that this stage has been reached in the progress of knowledge. The completion of our knowledge may yet require not only its extension, but in large degree its transformation. Yet we may assert that a great deal of our ignorance forms no bar to the completion of the investigations of Logic.

And in the second place, though Logic is in the main a reflection upon the nature of knowledge already gained, there is this paradox about knowledge, that we seem to some extent to know what knowledge ought to be, before we know anything as we ought. We have an ideal, of which we are sufficiently conscious to realize the imperfections of the actual, though not sufficiently conscious to be able to put it clearly and fully into words. This paradox is not confined to knowledge ; it occurs in art and in morality also.[1] We may recognize defect in an aesthetic whole without being able to rectify it, and yet we may be able to say in what direction its perfection must lie ; we may know that ' we have all sinned ', without having seen ' the glory of God ', and still be able to prescribe some of the conditions which that must realize. So also we may know that the form of our thought, even when we think best and most patiently, often falls short of the full measure of knowledge : that our way of thinking—our way of looking at things, if one may put it so—is wrong because it fails to escape contradictions and satisfy all doubts ; and that there must be some way of thinking (if the world is as a whole intelligible at all) in which contradiction and uncertainty will vanish. We may know all this, and know that we

[1] Cf. *supra*, p. 10, n. 2.

have not found that better way (for if we had, we should certainly not remain in the worse): and still we may be able to say something about it though we have not found it: to lay down conditions which our knowledge of any subject must satisfy because it is knowledge—i.e. to prescribe to some extent the form of knowledge, not only as a result of reflection upon instances of subjects being perfectly known or by abstraction from the activity of knowing perfectly in the concrete, but by way of anticipation, out of reflection upon instances in which we know subjects less than perfectly, and know the imperfection of our knowing. The extent to which we can thus anticipate is not unlimited; a man must get some way in science, before he will realize what science should be, and that it is not what it should be; just as a man must get some way in virtue before he will realize how much more it requires of him than he has achieved. Yet it remains true that thought can in some degree anticipate a form of knowing a subject which it has not exercised therein; and it is the business of Logic to set this form forth. So far again Logic has not to wait, in order to complete its task, until our investigation of what is to be known has been completed.

If this is true, we may say on the one hand, that no study of the nature of inference can be adequate which treats it as an operation performed with symbols, or one intelligible at any rate when we work merely with symbols. On the other hand, we may recognize that there are recurrent forms of inference, whose nature is the same in their different occurrences,[1] and that they occur commonly and are displayed in regard to subjects in many respects very diverse; we may also recognize an ideal of what inference should be if it is to convey knowledge: if we are to realize in making it not merely that the conclusion follows from the premisses, but that we are getting at indubitable truth.

Our discussion of inference up to this point must therefore be incomplete, in so far as (*a*) we have failed to deal with all those distinguishable recurrent forms of inference whose universal nature can be realized in an example; (*b*) we have failed to make plain the conditions of knowledge as well as the conditions of cogency.

As to the first count, there are certainly forms which have not

[1] Some might maintain that it is never *quite* the same when the matter is different, any more than the nature of man is quite the same in any two individuals. I do not wish to subscribe to this view; but even its upholders would admit that such differences may be negligible.

been examined. For example, there is the *a fortiori* argument. 'He that loveth not his brother whom he hath seen', asks St. John, 'how can he love God whom he hath not seen?' And there is mathematical reasoning, of which we have only said that it is not syllogistic; this from its importance may claim rather fuller consideration. But perhaps more remains to be done in the way of showing how far inference of these different forms enters into the building up of our knowledge, and what other operations of thought enter into it.

As to the second count: it is a charge brought against the analysis of syllogism, and the other inferential forms considered above, that such analysis only shows us the conditions of consistency in reasoning, and not the conditions of truth. To reason consistently is very different from discovering truth; for the consistent reasoner will reproduce in his conclusion the error there may be in his premisses.[1] Those who have brought this charge have sometimes supposed that what is wanted is other and better forms of inference. It would be much truer to say that what we want is to realize how much besides formal validity of inference must be present in an argument which is to convey knowledge. To realize what is needed is not indeed the same thing as to supply it; but Logic cannot help us to more. The critics of the Logic which was content to analyse the conditions of validity in some of the common inferential forms (and which often supposed—it must be admitted—that there were no other forms of inference) have not always believed this. Many of them, as has been said in the first chapter, still looked on Logic mainly as an instrument for the discovery of truth about any matter on which we might propose to reason, and hoped to find a new and better instrument than what the Logic which confined itself to such analysis afforded. This was the object with which Bacon wrote his 'New Instrument' or *Novum Organum*; and J. S. Mill, though he calls Logic a science, wrote his famous treatise in the hope that familiarity with the methods of reasoning used successfully in the physical sciences would enable men to prosecute the study of the moral and political sciences with more success.[2] Logic is not a short cut to all other branches of knowledge. But this we may say, that men who know the difference between con-

[1] Though formally a true conclusion may be got from false premisses, the error still infects the mind, and will lead to a false conclusion somewhere.

[2] Cf. *System of Logic*, VI. i., and *Autobiography*, p. 226.

sistency and demonstration, who know what is required before it can be said that they have *knowledge* about things, in the full and proper sense of that term, are less likely to remain content with the substitutes that commonly pass muster in men's minds for knowledge. By a study of the conditions of demonstration we may be led to see how far from being demonstrated are many of the beliefs we hold most confidently. To know what we do know, and what we do not—what, out of the things we suppose ourselves to know, we really *know* and are rationally justified in believing: this, as Plato long ago insisted,[1] is neither a small thing, nor an easy; and until we understand what *knowing* a thing means and requires, we are not likely to achieve it. This is why Logic should do more than present us with a study of the forms of consistent reasoning, and should attempt to exhibit the nature of knowledge and demonstration: not because such an exposition of the form of knowledge is itself an instrument for bringing our thoughts upon any subject into that form, but because it stimulates us to use such powers as we have, and to appraise the results which we have so far attained.

Now the most obvious criticism that can be made upon a Logic which confines itself to setting forth the formal conditions of valid inference is that it ignores the question of the truth of the premisses; the validity of the reasoning affords no guarantee that these are true. It is no doubt possible to direct men's attention so exclusively to the form of argumentation that they will bestow little upon the truth of the premisses from which they argue. It has often been complained that the study of Logic did this—or, as its critics would say, the study of Deductive Logic.[2] The epithet, however, implies a misunderstanding; it is a disproportionate attention to validity of form in general which the critics ought to deprecate. Validity of form is a thing worth studying, not only for its own sake, but in some degree lest we infringe it; yet it is psychologically possible, by studying it too much and too exclusively, to become distracted from due care about truth of fact. It is, however, probable that

[1] *Charmides* 171 D.

[2] The popular antithesis between Deductive and Inductive Logic has been so far avoided, and that deliberately; we shall have to consider presently what the nature of the difference between deductive and inductive reasoning is; but it may be said at once that it does not lie in using the forms of inference that are commonly expounded under the titles of Deductive and of Inductive Logic respectively. For inductive reasoning uses forms of inference with which treatises that would be called Deductive always deal; and treatises called Inductive discuss forms of inference which are certainly deductive.

in the times when men have been most remiss in the examination of their premisses, the state of the study of Logic has been as much a symptom as a cause of this ; and however that may be, so far as it lies with Logic to provide a corrective, it is very important for the logician to be clear as to the nature of the corrective he is to provide. And for that purpose he must distinguish two questions ; he may try to show what kind of premisses knowledge requires, or by what process of thought we may hope to get them. In modern times, the former of these questions has been too much neglected.

These last remarks may be a little expanded. And first as to the causes which for many centuries made men remiss in the examination of their premisses ; one sometimes finds the blame for this thrown upon the futility and misdirection of the scholastic Logic, which absorbed during the Middle Ages, and even later, so large a part of the energy of men's minds. It would be hard to deny that much of it was futile, and that much energy was misdirected ; but it is as likely that energy went into this channel because others were temporarily closed to it, as that others were robbed of it because it ran in this ; though no doubt there is action and reaction in such a case, and a habit which certain influences tend to form may in turn strengthen those influences.

It has been said that the mandate issued to the age of Plato and Aristotle was *Bring your beliefs into harmony with one another* ; that the mandate of the Mediaeval Spirit was *Bring your beliefs into harmony with dogma* ; and that the mandate of the new spirit which rebelled against the authority of the Church was *Bring your beliefs into harmony with fact*.[1] Such a mode of putting things may suggest some errors. It is impossible to bring one's beliefs into harmony with fact, except so far as facts are known to us ; our knowledge of facts is expressed in propositions which we believe ; and therefore to bring our beliefs into harmony with fact is to bring them into harmony with one another (though not conversely). It would be wrong to suppose that Plato and Aristotle forgot that among the beliefs they had to harmonize with one another were the beliefs they held about matters of daily experience, or that they were indifferent to the necessity of correcting and enlarging those beliefs by more or less systematic observation ; Aristotle in particular added largely to men's knowledge of facts. Again, it is clear that to bring one's beliefs into harmony with dogma is to bring them into harmony

[1] Minto, *Logic, Inductive and Deductive*, p. 243.

with other beliefs ; and that those who rated highest the importance of that task would least have doubted that they were bringing them into harmony with facts. Propositions do not cease to state facts because they are presented as dogmas. But it is true, as Minto wishes to bring out in the passage quoted, that dogma and the spirit which accepts dogma did during the Dark and the Middle Ages play a part in the history of thought far greater either than they played in classical antiquity or than they have come to play since the revival of learning. And such dogma was not necessarily ecclesiastical dogma ; it came from the scientific works of Aristotle, or other great men of old whose works were known, as well as from the Bible and the Church ; just as to-day there is orthodoxy in science, against which new scientific doctrines find it at times a little difficult to battle, as well as in theology.

The schoolmen knew, as well as Bacon or any other of their critics, that the study of the syllogism was not all-sufficing : that no syllogism could guarantee the truth of its premisses ; and that for a knowledge of the most general principles to which deductive reasoning appeals we must rely on something else than deductive reasoning itself. Bacon refers to the 'notorious answer' which was given to those who questioned the accepted principles of any science—*Cuique in sua arte credendum.*[1] And there are seasons in the process of learning when that is a very proper answer ; men must be content at many times and in many matters to accept the expert opinion of their day. But this is only tolerable if in every science there are experts who are for ever questioning and testing. When tradition stereotypes doctrine, it is as bad for knowledge as close guilds and monopolies are bad for the industrial arts ; they shut the door upon improvement. Authority plays, and must play, a great part in life—not only in practice, but also in things of the intellect. But the free spirit is as necessary, which insists on satisfying itself that what is offered upon authority has claims on its own account upon our acceptance.

Why was it that for so many centuries so much was accepted upon authority which afterwards fell to pieces in the light of independent enquiry ? Much knowledge of the human mind, historical and philosophical, would be needed in order to answer this question adequately. If a few observations may be made upon it here, it is with a full consciousness of the inadequate equipment of knowledge

[1] *Nov. Org.* I. 82.

upon which they rest. And it may be doubted whether we can hope fully to explain why some periods and places are richer than others in men of fruitful and original thought; at most we can hope to show what conditions are favourable to such men's work when they arise. Now to us, looking backward across the Middle Ages to the more brilliant days of Athens and of Rome, and looking also at the great increase of knowledge which the last three centuries have brought, the stagnation of the sciences in the period intervening is apt to seem a thing surprising. But how long was it before ancient science began to appear and to advance? The power of tradition and authority over the human mind is the rule rather than the exception.[1] And in the break-up of ancient civilization there perished not only much knowledge, but much material wealth; men were of necessity for long absorbed in the task of restoring this and restoring order; and it is not wonderful that they had little time to spend in questioning such scientific principles as had survived. Moreover, during the darkest times, the most powerful and the most beneficent institution that stood erect was the Church; the most comprehensive and well-reasoned theory of the world was that which the Church taught; the strongest minds, almost the only minds that thought at all, were enlisted in the ranks of the clergy (which was why independent thought took so largely the form of heresy), and the interest of men was directed rather to what concerned the soul than to nature around them. To this it must be added, that through a series of historical accidents, a great part of the literature of Graeco-Roman civilization had perished; but that of the works of Aristotle some few were known continuously, and the rest recovered, at least in translations, by the end of the first quarter of the thirteenth century.[2] The works of Aristotle, by their encyclopaedic range, by the effort after systematization displayed in them, and by their extraordinary intellectual power, were peculiarly suited to rivet themselves upon the mind at a time when ability was not wanting, but when detailed knowledge was slight, and there was little else to serve for an educational discipline. It is not surprising, if Aristotle and the Church (especially when the Church pressed Aristotle's philosophy into its service) acquired a preponderant influence over men's minds.[3]

[1] Cf. Bagehot, *Physics and Politics*.

[2] *v.* Prantl, *Geschichte der Logik*, III. p. 3.

[3] Professor W. G. de Burgh calls my attention to the language of Dante, *Il Convito*, iv. 6, about the authority of the *maestro di lor che sanno*.

Indeed, it is hard for us to imagine what self-confidence and courage were necessary, in order to question any part of that closely concatenated fabric of belief, upon appearing to accept which depended a man's comfort in society and perhaps his life in this world, and upon really accepting it—unless he could find for himself something better—his confidence with regard to the next. It is no small testimony to the inexpugnable power of reason, that this system broke down. And it began to break down largely through the recovery of other monuments of ancient thought and learning besides the works of Aristotle. This doubtless stimulated, though it could not produce, the powers of those men by whom the foundations of modern science were laid—men like Copernicus, Galileo, Harvey, Gassendi, Descartes. It was not the reform of Logic which liberated the mind, any more than it was Logic which had bound it.

It is, then, rather to the habit of believing on authority, the strength of which during that period it has been attempted in some degree to account for, than to the prevalence of an erroneous Logic (whose errors were not really what the 'inductive' logicians supposed), that the stagnation of science for so many generations must be attributed. Given that habit, it was natural that men should spend time and thought upon a barren elaboration of the more technical parts of Logic, and leave the traditional assumptions both of it and of the natural sciences unexamined. When the overmastering influence of authority began to decay, the science of Logic shared with other sciences in the revivification that comes from thinking out a subject freshly and independently.

But, as was said above, the particular matter which first attracted the attention of the reforming logician was the barrenness of an exclusive attention to the forms of valid inference; and the particular improvement proposed was the establishment of a Logic that should do for the discovery and proof of scientific principles what had already in part been done for the drawing of conclusions from them. This at least is how Bacon looked at the matter; and others have so looked at it after him, in this country more especially. Now it is a very interesting question, how sciences get their principles, and when they may be considered proved; but it is not quite the same as the question, what kind of principles knowledge requires.

The works of Aristotle dealing with inference are three—the *Prior Analytics*, *the Posterior Analytics*, and the *Topics*. Speaking generally, the first of these deals with syllogism from a formal

point of view—it pays no attention to the nature of the premisses, but only to the validity of inference; the second deals with knowledge, or demonstration: it asks not when a man is bound by the acceptance of certain forms of premiss to admit a certain form of conclusion, but when he can be said really to know a thing absolutely, and not merely on the assumption that certain premisses are true; the third asks how positions can be established or overthrown, what sort of considerations are useful in weighing their claims to acceptance, and on what sort of grounds men may be content to accept their principles in matters where certainty is not attainable. In the first and in the third of these treatises, Aristotle was analysing and formulating the actual procedure of his contemporaries; he did not, upon the whole, go ahead of the science, the disputation, the rhetoric, and the pleadings of his day. In the second, he was doubtless guided also by a consideration of the highest types of scientific knowledge then existing; but he was guided also by an ideal; he was trying to express what knowledge ought to be, not merely what the form of men's reasonings is.

It may be said that in scholastic Logic, the problems of the *Prior Analytics* bulked too large; that those who revolted against this raised, without realizing it, problems of the same kind as Aristotle had already discussed in the *Topics*; but that for a long time the questions of the *Posterior Analytics* received insufficient attention. It is these last which are the highest, and go deepest into the philosophy of the subject. The physical sciences employ many principles of great generality which they try to prove; but there are some assumptions about the nature of the world, which they accept without asking why they accept them. As instances of these may be mentioned what is called the Law of the Uniformity of Nature—the principle that every change has a cause upon which it follows in accordance with a rule, so that it could not recur in the same form unless the same cause were present, nor fail to recur when precisely the same cause recurred: or again, the principle that matter is indestructible: or that the laws of number and space hold good for everything numerable or extended. There are other principles less general than these, such for example as the law of gravitation, of which, as aforesaid, science offers proof; but whether the proof of these amounts to complete demonstration, and whether the assumption of the truth of those is justified—these are problems with which the special sciences trouble themselves little,

and which will not be answered merely by analysing the nature of the inferential processes that do as a matter of fact lead scientific men to accept the general propositions which they conceive themselves to have proved.

This is only an elementary book, and makes no pretence to give a complete answer to that most difficult of logical questions, *What is knowledge, in its perfect form?* But from what has been said in the present chapter, it follows that there are two problems to which some attention ought to be given. One is the question how, as a matter of fact, we do get our premisses: the other, what are the requisites of demonstration.[1] The first of these may be called the problem of Induction.

[1] *v. infra*, p. 524.

CHAPTER XVIII

OF INDUCTION

THE history of the word Induction is still to be written ; but it is certain that it has shifted its meaning in the course of time, and that much misunderstanding has arisen thereby. The Aristotelian term *ἐπαγωγή*, of which it is the translation, signified generally the process of establishing a general proposition not by deduction[1] from a wider principle, but by appeal to the particular instances, or kinds of instances, in which its truth is shown.[2] But if it is to be established thus, all the instances must be cited ; and induction meant primarily to Aristotle, proving a proposition to be true universally, by showing empirically that it was true in each particular case or kind of case : or, proving something about a logical whole, by appeal to the experience of its presence in every part of that whole ; as you might show that all horned animals ruminate, or that whenever the tail of a fish is unsymmetrical (or heterocercal) it is vertebrated, by a dissection of the intestines of every kind of horned beast, or of the tail

[1] The history of the term Deduction is also still to be written. *ἀπαγωγή* in Aristotle meant something very different (v. *Anal. Pri. 3.* xxv : there is also the use cited p. 315, n. 1, *supra*), and the nearest Aristotelian equivalent to Deduction is *συλλογισμός*.

[2] From what sense of the verb *ἐπάγειν* this use of the word sprang is not clear ; there are two passages (*An. Post. a.* i. 71ª 21, 24 : xviii. 81ᵇ 5), where the passive verb, in a logical context which makes it clear that the process of *ἐπαγωγή* is referred to, takes a personal subject ; as if it were meant that in the process a man is brought face to face with the particulars, or perhaps brought, and as we could say induced, to admit the general proposition by their help. In another place (*Top. a.* xviii. 108ᵇ 11 : cf. *Soph. El.* xv. 174ª 34), it is the universal proposition which is said to be 'induced' or brought forward or brought up (whatever the best translation may be) ; and perhaps the not infrequent antithesis of *ἐπαγωγή* and *συλλογισμός* might suggest that the usual object of the verb is the inductively obtained conclusion ; the conclusion is certainly what is 'syllogized', so that the conclusion may also be what is 'induced'. It has, however, also been thought that the process of bringing up or citing the instances, by means of which the conclusion is to be established, is what the word was primarily intended to signify (Bonitz, *Index Aristotel.*, s. v. *ἐπαγωγή*, seems to take this view) ; and anyhow the process described is one in which a general conclusion is established in that way, by citing the instances of its truth. Nevertheless, there is no passage where *ἐπάγειν* governs an accusative of the instances adduced.

of every kind of heterocercal fish. In such a proof, it would be assumed that the nature of each *species* of fish or beast might be judged from the single *specimen* dissected ; and it is to be noted that Aristotle thought that the process of induction began here with the *infima species* ; the species in his view (as we saw in discussing the Predicables) being essentially the same in every one of its particulars.[1] This form of argument he described in his own technical language as proving the major term of the middle by means of the minor ; and he showed how it could be expressed as a syllogism. From the premisses

The cow, the sheep, the deer, &c., ruminate
The cow, the sheep, the deer, &c., are horned

I cannot, as they stand, infer that *all horned animals ruminate*, because there may be other horned animals besides all that I have enumerated ; but if I know that this is not the case : if the members in my enumeration taken together are commensurate or equate with the term 'horned animals', then the possibility which forbids the general conclusion is excluded, and I may infer that *all horned animals ruminate* : as is shown by the fact that the minor premiss may be converted simply ; I may say that *all the horned animals are the cow and sheep and deer, &c.* ; and my syllogism becomes formally correct. In such a syllogism we are said to prove the major of the

[1] Induction certainly starts in one sense, according to Aristotle, with individuals ; for it starts with what we can perceive with the senses, and only the individual can be perceived : cf. e. g. *An. Post. a.* xviii. 81^{b} 5–9. But it may be said that what we apprehend in the individual is its character or type, and that it is to the individual as *such and such* an individual that we appeal : cf. *An. Post. a.* xxxi. 87^{b} 29. In *An. Post. β.* xiii. 97^{b} 7 sq., however, Aristotle describes a method of searching for definitions—the example which he uses is μεγαλοψυχία (magnanimity)—in which the instances cited in support of the definition of μεγαλοψυχία are not cited as types at all. This has come traditionally to be called the method of obtaining definitions by induction ; and the description of it seems based on those discourses of Socrates to which Aristotle refers as ἐπακτικοὶ λόγοι, inductive discourses ; but the term ἐπαγωγή does not occur in the passage. Still in the argument from Example, or παράδειγμα, the instance appealed to is not cited as the specimen of a kind ; and he calls this the rhetorical form of Induction. Hence, though the statement in the text is true, so far as concerns the proof by induction of the properties of natural kinds (for in regard to that, Aristotle's particulars are *infimae species*), it is difficult to maintain that he never regards induction as starting with individuals as such. How you are to tell what properties in a specimen are properties of the species is a question which is discussed in the *Topics* ; and certainly he would not have thought of proposing to prove that by a complete enumeration. The species of a genus are limited in number, and can all be cited ; not so the individual members of a species. Cf. *infra*, p. 384.

middle by means of the minor, because (as we saw) the minor means to Aristotle not primarily the subject of the conclusion, but the term of least generality and nearest to the individual; it is by the particular instances that the predicate *ruminant* is proved of the subject *horned animal*. And if we might regard the possession of horns as the cause of ruminating, then it would be the proper middle term by which to demonstrate *ruminant* of cow or sheep or deer; in Aristotle's own example, where longevity is proved of gall-less animals by means of man, horse, mule (and any other particulars that ought to be mentioned—though for brevity they are not enumerated), it is supposed that the absence of gall is the cause of longevity.

In symbolic form then we may express Aristotle's Induction thus:—

A B C D, &c. are *P*
A B C D, &c. are all the *M*
∴ All *M* are *P*

This, which he calls ὁ ἐξ ἐπαγωγῆς συλλογισμός, is commonly called now the **Inductive Syllogism**. If it is to be valid, our minor term must, as Aristotle says, comprise all the particulars; ἡ γὰρ ἐπαγωγὴ διὰ πάντων.[1]

We have now seen what Induction, as a formal process, meant in the mouth of the first author who used the term; and when Aristotle insisted that it must proceed through all the particulars, or (as it was afterwards put) *by complete enumeration*—the requirement which, to Bacon and the 'inductive logicians' of modern times, has given so much offence—he was quite right; for *if* you are going to establish a general proposition that way, you will clearly not be justified in making it general unless you *have* made sure that your enumeration of the particulars is complete; though, as has been said, it is not really an universal proposition then, but only 'enumerative': a thing which Aristotle fails to point out. The burden of the charge against Aristotle is, however, not that he held that, if a general proposition is to be established by enumeration of particulars, the enumeration must be complete: but that he recognized *no other* mode of establishing general propositions. And if this be so, then his Logic falls to pieces. For syllogism needs a general proposition for its major premiss; and as Aristotle himself insists, we cannot be said to know the truth of the conclusion,

[1] 'For induction is by means of all': *Anal. Pri.* β. xxiv. 68ᵇ 15–29.

unless we know first the truth of the premisses [1]; doubt of that will involve doubt of what is stated in the conclusion. so far as this is arrived at by inference, and not by direct experience independently of the inference. Now how can this condition be fulfilled, if our knowledge of any general principle rests on nothing better than an enumerative assurance that it holds good in every particular case? Let us take the principle that all matter gravitates, and symbolize it in the form 'All M is G'. If it is possible to know this without experience of its truth in every parcel of matter, one may use it in order to prove that this book must gravitate; and therefore may refrain from adding the book to one's kit in going up a mountain, or laying it upon a flower that is for show, or on the other hand may use it to keep one's papers steady in a wind. But if the principle can really only rest upon a complete enumeration, we must experiment with *this* book, before we can assert it; and then we shall know that this book gravitates by direct experiment, and our deduction thereof from the general principle will be superfluous, even if the enumeration be complete—as it would only be, if there were no other parcels of matter left to be experimented with; but even so, the deduction would be but a hollow show, and begging of the question. For let us symbolize any particular parcel of matter by μ. We propose to prove that μ is G, because all M is G, and μ is M; how do we know that all M is G? Only because μ_1, μ_2, &c. up to μ_n are G, and μ_1, μ_2 . . . μ_n are all the M, and therefore all M is G. Hence we use the fact that μ is G to prove the principle by which we prove that μ is G. And the upshot of this is that we can never prove anything by reasoning, until we already know it by direct experience; so that the use of reasoning, in order to infer that which we have not learnt by direct experience, must disappear. If we still try, by appeal to any general principle, to prove anything which we do not already know, we shall be appealing to a general principle which we do not know to be true, in order to prove a particular conclusion which we do not know to be true; for *ex hypothesi* our knowledge of the truth of the general principle depends upon the knowledge of what occurs in the particular case in question among others. Such a procedure hardly commends itself to a sane man. And if again it were said, that however little we may be logically justified, in advance of experience, in drawing inferences about some particular from a general principle, yet our experience when it

[1] *An. Post. a.* ii. 72^a 25–^{b}4.

comes is constantly confirming the inferences we thus draw, this, far from being a solution of the logical difficulty in which we have found ourselves, ought only to be matter of perpetual astonishment, to a creature that reflects at all upon his experience.

Such is the difficulty that arises, if there is no other means of proving a general proposition than by enumeration of all the particulars to which it refers[1]; and to this criticism Aristotle is obnoxious, if he recognized no other means. But did he recognize no other?

Now Aristotle undoubtedly says that we arrive at our first principles by a process of Induction.[2] He draws a famous distinction between the order of nature and the order of experience[3]; in the order of nature, the general principle is prior to the sensible fact; in the order of experience, it is the reverse. To us, the particulars of sense are known first: the intelligible principles by which these are explained are known afterwards; but Nature may be conceived as starting with principles or laws, and with these in her mind proceeding to the production of particular objects or events. Induction proceeds from what is first in the order of experience to what is first in the order of nature: from the apprehension of the sensible facts to the apprehension of the general principles, out of which we subsequently construct the sciences. Without sense-experience, there is no knowledge of intelligible principles; and the process of obtaining that knowledge out of sense-experience is Induction.

And this, taken together with his analysis of the Inductive Syllogism, might seem to settle the question of how Aristotle conceived that we come to know general propositions; if only we could suppose him capable of overlooking the difficulty in which his whole system would thereby have been involved. But so far from overlooking, he shows in one passage that he had considered it, and uses his distinction between what is prior in nature, and prior in our experience, in meeting it.[4] His view seems to have been this.

The business of any science is to demonstrate the properties of a kind—such kinds, for example, as geometrical figures, species of animals or plants, or the heavenly bodies. As we saw in the chapter on the Predicables, he was influenced much by the fact that geometry and biology were the two most progressive sciences of his day. Science is concerned with kinds, as what are identical in their

[1] Cf. what was said in discussing the *Dictum de omni et nullo*, pp. 301 sq.
[2] See e. g. *An. Post.* β. xix. 100^b 4.
[3] *φύσει πρότερον* and *ἡμῖν πρότερον*: cf. p. 88, *supra*.
[4] *An. Post.* α. iii.

many members, and eternal. In demonstrating their properties, it starts from a knowledge of their definitions ; such definitions cannot themselves be demonstrated ; and for them we are dependent on experience, which familiarizes us with the nature of any kind, or of its properties, by means of particular cases. But though experience may thus acquaint us with the definition of anything, yet the essential nature of a thing (which is what a definition gives) cannot possibly be an empirical fact. It may be an empirical fact that all sailors are superstitious ; but how can it be an empirical fact that a triangle is a three-sided rectilinear figure ? For to say that anything is an empirical fact implies that it might (so far as we can see) have been otherwise ; and certainly we can conceive that a sailor may be either superstitious or not superstitious ; but we cannot conceive that a triangle should not be a three-sided rectilinear figure, since if that—which is its essence—were removed, there would be no triangle left to be anything else. It will be asked, how do you know what constitutes the essence of anything ? The answer is, that the intellect sees it : sees it, as we might say, intuitively, as something necessary ; and this is the source of our assurance, in virtue of which we know the principles from which our demonstration proceeds more securely even than the conclusions we draw from them. But the intellect does not perceive it at once ; experience of things of the kind is necessary before we can define the kind. The use of these particulars is, not to serve as the proof of a principle, but to reveal it : as the counters, for example, which a child uses in learning the multiplication table, though one among innumerable instances of the fact that three times three is nine, are to be appealed to not because the general proposition could not be asserted unless it were tried and found true in the case of these counters as well as of all other countable things : for had the child learned with nuts, it would have been quite unnecessary to confirm the generalization by an examination of the counters ; but because they serve as a material in which the child can be brought to realize the truth of a numerical relation, which it apprehends forthwith with a generality that goes far beyond these particular counters. They are a means used because some countable material is necessary in order to realize the general truth ; but the general truth is not accepted simply because it is confirmed empirically by every instance.

Now we need not ask at the moment whether the sort of intellectual insight with which we do apprehend the necessity of numerical

or spatial relations[1] can really serve us in determining the essence of gold or of an elephant or a tortoise; our present purpose is only with the nature of Induction, and the different senses in which the term has been used. And the purpose of the preceding paragraph is to show that in spite of the analysis which Aristotle gave of Induction as a logical process, yet when he said that we get our first principles by induction, he had something else in his mind. Where your units are species, and you want to prove something about the genus to which they belong, there you may proceed by appealing to the fact, that it is found true of every species in the genus; there your reasoning may be thrown into the form of the 'inductive syllogism',—which is inconclusive unless every species is included in the premisses. But even there, from the fact that he regarded the conclusion as an universal and not merely an enumerative proposition, we must suppose Aristotle to have thought that the mind grasped a necessity in that relation between the terms of the conclusion, at which it arrived by a process of enumeration; directly or indirectly, the connexion of longevity with gall-lessness was to be seen to be necessary, and freed from the appeal to all the species. And where your units are individuals, and you want to discover the essential nature of the species to which they belong, there you do not work by an inductive syllogism that summons all the instances to bear witness to the truth of your definition; for how could you summon the numberless members of a species? There is still a use for experience; we may still say that we know these things by induction; but the induction now is a psychological rather than a logical process; we know that our conclusion is true, not in virtue of the validity of any inductive syllogism, drawing an universal conclusion in the third figure because the subject of the conclusion is coextensive with the particulars, taken collectively, by means of

[1] There are philosophers who would not agree with what has been said of the nature and grounds of our assurance of the truth of mathematical principles. Some hold that they are only generalizations from experience, deriving their high degree of certitude from the great number and variety of the instances in which they have been found to be true. This doctrine is maintained by J. S. Mill in a well-known passage of his *System of Logic*, Bk. II. cc. v–vii, to which he refers in his *Autobiography* (p. 226) as a crucial test of his general philosophical position. For a partial examination of the passage, crushing so far as it goes, see Jevons's *Pure Logic and other Minor Works*, pp. 204–221. Others again hold that at any rate geometrical axioms are only the simplest and most convenient assumptions that fit the facts of our experience: *v.* H. Poincaré, *La Science et l'Hypothèse*, c. iii. ad fin. 'De la nature des axiomes', pp. 64–67.

which we prove it : but in virtue of that apprehension of the necessary relation between the two terms, which our familiarity with particulars makes possible, but which is the work of intellect or *νοῦς*.

Such seems to have been Aristotle's doctrine : and thus he avoided the bankruptcy that would have ensued, had he taught that all syllogism rested on universal propositions, and that universal propositions rested on nothing but showing by enumeration that they held true in every particular instance that could be brought under them. But it may be said that thus he only avoids the Charybdis of moving in a logical circle to be snatched up by the Scylla of an arbitrary assumption. We are to accept the general propositions upon which every subsequent step of our inference rests, because our intellect assures us of their truth. This may satisfy the man whose intellect gives him the assurance ; but how is he to communicate that assurance to others ? If a principle is not arrived at from premisses which another admits, and between which and it he sees a valid process of inference to lie, why should he accept that principle ? No evidence is offered, whose sufficiency could be tested. The *ipse dixit* of an incommunicable intuition takes the place of any process of reasoning, as the means whereby we are to establish the most important of all judgements—the general propositions on which the sciences rest.

Of this charge Aristotle cannot altogether be acquitted ; yet we may say this much in his defence. Such an intellectual apprehension of the necessary truth of the principles from which demonstration is to start forms part of our ideal of knowledge [1] ; doubtless it seldom enough forms part of the actuality. But Aristotle idealized ; he spoke of what, as he conceived, science in the fullest sense of the term involved, and forgot to state, or failed to see, that the sciences did not attain it. And the prominence which he gave to the question ' What sort of premisses does knowledge require ? ' led him to

[1] With this proviso, that for perfect knowledge all the parts of truth ought to seem mutually to involve each other. In mathematics, where alone we seem to achieve this insight into the necessity of the relations between the parts of a systematic body of truth, we find our theorems reciprocally demonstrable ; and if twice two could be three, the whole system of numerical relations would be revolutionized. Yet we do not need to wait till we discover how all other numerical relations are bound up with the truth that twice two is four, before we are as fully convinced of this truth as we are capable of becoming. Whether in every science we should desire that each principle should thus be apprehended as necessarily true, even when cut off from its implications, may be doubted.

relegate to an inferior position the question 'How can the sciences as they are validate their premisses?'

He did not overlook this last question altogether; indeed he devotes to it a considerable portion of the longest of his logical treatises, the *Topics*; for when he asks by what sort of considerations you can prove or disprove that a proposition gives in its predicate the definition, or a property, of its subject, he is asking how you can prove scientific first principles.[1] And he knew this; and among the uses of Dialectic, or of the disputation whose methods he elaborates in the *Topics*, he places as its most proper use the examination of the truth of scientific principles.[2] But he ought to have seen that, outside mathematics, we seldom have any other means of establishing general propositions upon the evidence of particular facts than those of the kind which he discusses in the *Topics*. For the rest, his account of the logic of the reasoning by which the sciences do as a matter of fact support the general principles which they accept contains hints which are in advance of much modern 'inductive logic'; though there is much in his conception of the character of the general principles which science seeks to establish, that is now antiquated. Science seeks to-day to establish for the most part what are called 'laws of nature'[3]; and these are generally answers rather to the question 'Under what conditions does such and such a change take place?' or 'What are the most general principles exemplified in such and such a change?' than to the question 'What is the definition of such and such a subject?' or 'What are its essential attributes?'[4] It is more in respect of the

[1] Though Aristotle does not mention among the premisses of demonstration propositions giving the properties of kinds, and says that it is the business of science to prove these, yet he allows incidentally that some such propositions are indemonstrable (cf. *Anal. Post.* α. iv. 73ᵃ 37 '3) —e. g. that a line is straight or curved, a number odd or even. In point of fact, as Professor Cook Wilson has pointed out, the sciences in such cases assume that the genus displays these alternative properties, but prove which it is that belongs to some species of the genus.

[2] Cf. *Top.* α. ii. 101ᵃ 34–ᵇ4.

[3] The term 'Law of nature' is used especially of these most general principles, though sometimes of derivative principles as well. Cf. J. S. Mill, *System of Logic*, III. iv. 1, where Laws of Nature in the strict sense are said to be 'the fewest and simplest assumptions, which being granted, the whole existing order of nature would result'.

[4] I think this contrast is substantially true; though it is possible to bring many scientific investigations to-day under one or other of the types of question which Aristotle says we enquire into, yet looking to his examples, one must confess that (as is natural) he put the problems of science to himself in a very different manner from that in which scientific men put them now. Cf. *An. Post.* β. i. 89ᵇ 23 *τὰ ζητούμενά ἐστιν ἴσα τὸν ἀριθμὸν ὅσαπερ ἐπιστάμεθα.*

problems to be answered, than of the logical character of the reasoning by which we must prove our answers to them, that Aristotle's views (as represented in the *Topics*) are antiquated.

We may briefly indicate the nature of 'dialectical' reasoning, as Aristotle conceived it, and of the 'topics' which it employed. Dialectic is contrasted with science. Every science has its own peculiar subject-matter: geometry investigates the nature and properties of lines, surfaces, and figures in space, geology the conditions which determine the character and distribution of the materials which form the crust of the earth, physiology the functions of the organs and tissues of living bodies, &c. Each science, in explaining the facts of its own department, appeals to *special principles*, or *ἴδιαι ἀρχαί*; to the specific nature of its own, and not another, subject-matter—to laws in accordance with which that particular class of facts is determined, and not another class. The geometrician makes use of the axiom of parallels, of the notion of a straight line, of the definition of a cone or circle; but the nature of chalk or granite is indifferent to him. The geologist will use such principles as that stratified rocks are sedimentary, or that mountains are reduced by denudation; but he draws no conclusions from the definition of a cone. The physiologist in turn has his own problems to explain, and his own principles to explain them; that every tissue is composed of cells which multiply by division is a physiological principle of which we hear nothing in geology, while the laws of denudation contribute nothing towards the explanation of the growth of living bodies.[1] Dialectic, on the contrary, has no peculiar subject-matter; all the sciences submit their principles to its investigation; the dialectician may ask whether a geometer would be right in saying that it is a property of a rectilinear triangle to have its exterior angles equal to four right angles: whether the geologist has rightly affirmed all

ζητοῦμεν δὲ τέτταρα, τὸ ὅτι, τὸ διότι, εἰ ἔστι, τί ἐστιν. ('The subjects of investigation are equal in number to the subjects of knowledge: and we investigate questions of four sorts—facts, their reasons, whether something exists, what it is.')

[1] One science does often to some extent use the results of another. In particular, of course, all the other sciences resolve all they can into terms of chemistry and physics. Yet looking (say) to Physics, Chemistry, Physiology, and Political Economy, no one will deny that they must continue to rest each in part on different principles, even if the later mentioned may have to take note of some facts whose explanation involves the principles of the earlier mentioned. Aristotle noted such partial use by one science of the results of another; though the state of the sciences in his day prevented him from illustrating it as it would be illustrated now, and his remarks on the subject are open to a good deal of criticism. Cf. *An. Post.* *a.* xiii. 78^{b} 32–79^{a} 16.

stratified rocks to be sedimentary: whether the physiologist would do well to accept Spencer's definition of life, as 'the continuous adjustment of inner to outer relations'. And in debating such questions, the dialectician will invoke not *special*, but *common principles*, *κοιναὶ ἀρχαί*[1]—i.e. not principles whose application is confined to the science he happens to be investigating, but principles of universal application: as, for example, that what is common to the genus is not a property of the species—whence it follows, that since all plane rectilinear figures have their exterior angles equal to four right angles, this is not a property of a rectilinear triangle, or in other words, that it is because a plane figure is rectilinear, and not because it is three-sided, that this can be predicated of it; it is for the geometer to show that all plane rectilinear figures have their exterior angles equal to four right angles; the dialectician's business is to show that therefore it cannot be called a property of a triangle, as such. Or again, the dialectician may ask, with regard to Spencer's definition of life, whether the distinction between 'inner' and 'outer', on which it rests, is clear; for he knows that the terms of a definition should be clear, though he does not necessarily know physiology; and if Spencer, or his disciples, could not show precisely what it means, he would say the definition must be faulty; and if they replied that 'inner' meant within the organism, and 'outer' outside it, he would ask whether all material systems which change inwardly in response to changes outside them are living bodies; for he knows that a definition should not apply to anything except the species defined, and if this expression does, it cannot be a definition; or he might ask whether many of the peculiar processes of living bodies are not apparently initiated from within the body; and if the answer

[1] Cf. *Anal. Post.* *a.* x. 76[b] 11-22, xi. 77[a] 26-34, xxxii. 88[a] 31-[b]3, [b]9-29. In the second of these passages, Aristotle gives as examples of 'common principles' the Law of Contradiction, that the same proposition cannot be at once true and false, and the mathematical axiom that the differences between equals are equal. The latter is not really 'common', but special to the sciences of quantity; and if he wished to be consistent with what he says in *β.* xvii. 99[a] 6-16, Aristotle should have allowed that it means something a little different in geometry and in arithmetic. By no means all of the *communes loci* in the treatise called the *Topics* are 'common principles'—e.g. the topics given in *γ*, *περὶ τοῦ αἱρετωτέρου*, which are principles to be appealed to in determining which of two goods is to be preferred: as, that the more lasting good is preferable, or the more secure, or the greater, or the nearer. Most of them however are such, though it must be admitted that Aristotle does not describe his topics as common principles, or *κοιναὶ ἀρχαί*: and I think that the distinction which he intends to convey in the *Posterior Analytics* by the antithesis of *ἴδιαι* and *κοιναὶ ἀρχαί* is really what has been stated in the text.

was affirmative, he would again object to the definition; for though it is not his business to know whether any of the peculiar processes of living bodies are initiated from within or not (and therefore he has to ask the physiologist how that matter stands) it is his business to know that a definition must include everything essential to the thing defined; so that if there are such processes, a definition of life which excludes them must be a wrong one. Or, lastly, the dialectician might ask the geologist if there are not some igneous rocks that are stratified: not knowing, as a dialectician, the answer to that question, but knowing that, since igneous rocks are not sedimentary, the existence of igneous rocks that are stratified would upset the geologists' proposition; while if the geologist were able to answer the question in the negative, he would so far have come out victorious under examination.

All these general principles, to which the dialectician appeals, are called *topics*[1]: it is a topic, that what belongs to the genus is not a property of the species; or that what in some particular instance is absent from a species is not a property of it; or that the terms of a definition must be precise, or that it must be commensurate with what is defined. All these principles hold good in any science; it matters nothing what the species may be, or what the property, or what the definition. A man therefore whose mind is stocked with principles of this kind has points of vantage, as it were, from which he may proceed to attack or defend any definition, any predication of a property; they are topics in common, 'commonplaces', considerations to which you may turn in examining the statements of any science. Just as a man who knows nothing of the truth of its premisses may be able to detect a flaw in a syllogism, so the dialectician, without a scientific knowledge of a subject, may know what sort of questions to ask, if he wishes to test a scientific man's right to affirm the principles he enunciates.

Aristotle's *Topics* is written with reference to his doctrine of Predicables. He regards every proposition as asserting (or denying) some accident, property, differentia, genus or definition, of its subject; and he asks, to what considerations are you to look, if you would know whether such and such a predicate does stand to such and such a subject in any one or other of these relations? Each of these considerations is a topic. He details an astonishing number of them. They are of very different degrees of importance and value. Some are drawn from language. Look, he says, for example, to

[1] τόπο, *loci*, *communes loci*.

conjugate words—the various words, that is, from a common stem; if noble is a property of just, then *justly* is *nobly*; perhaps a man who affirmed generally that justice is noble might admit that it is possible in some cases to act justly and not nobly.[1] Others are based on the principle that contrary things have contrary properties; so that you cannot say that the just is the equal, unless you can say that the unjust is the unequal. Some aim only at enabling you to determine whether an expression is elegant, according to accepted rules. But others are principles of great importance. For instance, there is what we might call the topic of Concomitant Variation[2]; that is not a property of a subject which does not increase or decrease with an increase or decrease in the subject, and conversely, if you find two things increasing and decreasing together you may assert such connexion between them.[3] Considerations of this kind enable you to judge how different concepts are related to one another; and relations between concepts furnish the principles with which the special sciences work.

It may be admitted that this treatise contains much that is trivial; that it throws together considerations, or principles, of great and of little cogency; that the problems of science assume other forms than determining the definition of a subject, its properties, or its accidents (although such problems occur too, and many problems which we should not express in those forms can be translated into terms of them). It may also be admitted that Aristotle had his mind fixed too exclusively upon debate. The answers to the questions asked were to come from the respondent—the other disputant; but in building up the sciences, they must come from the field and from the laboratory. Aristotle would have a man test any scientific doctrine that is put forward by interrogating its maintainer; the man of science must test those which he himself or a fellow worker puts forward by interrogating nature. It would be easy to do Aristotle an injustice on this head. It may be assumed after all that the respondent testifies to what he has seen; and Aristotle was alive to the importance of collecting and recording facts.[4] But the *Topics* is a treatise on the art of disputation; disputation aims after all more at silencing an opponent than at establishing truth; and though we are told that Dialectic has its use as much in the examination of the principles of the

[1] Cf. *Top.* ε. vii. 136[b] 15.
[2] τόπος ἐκ τοῦ μᾶλλον καὶ ἧττον.
[3] e. g. *Top.* ε. viii.
[4] *Anal. Pri.* α. xxx.

sciences as in the conduct of a disputation, it is in the latter spirit that it is expounded. Nevertheless, in the distinction drawn between scientific and dialectical reasoning, as illustrated above, and in its account of the general nature of the considerations to which one must appeal in any defence of the principles of a science, the *Topics* is a work of great logical value.

What, then, has Aristotle to say about Induction ?

1. He gives the name to a formal process of inference, by which we conclude a proposition to hold universally of some class, or logical whole, because an enumeration shows it to hold of every part of that whole. This is what has been since called *Induction by Complete Enumeration*, or *Perfect Induction* ; and he shows how it might be thrown into the form of an *Inductive Syllogism*.

2. He points out that our knowledge of scientific principles springs historically out of our experience of particular facts ; though its certainty rests ultimately upon an act of intellectual insight. And he gives the name of Induction to the process in which the particulars of our experience suggest to us the principles which they exemplify. But this is not a formal logical process from premisses to conclusion ; and it is not the induction (in this sense) which leads us at the end to accept such principles, but our intellect, or *νοῦς*.

3. He shows where (presumably in default of the necessary insight and assurance from our intellect) we may look for reasons for accepting or rejecting any principles which a science puts forward. He does *not* give to this procedure, which *is* of a formal logical kind, the name of Induction, but calls it Dialectic ; nevertheless what he says on this head is of much the most importance from the point of view of scientific method, and comes much closer to what modern writers understand by Induction.

Thus he admitted that our knowledge of general principles comes from our experience of particular facts, and said that we arrive at them by Induction ; but the only formal logical process which he described under the name of Induction was that ' Perfect Induction ' which clearly neither is nor can be the process by which the sciences establish general propositions ; while the kinds of process which they really do employ, so far as they appeal merely to the evidence of our experience, he described under a different name. It is not surprising that some confusion has resulted.

The critics of whom Bacon is the coryphaeus, recognizing with Aristotle that we discover universal truths by induction, attacked

him for saying that we only discover them by complete enumeration, which he had not said; and finding the name Induction given to no other formally valid process than this[1], supposed he had nothing else to say of the processes by which such truths are reached. Bacon himself attempted to systematize the process of discovering and proving them in a way which undoubtedly possesses value, and no less undoubtedly owes much to Aristotle; but as the Aristotelian doctrines on which it is based do not occur in the *Organon* in connexion with ἐπαγωγή, he hardly realized how much he was borrowing. His analysis is offered in connexion with an unworkable theory of the nature of the problems which science should set itself to solve. To put it summarily, he thought that a list of the several sensible properties of bodies should be drawn up, and that men should then try to discover on what particular principle of corpuscular structure in the bodies exhibiting it each property depended. There was nothing in any particular principle of structure which would lead you to anticipate that its presence would involve any one sensible property more than another; you could not tell, apart from experience, that a particular motion of the component particles of a body would exhibit itself to the senses as heat, or that a particular disposition of its surface particles would show as white, and another particular disposition as black. Suppose we were to symbolize the sensible properties of bodies by Greek letters, and the principles of corpuscular structure in them on which these depend by Roman letters: how are you to prove whether a property α is connected with *a* or *b* or *z*? Bacon's answer is as follows. He called the principles of corpuscular structure Forms: whatever be the Form of a given property α, it must be so related to α as to be present in every body in which α is present, to be absent from every body whence α is absent, and to increase or decrease in any body as α increases or decreases. Our problem then is, as he says, *ut inveniatur natura alia* (the Form) *quae cum natura data* (the sensible property) *perpetuo adsit, absit, crescat atque decrescat.*[2] How are we to solve it? No mere enumeration of instances in which a sensible property α and a Form *a* are present together will prove that they are thus related, and that *a* is the Form of α; for your enumeration must be finite, but your conclusion is to be universal. You may

[1] It was also given to Induction by *simple* enumeration—i. e. to any attempt to prove a general proposition by merely citing a number of instances of its truth; but this is not a formally valid process.

[2] *Nov. Org.* II. 4.

find a hundred bodies exhibiting both *a* and α: yet the presence of one may be quite unconnected with the presence of the other, and you may find a body to-morrow exhibiting one without the other. We must proceed then by *exclusions*. Where a hundred instances will not *prove* an universal connexion, one will *disprove* it. This is the corner-stone of his method: *maior est vis instantiae negativae*.[1] If we had drawn up an exhaustive list of the different principles of corpuscular structure present in bodies in different combinations, all we should have to do would be to find instances in which any of these was present in a body that did not exhibit the property α, or absent in one that did exhibit it, or in which it increased or decreased without a corresponding variation in the degree of the property, or vice versa. We could then confidently reject that Form; and when we had thus rejected every other Form, then we could confidently affirm that principle of corpuscular structure which alone had not been rejected to be the Form (or cause of the presence) of a given sensible property α. Our assurance would rest not on the positive testimony of its presence along with α in a number of instances, but upon the fact that we had disproved all possible rival theories.

It will be seen that this procedure presupposes that we know all the possible Forms, among which that of any particular sensible property is to be sought; and Bacon, though he promised to do so, never showed, and could not have shown, how we were to secure that. The procedure is formulated too under the belief, that the immediate task of science is to draw up a complete list of all the distinct sensible properties found in nature, and then look for what we should perhaps now call their physical basis. This belief was mistaken. But the fundamental principle of the method by which Bacon proposed to 'interpret nature', the principle on account of which he gave it the name by which he called it, *Exclusiva*, is correct; it is that where you cannot (as in Mathematics) see that a proposition *must* universally be true, but have to rely for the proof of it on the facts of your experience, there there is no other way of establishing it than by showing that facts disprove its rivals.[2]

Bacon called this method inductive; it may be as well to point

[1] *Nov. Org.* I. 46. Cf. Aristotle, *Anal. Pri. a.* xxvi. 43ᵃ 14 ἅμα δὲ δῆλον ὅτι καὶ τὸ ἀνασκευάζειν ἐστὶ τοῦ κατασκευάζειν μᾶλλον ('And it is plain at the same time that it is easier to refute than to establish'): and more fully, *Top. η.* v.

[2] There are many very valuable remarks in Bacon's account of his 'Exclusiva' about the kinds of instances which are of most evidential value (and which he therefore calls *Prerogative Instances*); but a discussion of them would hardly be relevant to the present argument.

out at once that *formally* the reasoning involved is just that of a disjunctive argument, with hypothetical argument employed in the disproof of rejected alternatives. The alternative hypotheses (with Bacon, the alternative hypotheses as to the Form or physical basis of a particular sensible property) are so and so: such and such of them are false, for, if they were true, the facts would be other than we find them; therefore the one remaining is true. How we are to discover what the alternative hypotheses are, he does not explain to us; we are to prove that the rest are false by appeal to the facts of our experience; these facts he would have men methodically collect and tabulate, and in making use of them he relies upon the general principle that nothing can be the Form sought for which is ever present in the absence of the property whose Form it is alleged to be, or absent in its presence, or variable when it is constant, or constant when it varies; when he has got his premisses, his conclusion follows according to the ordinary principles of disjunctive reasoning.

Bacon wrote in the dawn of modern science, and proclaimed with splendid confidence its future triumphs. His predictions have been fulfilled, perhaps to the extent, though not on the lines, that he anticipated. *Spes est una*, he wrote, *in inductione vera*[1]; and as men watched the continuous progress of the inductive sciences, they came to think that induction was really some new form of reasoning, ignorantly or perversely rejected by our forefathers in favour of the deductive reasoning, which they associated with the name of Aristotle, and now held to be in comparison an idle thing. To praise induction became a sign of enlightenment; but the praise of it ran ahead of the understanding.

Those who did most to advance the sciences had not the need or inclination to pause and analyse the arguments which they were so successfully building up; nor would it imply any disrespect to add, that many of them probably had not the power of doing so. It is no more necessary that a great scientific genius should be able to give a correct account of the methods he uses than that a great artist should be able to expound the philosophy of art; those can often do things best who are quite unable to explain how they do them. The chief scientific name in the history of speculation upon the logic of the inductive sciences in this country is that of Sir John Herschell; four writers in all, if we exclude those still living, have

[1] *Nov. Org.* I. 14.

made the principal contributions to the subject. David Hume, in a brief section of his *Treatise concerning Human Nature* (Of the Understanding, Part III, Sect. xv), gives 'Rules whereby to judge of causes and effects' which contain the pith of much subsequent writing; but the work, as he said himself, 'fell stillborn from the press'; this section was not incorporated in the later and more popular 'Enquiry'; and it had no influence on the exposition of Induction. Sir John Herschell's *Discourse concerning the Study of Natural Philosophy* and the various works of Dr. Whewell did, on the other hand, much to stimulate interest in the subject; especially since Whewell propounded an explicit theory of it. The help which he had derived from both is acknowledged by J. S. Mill, whose *System of Logic* for many years held the field as an exposition of inductive reasoning. To that more than to any other work is to be traced the prevalence of the opinion, that inductive reasoning, or Inductive Logic as the theory of it, is a discovery of the moderns —an opinion which certainly contains less truth than falsehood. The name induction may be said with him to have stood for more than a particular form of inference; it was the battle-cry of a philosophical school, the school, as it is called, of experience. But as a result of this, and of its previous history, it has become one of the most confusing terms in Logic. It stands firstly for that induction by complete enumeration which Mill denies to be properly induction at all, but from which his influence was unable to withdraw the name after the prescription of so many centuries. It stands secondly for the logical processes employed in the inductive sciences, so far as these infer from particular facts the principles that explain them; as to what the nature of these logical processes is, Mill had a theory different from Whewell's, and others have since had theories different from Mill's. Thirdly, Mill, who admits that there are certain common principles assumed as true in the reasonings of the inductive sciences, gives the name to what he conceives to be the logical process by which these principles themselves are reached: a process that, in his view, starts barely from a great number of particular facts, and without the help of any general principles at all bases upon these facts the general principles whereon all other inductive inference rests. Many of Mill's critics have thought, and have thought rightly, that if the process by which these principles are reached were as he describes it, it could only be called an illogical process.[1]

[1] The second part of Jevons's *Principles of Science* ought perhaps to have

It would have been possible to omit the foregoing historical sketch, and to offer a purely dogmatic account of what Induction is, and what it is not. But against such a course there were two reasons. In the first place, a new writer has no right to do such a thing. It is indeed necessary for him to put forward that account of the nature of the reasoning of the inductive sciences, which he believes to be true; but not as if he was only delivering an accepted tradition. And in the second place, unless the reader knows something of its history, he can hardly fail to be confused by the diversity of senses in which he finds the word Induction used. Men have rightly felt that an antithesis could be drawn between the inductive and the deductive sciences; though they can be classed only according to their predominant character, since no sciences, except the mathematical, are exclusively the one or the other. On the strength of this they have most unfortunately erected an antithesis between Inductive and Deductive Logic: unfortunately, partly because Logic is one; the science which studies the nature of our thought embraces equally the processes of thought that enter into the construction of the deductive sciences and of the inductive: but unfortunately also, because it has led to much misunderstanding of the nature of inductive reasoning itself. What 'inductive logicians' have called Deductive Logic, contrasting their own Inductive Logic with it, expounded forms of argument that belong to their most typically inductive enquiries. Their 'inductive methods', as has been said above, and as will appear more fully by and by, are, so far as the argumentation is concerned, but a mixture of hypothetical and disjunctive reasoning; and these forms were supposed to be deductive. Nothing but confusion can result from grouping together all the processes traditionally called deductive, and opposing to them collectively those of inductive science. If any clear antithesis can be drawn between Deduction and Induction, we must not identify them with the forms of argument expounded respectively by Deductive and Inductive Logic. The names are perhaps unfortunate. Things may be different without being

been included along with the four works mentioned above (cf. also Lotze's *Logic*, E. T., Bk. II. c. 7). Among contributions on the part of living writers to the criticism of Mill's doctrines (for the great acceptance which his views obtained has made criticism of him a prominent feature of much subsequent writing on Induction) may be mentioned Mr. F. H. Bradley's *Principles of Logic*, Bk. II. Part ii. cc. 2 and 3, and an excellent discussion in Professor Welton's *Manual of Logic*, vol. ii. § 155.

contrary; but a difference indicated by terms formed from the same stem with different prefixes is apt to be thought a sort of contrariety. Hence we incline to think of Deduction and Induction as processes moving between the same points, but in opposite directions; Deduction, we think, argues from general principles to particular facts, Induction from particular facts to general principles. Even if this were true, such a statement tells us nothing of the difference in the nature of the reasoning between the two cases; and in point of fact, though there are arguments of those two kinds, the distinction is by no means the most important that can be drawn, does not coincide with the distinction between the arguments traditionally assigned to Deductive and Inductive Logic respectively, and leaves out some of the operations of reasoning that best deserve to be called scientific.

In the inductive sciences we do argue from particular facts to the principles displayed in them; in subsumptive syllogism we may draw conclusions about particular facts from the principles which we have inductively discovered. Thus a study of certain facts leads us to conclude that the air exerts a definite pressure upon the surface of bodies exposed to it, and from this we may deduce that if we pour mercury into an open glass bulb, it will exert that pressure on the surface of the mercury. This was part of the reasoning which led Torricelli to the construction of his barometer. So far, there is a contrariety and an antithesis. But if we look beyond this simple statement, and compare the structure of a syllogism with the structure of the reasoning by which the principle that the air exerts this pressure on the surface of bodies exposed to it is reached, we shall see a difference, but not a contrariety. Moreover, though propositions about particular facts are included among the premisses of inductive argument, it appeals to universal principles as well; only these universal principles do not, like those of the syllogistic reasoning contrasted with it, tell us between what determinate characters in things universal relations hold, but rather what kinds of relations between determinate characters of things a causal system involves. Induction and deduction then cannot be contrasted in respect that the premisses of the one are propositions about particular facts, of the other universal; for some universal principles are included among the premisses of both; the difference is in the nature of these. Nor again can they be contrasted in respect that the premisses of the one and the conclusions of the

other are propositions about particular facts; for though subsumptive syllogism may apply the rule in the major premiss to determine a conclusion about some individual thing, it may also (as when used in geometry) apply it to determine an universal conclusion.

But syllogistic reasoning is not confined to the deductive sciences, being common in the inductive sciences as well; nor is subsumptive syllogism the only reasoning traditionally called deductive. Between the use of general principles to determine conclusions about particular things in which they are displayed, and the use of an examination of particular things to determine conclusions about the general principles displayed in them, there is indeed a contrast. But it is not the most important contrast distinguishing deductive and inductive reasoning. The most important contrast is that which Aristotle intended to signalize when he opposed Demonstration to Dialectic. In demonstrative reasoning we have a real insight into the connexions of things. Where this is possible, though Aristotle thought that we used syllogism, yet, as we have seen, there is not really any subsumption. The conclusion need not be less general than the premisses: there need be no application of a rule invoked *ab extra*: the connexions may be traced in an individual subject, though between characters in it that are universal.[1] But we may use premisses that state connexions which we do not see to be necessary between certain characters in things; still, taking these as true, if combining them together we can see what consequences their truth involves in some actual or imagined complex of things, we so far have insight into facts. Explanation, therefore, though the premisses from which it proceeds are often not seen to be necessary, is yet to be ranked with demonstration in respect of the nature of the reasoning. On the other hand dialectical reasoning involves no such insight. There are, according to Aristotle, dialectical as well as demonstrative syllogisms; and the dialectical nature of a syllogism, as we saw,[2] turns on the character of the premisses, as well as the form of the statement. What concerns us here is the syllogism that argues from signs, not causes—e.g. that the rick is overheated, because it smells thus. If all ricks that smell thus are overheated, the argument is sound; but it explains nothing. Were one however

[1] Cf. *supra*, pp. 310–311; *infra*, pp. 437, n. 1; 524, n. 2; 545, n. 2.
[2] *Supra*, pp. 305–306.

to argue that it is overheated because being damp it allows the organisms in the grass to effect too rapid an oxygenation, though this argument agrees with the former in that it can be stated syllogistically, it differs in that it explains. Now the syllogism whose middle term is a sign Aristotle called dialectical, because it gave no understanding of the connexions in the characters of things. And this is equally true about induction. I establish inductively some general principle of connexion in nature when I can appeal to the evidence of particular facts to show that apparently either this principle holds, or none. But herein I do not make the connexion intelligible. I should establish it demonstratively, if I could show that it is involved in the existence of other known connexions. Thus repeated observations of ice floating on water, in various times and places, of various sizes and shapes, may lead me to conclude that ice is lighter than water; for as it floats irrespectively of size or shape, time or place, I can connect its floating with nothing but a less specific gravity. That it should be lighter, however, remains a brute fact, nowise apparently necessary. But if I could show that water expands in becoming ice, then, though this indeed is still a brute fact, yet, granting this, I see that the ice must float; so far, I have explanation, insight into the necessity of the connexion of facts, demonstrative thinking. And here it will be observed that we are concerned not with a contrast of opposite directions in the reasoning process between the same two points (as we saw that the contrast of deduction and induction was apt to be thought), but with the contrast of different ways of reaching the same point [1]: of establishing a general proposition either by insight into the necessity connecting the terms of the system of nature, or by appeal to particular facts which, though we can find no system of connexion in them unless our proposition is true, give us no insight into the necessity of it. It is in this contrast, rather than in the former, that the distinctive character of inductive reasoning is brought out. But it escapes us in any simple and collective opposition between the forms of reasoning traditionally assigned to Deductive Logic and those which 'inductive logicians' have called inductive.

[1] I owe this remark to Mr H. A. Prichard.

CHAPTER XIX

OF THE PRESUPPOSITIONS OF INDUCTIVE REASONING: THE LAW OF CAUSATION

'WHY is a single instance, in some cases, sufficient for a complete induction, while in others myriads of concurring instances, without a single exception known or presumed, go such a very little way towards establishing an universal proposition? Whoever can answer this question knows more of the philosophy of logic than the wisest of the ancients, and has solved the problem of Induction.'[1] However we may think of the knowledge possessed by the wisest of the ancients, the question which Mill asks is no doubt an important one. By what right do we ever generalize from our experience? and how can we tell when we have a right to do so? To these questions we must now attempt an answer. Afterwards we may note what other processes of thought besides generalization enter into the sciences; and then we shall be able to realize better the true nature of that antithesis between induction and deduction which was spoken of at the end of the last chapter.

The present chapter will address itself to the question, by what right do we ever generalize from experience. This is the primary question. Syllogism never generalizes. Unless it is provided with universal propositions for premisses, it cannot arrive at them in its conclusions, and even so, its conclusion is never *more* general than its premisses.[2] It is just this fact which raised the difficulty,

[1] Mill, *System of Logic*, III. iii. 3, concluding paragraph. Strictly speaking, a single instance never is sufficient—if we had really to rely on it alone without help from conclusions already drawn from other parts of our experience. Cf. Jevons, *Pure Logic and other Minor Works*, pp. 295-299; and also Lotze, *Logic*, §§ 252, 253.

[2] The third figure, when both premisses are singular propositions, may seem to furnish an exception to this statement, and it would hardly be a sufficient answer to recall the fact that this is the inductive figure; for the question is whether a syllogism can generalize, and it is hardly consistent with saying no, to add that it can only do so when its character is inductive. But the statement may stand, because all conclusions in this figure are particular or contingent. We may aim at generalizing—at finding a judgement which is true universally; but we have failed, with such premisses, to do it.

how to get the universal propositions which syllogism needs to start with. If experience gives us only particular facts, how are we to get universal conclusions out of them? A mere enumeration of particulars will justify a statement about no more than the particulars which have been enumerated, whereas we claim in any generalization to go beyond the observed facts on which the generalization is based, and to draw a conclusion true in any possible instance of some sort whatever. By what right do we do this?

The answer is that all induction *assumes* the existence of *connexions* in nature, and that its only object is to determine between what elements these connexions hold. The events of our experience are no doubt particular, but we believe the principles which they exemplify to be universal; our difficulty lies in discovering *what* principles they exemplify; in that, a close study of particular facts will help us; but were we to be in doubt whether there are any such principles or not, no amount of study of particular facts could resolve our doubt.

There are many ways in which this assumption may be expressed. It will be well to consider some of these, and to ask what precisely it is that we assume. We may then show that (as has just been said) it is hopeless to attempt to prove the assumption by any appeal to experience; and ask ourselves what justification we have for making it.

The commonest expression for it is the *Law of Universal Causation*, or (more briefly) the *Law of Causation*; again, we say that we believe in the *Uniformity of Nature*; but the same belief is implied in the distinction between *essential* and *accidental* circumstances, or in asking what circumstances are *relevant* to the occurrence of an event, or what are the *material* circumstances in the case. For only that can be called material, or relevant, or essential, without which the event would not have occurred, or whose non-existence or non-occurrence would have made some difference to it; and the existence or non-existence of any particular circumstances can make no difference to an event, unless there is some connexion between them and it. Were everything in nature loose and unconnected, it would be impossible to say that an event occurred *because of* any one thing rather than another. All these phrases therefore imply Causation, and imply Uniformity.

Both the *Law of Causation* and the *Uniformity of Nature* are phrases open to misunderstanding. There is a sense in which it is

the business of induction to *discover* laws of causation; in the plural, the term refers to the various particular principles of connexion exemplified (whether we detect them or not) in the course of nature; it is equivalent to *Laws of Nature*, or *Natural Laws*, such laws, for example, as that matter gravitates, or that organisms reproduce themselves after their kind. Used absolutely and in the singular, however, it means the principle that there are such particular principles, and hence we speak of the Law of *Universal* Causation, intending to assert that *every* event has a cause, and that no change occurs except under conditions with which its occurrence is connected universally. And it is because we believe its occurrence to be connected universally with such conditions, whatever they are, that we speak of the *uniformity* of nature. We do not mean to deny variety, but only to assert the unbroken reign of law. That which collectively we call nature is a vast assemblage of substances of divers kinds diversely intermingled: interacting with one another in ways that depend upon their abiding character and their shifting situation. Even what we call single things are highly complex, and their properties and behaviour depend upon their composition, and upon the situation in which they are placed relatively to other things; we may believe that whenever one complex thing of precisely the same kind is placed in precisely the same situation as another, it will behave in precisely the same way; nor is more required by the principle of the Uniformity of Nature; and yet we may doubt whether such precise repetition ever occurs. Watch the movements of a waterfall, how it breaks into a thousand parts which seem to shift and hang, and pause and hurry, first one, and then another, so that the whole never presents quite the same face twice; yet there is not a particle of water whose path is not absolutely determined by the forces acting on it in accordance with quite simple mechanical laws. No one would suppose that because these mechanical laws are unchanging, the waterfall must wear a monotonous and unchanging face; and so it is, on a larger scale, with the course of nature. Nature is uniform in the sense that under like conditions like events occur; and in fragments, as it were, she is ever presenting us with the repetition of conditions that have been fulfilled before; so that in fragments there is recurrence of like events enough. But sooner or later, because the assemblage of things is not quite the same as before, the likeness in the course of events is broken; from the beginning it was probably not complete.

Were it indeed possible for the procession of events to bring back precisely the state of things which had existed at some moment in the past, then it must follow, from the principle of the Uniformity of Nature, that the same procession would recur, and terminate again by reinstating the phase in which it had begun ; so that the history of the world as a whole would really repeat itself indefinitely, like a recurring decimal, and to a spectator who could watch it long enough, might seem as monotonous as the music of a musical box which, in playing, somehow wound itself up, to pass always from the conclusion to the recommencement of its stock of tunes. But nothing of this kind occurs ; the *Magnus Annus* is but a fancy ; and the uniformity of nature is consistent, as Mill said, with her infinite variety.

But it may be said, the Law of Causation is one thing, and the Uniformity of Nature is another ; every event may have a cause ; but the same cause need not always produce the same effect, nor the cause of the same effect be always the same. The human will, for example, is a cause ; but it does not always act in the same way under the same circumstances ; to-day in a given situation I may act meanly : yet it is possible that in a situation of the same kind I may act better to-morrow. The Law of Causation, if you will, is a presupposition of inductive reasoning ; but whether causes act uniformly, whether the same cause in the same situation always has the same effect (which is what the Uniformity of Nature means) can only be determined by experience.

To understand the relation of these two principles, we must enter a little more fully into the difficulties connected with the notion of cause. Most people, if asked what they meant by the word, would probably say, that they meant a thing (or person) producing a change in something else. But great trouble arises when we ask what producing is. If I say that a wave, by striking it, produces motion in a boat, I suppose myself to mean more than that the motion of the boat ensues immediately upon the contact with it of the wave. I imagine the wave to exert a force upon, to have a power to move, another body. But what this power in it is, we do not understand. Some persons indeed, when challenged on this head, will answer that they do understand it, because they have experience of exerting power themselves ; they know themselves as causes in their voluntary actions. But the answer is unsatisfactory, for the wave is not intelligent, and we cannot suppose that the action of an

intelligent being, and what we call the action of a machine, or other body whose movements are mechanical, are the same thing. Moreover, as Hume pointed out, the connexion between the movement of my limbs, and what I regard as the psychical cause thereof, is no more intelligible to me than that between their movement and the movement of a body which they strike. He therefore, and many since, have attempted to eliminate the notions of power, agency, or force, and to reduce the causal relation to uniformity in succession.[1] Words like *agency* or *power*, on this view, are *voces nihili*; we think we mean something more by them than habitual sequence, but we do not. We must say the same of *connexion*. We observe events together or in succession; their conjunction is plain, but not any connexion; and uniformity in their succession is all that causation means.

A little consideration however will show that we do not mean by the causal relation one of habitual sequence. We take uniformity in the succession of events—i.e. likeness in the conditions upon which like changes succeed—to be a *sign* of causal relation, but not the same with it. For when I say that a wave striking a boat causes it to move, I imply that the relation subsists between the blow of this wave and the ensuing movement of this boat; whereas uniformity can only be exhibited in the sequence of several such movements of this or other boats upon the blows of several waves. Connexion is between individuals; uniformity of succession is in the sequences of each member of one group of similars upon a member of another group of similars. We mean by the causal relation something that might hold between terms that were unique, and does hold between terms that are individual, even though there are other individuals of the same nature.

And there is another objection to defining the cause of anything as its invariable antecedent. Antecedent and consequent are events. But we cannot treat the world as a mere procession of events; there are also things to which the events happen. It is instructive to observe how Mill is forced to recognize this. In spite of having defined cause as 'the invariable and unconditional antecedent', he speaks of 'indestructible natural agents', such as the earth, as 'permanent causes', since the earth affects the movements of any pendulum upon its surface, and they cannot get out of the

[1] *Treatise of Human Nature, of the Understanding*, Part iii; and *Enquiry concerning Human Understanding*, §§ iv–viii.

range of its influence; he also calls oxygen and hydrogen causes of water.[1] Now the earth is not more antecedent than consequent in time to the movement of a pendulum which it attracts; and oxygen and hydrogen are ingredients necessary to the formation of water, but they do not happen like their combination. Cause no doubt implies change and succession. But there can be no change without something which changes, i.e., which persists through a succession of states. It would not be change but substitution, if one event succeeded another, and there were nothing but the events; just as a child does not change into the changeling which is substituted for it. Whatever difficulties there may be in understanding what a substance is, or the relation of a thing to its attributes, it is a desperate remedy to offer us instead a 'stream' of events, loose and disconnected, in relations of simultaneity and succession.

That causation is more than uniformity in the sequence of events of one sort upon events of another was felt by Mill when he defined the cause of an event as its invariable and *unconditional* antecedent. 'What writers mean', he tells us, 'when they say that the notion of cause involves the idea of necessity' is that the sequence of phenomena must be not only invariable but unconditional; and 'we may define the cause of a phenomenon to be the antecedent, or the concurrence of antecedents, on which it is invariably and *unconditionally* consequent'.[2] If we examine his explanation of this addition we shall see that without the notion of connexion or necessity his definition becomes altogether futile. For he distinguishes between positive and negative conditions, and the negative conditions of a phenomenon 'may be all summed up under one head, namely the absence of preventing or counteracting causes'[3]; and he explains an unconditional sequence to be one 'subject to no other than negative conditions'[2]. The cause of a phenomenon therefore is the antecedent, or concurrence of antecedents, on which it is invariably consequent in the absence of preventing or counteracting causes. Now what is a *counteracting* cause? If the cause of anything is only that whereon it uniformly follows, a counteracting cause should be that whereon it uniformly does not follow; and the invariable and unconditional antecedent of anything should be that whereon it habitually follows in all cases except those in which it habitually does not. By such a definition we might call anything

[1] *System of Logic*, III. v. 8, viii. 6: x. 4.
[2] *Ib.* III. v. 6.
[3] *Ib.* III. v. 3.

the cause of anything, and say that experience supported us. But clearly, when he speaks of preventing or counteracting causes, Mill is forgetting the analysis of the causal relation into uniformity of sequence, and is thinking of the necessity and the connexion which he professes to repudiate.

And we do mean something by the words *necessity* and *connexion*, and know what we mean, even though we are unable to see between what changes in a changing world connexion lies. Within the fields of geometry and mathematics, and in philosophical enquiries, we see this necessarily to involve that, and the connexions are apprehended with their terms. In that which changes we realize that there must be connexion between successive states, without knowing what is connected with what; we understand that a cause produces its effect necessarily, without understanding that it must produce just this or that effect.

But need a cause therefore act uniformly? In a sense, yes. If one thing the same in nature at different times, or two things the same in nature, are to act in situations the same in their nature, they must act on both occasions in the same way. This is not a generalization from experience: it follows from the sameness of thing and of situation. But to what extent things come again to the same situation, and whether there exist many things of one kind, we must learn from experience.

That a cause must act uniformly in the above sense we may the more readily realize, if we ask what is involved in the supposition that it does not. It will be found that this is tantamount to denying the existence of causal connexions altogether. For suppose that every event had a cause, but that there was no reason why the same event should have the same cause or the same cause produce the same effect on different occasions. There need therefore be no appearance of order in nature at all, but events might happen just *as if* all changes were fortuitous. As it is, we believe that plants produce seed after their kind; we do not expect to gather grapes of thorns, or figs of thistles; where we see garden fruit upon a wild stock, we look for a graft, convinced that the same stock will only bear different fruit in virtue of some material difference in the conditions. If any plant might produce any seed, or any seed any plant, and it was impossible to discover, in such circumstances as graft or soil—because no reason of the kind existed—why the same plant produced now one seed and now another, or the same seed

now one and now another plant, then we should just deny that there was any cause for that which happened. We should not say that there was always a cause, though the cause need not act uniformly. If two plants, whose nature is really the same, can determine the growth of totally different seeds, how can we call either the seed *of* that plant at all? Grant that a seed may sometimes be produced by a plant of its own kind, and sometimes by a plant of another kind, without any difference of circumstances, and merely because causes do not act uniformly, and you have really granted that anything may produce anything; flint and steel may produce seed instead of a spark, and oil raise the waves or quench a conflagration. But to say that anything may produce anything is to empty the word 'produce' of all its meaning. For the causal relation is a necessary relation, such that if you have one thing you *must* have another. To add that it does not matter what the other is, destroys the force of the *must*. The distinction between essential and accidental, material and immaterial, relevant and irrelevant, will vanish. So long as causal connexion is between determinate terms, there is a meaning in it. That is essential to health, without which health is impossible, and that is accidental to it which (though doubtless it has its effects) has no effect upon health. But if exercise, which is essential to my health to-day, should suddenly and without any change in my condition give me epilepsy to-morrow, and the loss of a letter in the post somewhere in the antipodes on the following day should give to one man epilepsy and cure another of it, then it would be impossible to say that anything was accidental, or anything essential, to the same result for two minutes together. And the discovery of causal connexions in the succession of events now would certainly be of no use in enabling any one to forecast the future; because the connexions themselves might have altered in the meantime. It is difficult to see how all this differs from denying that there are any connexions.

Uniformity of action is not indeed the fundamental element in the causal relation, for it depends on repetition of the action; the causal relation has nothing to do with *number* of instances, so far as its *existence*—though much so far as its *detection*—is concerned; it is bound up altogether with the *nature* or *character* of things, and the nature of anything is not a question of the number of such things that may be or have been fashioned. Yet if a thing is to have any determinate nature and character at all, there must be uniformity

of action in different things of that character, or of the same thing on different like occasions. If a thing *a* under conditions *c* produces a change *x* in a subject *s*—if, for example, light of certain wave-lengths, passing through the lens of a camera, produces a certain chemical change (which we call the taking of a photograph of Mount Everest) upon a photographic film—the way in which it acts must be regarded as a partial expression of what it is. It could only act differently, if it *were* different. As long therefore as it is *a*, and stands related under conditions *c* to a subject that is *s*, no other effect than *x* can be produced; and to say that the same thing acting on the same thing under the same conditions may yet produce a different effect, is to say that a thing need not be what it is. But this is in flat conflict with the Law of Identity.[1] A thing, to be at all, must be something, and can only be what it is. To assert a causal connexion between *a* and *x* implies that *a* acts as it does because it is what it is; because, in fact, it is *a*. So long therefore as it is *a*, it must act thus; and to assert that it may act otherwise on a subsequent occasion is to assert that what is *a* is something else than the *a* which it is declared to be. It may be replied that no two things ever are the same, and—what that reply must commit you to—that no one thing ever is the same for two successive moments. The fact of change is not disputed, nor the difficulty of finding two things that are qualitatively the same. But if the second has a different effect, that must be because of its qualitative difference from the first, and not merely because it is a second; and so far as it is qualitatively the same, the effect must be the same also: it being understood of course that to sameness of effect qualitative sameness is equally necessary in all the material conditions. To deny this is to deny the possibility of reasoning altogether. If we cannot truly make the same assertion about a number of things, then, as Aristotle observes, there will be no universal, and so no middle term, and no demonstration.[2] For an universal judgement connects a certain attribute with a certain subject in virtue of their nature and without regard to the frequency of their existence. If we can do this, we can make the same assertion about all things of such and such a kind; if we cannot do it, we are left with nothing but particular things whose attributes must be ascertained by inspection or experience of themselves; and not by transference of what we have in one instance found true of such a *kind* of thing to others of the kind.

[1] Cf. *supra*, p. 13.

[2] *Anal. Post. a.* xi. 77ª 5–8.

What holds for the relation of subject and attribute holds in this respect *eo ipso* for that of cause and effect. To suppose that the same cause—other things being equal—can have different effects on two occasions is as much as to suppose that two things can be the same, and yet so far their attributes different. To reply that two things cannot be the same, and that the same cause cannot be repeated, is either to miss the point, or to abandon reasoning. If it is meant that two complex things cannot be qualitatively the same, nor can conditions precisely the same in kind ever recur, such an objection misses the point. One need not maintain that such identity, or such recurrence, in fact occurs, though it is not perhaps inconceivable that it should; all that is maintained is, that so far as things are qualitatively the same they have the same attributes, and so far as conditions precisely the same in kind recur, they must, if there is such a relation as cause and effect at all, have the same effect. If, on the other hand, it is meant that there can be no qualitative sameness in what is numerically different, we can only say that if so, there is no reasoning. For in reasoning we trace connexions between things in virtue of *what* they are, not of their being numerically these individuals or those; and if we make this distinction, we must conceive that what one thing is another might also be; so that to deny the conceivability of qualitative sameness, or sameness in what things are, is to deny that we can distinguish what they are from individual things. But this denial that any identity is conceivable between different things is what is really at the bottom of the attempt to resolve the causal relation into uniformity of sequence. For the causal relation which connects a with x connects a cause of the *nature* a with an effect of the *nature* x. The connexion is between them *as* a and x, and therefore must hold between any a and any x, if they really are a and x respectively; in other words, it must be uniform. The denial of this is just the denial of universals. How then could we speak of things *of a kind*, or hold our sequences uniform except in the fact that they are sequences, since any other uniformity must consist in the same antecedent having the same consequent on different occasions?[1] The cause of an event might then indeed be anything to which it stood in a relation of sequence at all, and need no more be the same on different occasions than its antecedent need be; for we should have agreed that it was impossible

[1] Strictly speaking, even sequence could not be a feature *common* to two successions.

that the sequence of the same x upon the same a should ever be repeated.

Must it then be maintained that if man is a cause of any change, his acts are necessary, and the will not free? The problem of the human will is so notoriously difficult, that one might perhaps be content here to leave it on one side, merely pointing out that what has been said will still be true of causal relations displayed in things material, though it were otherwise with an intelligent cause. But it is worth while to notice that the interpretation of freedom which could make a man's acts undetermined by his character has been often rejected for the very reason that they then would not be his, and that if he might indifferently do or forbear an act, no matter what his character was, he would be nothing in particular. And it may be suggested that error lies not in holding a man's actions to be necessary, but in holding them to be mechanical.

It may throw some further light on the presuppositions of induction, to consider this distinction. A machine is a collection of material parts, interacting in various ways, so as to produce in some part or parts of it a desired movement, such as the regular revolution of the hands of a watch, the up-and-down or circular movement of a saw, the lowering and raising of the hook of a crane, with what is attached to it. But though intelligence has directed the building it does not direct the action of the machine; and if there has been a mistake in the building, whereby it fails to achieve the result desired, the machine will not correct the mistake, but will produce an effect rendered necessary by its construction, however far from that intended. For it moves in virtue of the 'laws' of matter. Any two of its parts (and in the last resort the parts which have to be considered might be molecules or atoms, or even smaller) interact together in divers ways, according to their kind, proximity, velocity, and what not; and every two are in some relation of interaction; and we believe that, if our powers of calculation were sufficient, we could derive the 'action' of the machine from the interactions of all its parts, as their necessary resultant. In this way would the changes of any closed material system be explained; and if no material system that we know is closed—i.e. shut off from interaction with bodies outside it—yet there are many in the explanation of whose changes most of what is outside them may be treated as constant.

Now consider what would be involved in the view that a man's

actions are mechanically explicable. One form which this view takes is to suppose that consciousness is an otiose thing, and that those movements of the body which we commonly think to be intentional are mere results of the state in which the body and things surrounding it were at the previous moment. We should then be, in Huxley's language, conscious automata; and the laws of matter and motion would of themselves have sufficed (if we may borrow an illustration from Professor James [1]) to produce the manuscript of Shakespeare's works—and indeed every edition and performance of them—though Shakespeare had been but a lump of organized matter as devoid of thought and feeling as the pen he wrote with, or the automaton of Vaucanson. Such a conclusion is paradoxical, but that does not of itself constitute a refutation. It is however impossible as a final account of things, because the facts of consciousness undoubtedly exist, and the theory can give no account of them. For it demands not only that a physical event should be physically determined, but that physical conditions should determine only a physical result. Mass and energy are to remain constant in amount, but to undergo redistribution in accordance with certain laws, which can be expressed in mathematical formulae enabling us to calculate the precise degree of change in one direction that will be involved in a given degree of change in another.[2] In these redistributions there is no room for knowledge or feeling among the 'forms of energy'; for mechanical conditions are to have their complete mechanical equivalent, in terms of matter and of motion, potential or actual. We cannot then bring conscious processes within the sweep of a mechanical explanation by regarding them as thrown off in the process of physical change, which they nowhere influence. But it might still be held that they are themselves explicable on the same lines as physical processes. On this view there would be psychic elements, however originated, affecting, suppressing, intensifying or fusing with one another in such ways that, from the laws displayed in their several interactions, we might deduce the resultant complex of our conscious states. Now

[1] *Principles of Psychology*, i. 132.

[2] Hence the saying of H. Poincaré, that a physical law is a differential equation, *Address on the Principles of Mathematical Physics*, St. Louis, U.S.A., Sept. 1904: *v. The Monist*, Jan. 1905, p. 3. Recent physical research has suggested doubts about the constancy of mass, or in other words about the unconditional truth of Newton's Law of Gravitation (cf. H. Poincaré, *Science and Method*, III, 'The New Mechanics'). But there would be some law of its variation, if it were not constant. Cf. *infra*, p. 418.

such a process would be as unintelligent in the soul as it is in a material system. What we call knowledge and what we call error would differ merely as two equally necessary results of pre-existing psychic conditions, like the action of a machine running truly and of one running awry. But in an intelligent operation of whatever sort the conclusion wherein it ends is not explicable as a result of the preceding stages. There is an advance to something new, not the old in an equivalent form. And this advance is a development, wherein the soul comes to be what it had it in it to be but was not. When the development is made, we see it as the realization of what went before. The man who comes to understand a puzzling set of facts finds in what he has come to apprehend the solution of his previous puzzles, the artist finds in his finished work that which he was feeling after, he who has discovered what he ought to do finds therein the answer to his questions. At the outset he did not know what the solution, the finished work, the right course of action would be; and he knows it now not as a result of his previous ignorance, or even as a result barely of his previous knowledge of other things, but because he is intelligent.

It follows that he, unlike a machine, is an unity, whose later states and actions are not calculable from the earlier, and involve in their explanation a soul, or an intelligence, distinct from any particular act or state. What the powers and resources of this unitary principle are is only learnt as they reveal themselves in its activity. Since it is not resoluble into an aggregate of interacting elements, its actions are not those of such elements, but are the manifestation of itself, of its own being, even though in different circumstances it would have acted in some ways differently. Again, since it is not compounded out of elements whose modes of interaction with each other can be exhaustively expressed in the formulae of 'laws', each soul may be unique. If indeed we all were purely rational, we should all think or act in the same way in the same situation; for reason is one. And if anything which passes in the soul is mechanically determined, the laws of that may be discoverable. But the nature of a finite soul is not formed by adding to complete rationality something not rational but mechanical; rather it is brokenly and partially rational, and the broken and partial rationality of each will develop differently. Moreover we are constantly confronted with novel situations, in which the rational course required is something special, not deducible from general laws. To say all this is

not to deny necessity in the development of a soul's powers; but it is to deny uniformity, so far as no second soul of the same nature is placed in the same situation.

The amount of success obtained by psychologists and physiologists in their attempts to discover laws of psychical development or change is not such as to refute this uniqueness; and certainly psychologists have failed to explain such matters as memory, or space-perception, precisely in this particular, that they have not made it intelligible why the conditions under which these arise should lead to them. On the other hand the natures of men are found to be in large measure the same, and to that extent generalization is possible. For generalization, and the use of what we learn from experience of one thing to anticipate experience of another, depend on the natures of things being repeated in instance after instance. So far as things have not the same nature, they will not act in the same way. This is but the other face to the fact, that so far as they have the same nature, they will.

If we may accept the foregoing account of the difference between an intelligent being and a material system[1], it follows that the causal relation is always necessary, and uniform on condition, but only on condition, that there are more instances than one of the same kind. Now that there are things of a kind is obvious; but things of one kind may and do exhibit individual variations. In the physical sciences we suppose these variations to be due to diversely combining elements of various sorts, but qualitatively the same in all instances of each sort. And the success with which changes in bodies have been accounted for on the assumption that they are composed of such elements, whose mutual interactions are statable in quantitative laws, or connected with what may be so stated, leads men to accept the mechanical view of physical changes as correct. Inanimate bodies certainly do not behave as if they were intelligent; many living bodies do not. Even for the execution of an intelligent plan, a material seems needed in which changes are mechanically determined, and the effects of causes calculable from laws.[2] A body certainly is not a mind, whether a mind can influence its movements

[1] Cf. with the foregoing a paper in the *Hibbert Journal* for April 1914, vol. xii. No. 3, *Mechanism, Intelligence and Life*. I have not here, as I have there, discussed at all the question whether living bodies are mechanical systems, or more akin to what is intelligent. On the relation of uniqueness to causality and to freedom, cf. further there, pp. 622–624.

[2] Cf. Lotze, *Outlines of Practical Philosophy*, § 21.

or not. So far as not influenced by a mind, these movements must exemplify mechanical necessity. They are displayed in things manifestly of many, but not indefinitely many, sorts. What the laws are according to which a thing of one sort produces a change in a thing of another sort, we learn from experience; but that there are such laws, that the causality in change involves uniformity, is evident to reflection.

But if we are to speak of laws exemplified in the changes of things, it is important to distinguish *conditional* and *unconditional* laws. A law indeed is not a cause, and does not act; it is a principle of action displayed in the things that do act, or are causes. The statement of a law is the statement of such principle of action, or of a connexion between the action or change of action in one thing, and change in another. The first law of motion is a principle of action; a body, left to itself, persists in its state of rest or uniform rectilinear motion; but it produces no change in another body, in virtue of that law, until they collide. The second law of motion is a principle of the action of one body on another. But it is the body moving with a certain momentum in a certain direction, and not the second law of motion, which causes another body to move accordingly.[1] Nevertheless, if the causal relation is necessary, a law must be necessary also; a statement of the way in which a cause does act is a statement of the way in which it must act. A law then can admit of no exception; or, what is the same thing, a law which admits of exception, and holds good only for the most part, is not the true law, and indeed as formulated is no law. A true law is true unconditionally; i.e. there are no conditions, beyond what are included in the statement of the law itself, variation in which can affect its exemplification. If there are such, the law is only conditionally true, i.e. exemplified under certain conditions, and not otherwise.

The first law of motion is an example of a natural law which would perhaps be regarded as unconditionally true—that every body persists in its state of rest, or uniform rectilinear motion, until it is interfered with by some other body. The same might be said of the law of universal gravitation, that all bodies attract one another with a force that varies directly as the mass, and inversely as the square of the distance.[2] Compare with these the principle that acquired characters in a plant or animal are not inherited. Supposing

[1] Cf. *infra*, p. 502, n. 1.
[2] But cf. *supra*, p. 411, n. 2.

this to be true (for it is still *sub iudice*), yet it need not be unconditional. We are not in a position to say that living things could not be so organized, in respect of their reproductive system, as to make acquired characters heritable, but only that, with the organization which we find, they are not heritable. That organization therefore may condition the truth of our principle. Just as the prevailing necessity for sexual union in the reproduction of all multicellular organisms does not exclude arrangements in some species which make them parthenogenetic, so there might possibly be conditions under which the non-heritability of acquired characters held good no longer. And as conditions may change, those realized at one time not being realized at another, so the conditional principles prevailing, or exemplified, may be different at one time from what they are at another. It appears to be the case that living matter can only be produced from other living matter; there is no spontaneous generation of it from the inorganic; *omne vivum ex vivo*. But many scientific men have supposed that though this is true and necessary now, yet in an earlier period of the earth's history, under very different conditions of temperature and so forth, it was not so.

Conditional principles are necessarily *derivative*: i.e. their truth, so far as they are true, follows from some unconditional laws, which *under given conditions* involve them as their consequence.[1] They therefore admit, theoretically if not as yet actually, of explanation. But derivative principles, or principles admitting of explanation, are not necessarily conditional. For when we call a principle conditional, we mean that the truth of our principle depends upon conditions which are *not stated in it*. If we bring the conditions into the statement, then, though it remains derivative, it is conditional no longer. Supposing that we knew precisely those conditions of organization in animals and plants which made acquired characters non-heritable; then the statement that in animals or plants *of that organization* acquired characters were not inherited would be unconditionally true, although no doubt the fact would

[1] Not as their effect. The relation of cause and effect is displayed between things acting together according to their several natures and the complex change which they produce; and it involves time. But the fact that the laws according to which each thing severally acts involve that in a given combination the things produce this complex change is not itself a change produced whether by the things or by the laws of their several action; and there is no before and after between the simpler and the derivative principles, as there is between the action of the cause and its effect; their relation is not a time-relation. Cf. *infra*, p. 502.

admit of explanation. It would probably not be called a law of nature, because derivative; but it would have all the necessity of a law of nature.[1]

The necessity then of the causal relation, and the uniformity that follows from it in a world containing many things of the same kinds, involve the truth, without exception or qualification, of all unconditional laws; but conditional principles admit of apparent exceptions, without derogation to such necessity and uniformity; and if we are ignorant of the conditions within which these conditional principles hold good, we cannot tell when the exceptions may not occur. To return to our previous illustration: if we do not know under what conditions of organization acquired characters are and are not heritable, we must be prepared to admit evidence that in some cases they have been inherited. Where, however, exceptions occur to some conditional principle, they constitute no exception to the Uniformity of Nature; but only imply that the conditions, under which that principle held good, are not fulfilled in the exceptional case. And the exception leads us, not to deny that 'nature is uniform', but to revise or to determine more precisely the particular statement of principle which we have found invalid. It is only unconditional laws that can have no exception.

It becomes therefore important to determine, if possible, when we have discovered an unconditional law. We may disregard here those derivative laws, which we may be capable of explaining from others more general than themselves; for they can only be unconditional if the more general laws from which they are derived are so. Now, if we have no better reason for accepting a law as unconditional than that by assuming it to be true we can account for the facts of our experience, then, though we might provisionally accept it, we can hardly be content with our warranty; for perhaps some other law might also account for the facts. But if (and this, as we shall see hereafter, is a distinction of the first importance in inductive theory) —if, without assuming it to be true, it is impossible to account for the facts of our experience, we should have to suppose it unconditional; though such impossibility may be hard to establish. Still, we should not be fully satisfied; for had the facts been otherwise, we need not have admitted the law; and we do not see, except on the hypothesis that the law is true, why the facts might not have been

[1] Cf. *supra*, p. 386, n. 3, and *infra*, c. xxii.; the non-reciprocating causal relations there discussed are all conditional.

otherwise. Complete satisfaction would only come, if the law which the facts had forced us to recognize should, when considered, appear self-evident.

Are there any unconditional laws known to us? There is no doubt that the fundamental principles of physical science are often so considered. It is often held that we have discovered certain physical laws prevailing throughout the material universe, in accordance with which every event in the material order takes place; that these laws are mechanical; and that nature is, in truth, and in the last resort, a purely mechanical system. And this view is supposed to be confirmed by the character of the principles with which physical science works. A great deal is purely mathematical; and about mathematical principles at any rate we can say that they are unconditional because self-evident; no apparent exception would make us doubt them or revise them; we should only doubt the alleged fact which was supposed to constitute the exception. And some of the most general physical laws have often been held to possess the same self-evidence; the first law of motion, and the laws of the conservation of energy and the conservation of mass, are instances. That anything should occur in the material system unconformably with these principles would then present the same kind of contradiction as that two and two should make five. The explanations of physical science, at least so far as they rested on laws of this kind, would be complete and final.

We have however seen that there are grave difficulties in a purely physical or mechanical theory of the world. Consciousness is to it unaccountable; and it cannot therefore be a complete or final theory. Hence many philosophers have suggested that in the last resort, instead of vainly attempting to explain consciousness in terms of physical law, we must find in physical law a manifestation of intelligence. This view may take the form of saying that an intelligent Being sustains the physical world, and directs its changes on mechanical principles, because it is important that men should be able to calculate and count upon them; and it is added that the Being who maintains these principles may depart from them, if on any occasion any better purpose is to be served by departing from them than by acting on them.

If, as seems necessary to admit, men, not as physical machines but as intelligent beings, produce movements in bodies, we cannot deny the possibility that some intelligence not connected with a

living body may do the same. And if the so-called physical laws themselves depend on the will of an intelligent Being, and his plan involves departure at any time from his own rules of action, what would appear to us exceptions to these laws would occur. But such departure cannot be arbitrary. Any one understanding the plan would see that the exceptions to the law were as necessary as the illustrations of it. The law therefore would not be unconditional, and fuller knowledge would introduce into the statement of it the required qualifications. Only, since intelligent action differs from mechanical, it would not be possible to express these qualifications in physical terms, and to substitute for the inaccurately stated law another really unconditional, and yet connecting changes in a mechanical way.

Other philosophers have sought to show in a different way that physical law is a manifestation of intelligence. They have pointed out that the material order is an object of apprehension, and therein stands related to the minds that apprehend it; and they have urged that the world and minds together form the complete reality, or *res completa*, and cannot be understood except together. There is indeed a special difficulty here in the fact that what understands is itself mind, so that one term in that relation has to understand both itself and the other term. With the problems of such Idealism we are not here concerned. But we may point out, in regard to the unconditionality of physical laws, that if they are known to be unconditional, the knowledge of them is not itself a condition of their truth. It is possible that we may some day know that matter gravitates as Newton supposed only under the condition that it is moving with less than a certain velocity, and so not unconditionally. But matter moving with less than that velocity would gravitate thus unconditionally. Whatever transformation our view of the material order may undergo, yet the interconnexions of events within it, the connexions of cause and effect there traced, will have to be taken over as it were *en bloc*, unbroken and undistorted, by any interpretation of the world which takes knowledge as well its objects into the account, and holds matter dependent on its relation to mind. What we call a moving body may be something else at bottom than a moving body; but its motion would not because of that any the less appear determined in accordance with physical laws.

It is different if a body is the subject of the *action* of a mind or spirit. That would condition the movement as the knowledge of

it would not. And if that is possible, the laws of motion which physics has formulated cannot be unconditionally true; for they make changes in the movements of bodies to be dependent altogether on other bodies, ignoring the influence of anything besides. Now if we could see the necessity of physical laws, as we can of mathematical relations, we should have to allow that they are unconditional. But this we cannot do. Some indeed have thought the first law of motion self-evident, as only saying that a body cannot change its state of rest or motion without a cause. But it says more than this; viz., that the cause can only be another body; and this is not self-evident, for we do not understand how one body causes a change in the state of rest or motion of another. It is true that neither do we understand how a mind or spirit does; yet we may have to admit it on the evidence of facts. But there is a little more to be said for the first principles of physical science. Intelligent action leads to something new, mechanical action does not; in a material system there is no development. Therefore the principles that express the inertia of matter, and the constancy of the ultimate facts like matter and energy, may be unconditionally true of a system purely physical. They will not therefore be unconditionally true of a physical system in relation to intelligent agents.

Nevertheless there is a great difference between what is meant when in the sciences a physical principle is called conditional, and what is meant in calling them conditional on something non-physical. We conceive that for any principle conditional in the former sense, such as the non-heritability of acquired characters, the conditions on which it depends might be found, and would be *in eodem genere* with the principle itself; i. e. the principle stated so as to include these conditions (and in that form called unconditionally true) would be derivative in an intelligible way from principles more general, but from principles holding, like itself, within what is material. But if the ultimate physical principles are called conditional, it is not because they can be derived from any *physical* principles more general than themselves, and the kind of explanation possible of the other sort of conditional principles, viz. showing that the facts exemplifying them really only exemplify simpler principles of the same sort with themselves, is here precluded. And if there are spiritual conditions upon which the movements of bodies to some extent depend, physical science cannot deal with these. For as a mind or spirit does not act mechanically, we

cannot from observed changes form an hypothesis as to its mode of action, and thence calculate the effects which it should produce in another situation.

For this reason, physical science will ignore such conditions. It is of no use to consider in our calculations a factor which is incalculable. The man of science, even if he believes that such conditions exist, will reasonably consider that he has no means of determining their influence, and that he can only discover the extent to which physical principles will account for physical changes by proceeding as if they would do so altogether. The principle of the Uniformity of Nature is sometimes understood as claiming that. It need not be. What has been maintained in the foregoing discussion is that the Law of Causation is presupposed (not reached) by induction; that so far as things and situations are repeated, it carries with it uniformity; but that it consists with this uniformity that there should be unique things, and principles only conditionally true, and so admitting of exceptions. An unconditional principle admits of no exceptions; and a self-evident principle is unconditional. The fundamental principles of physical sciences are often treated as unconditional; but they are not self-evident, and much occurs in this world which is not explicable from them. If they were self-evident, what follows from them would have to be retained and not contradicted in any complete explanation of the world that took into account what physical science leaves on one side. But if the first principles of physical science are only conditionally true, yet so far as the conditions under which they do and do not hold good are unascertainable, physical science may reasonably push ahead ignoring such conditions.

We argued indeed that it is no more than a corollary of the Law of Identity, that the same thing unaltered on different occasions, or two things of the same nature, should under the same conditions produce the same effect. But this does not show that anything remains unaltered, or that any two things have the same nature. It involves an assumption—if assumption it be—that what is real is intelligible or rational. Any one who questions this, to the extent that he does so, despairs of reason and thought; and his questioning cannot be set at rest by reasoning. The assumption however does not require us to deny uniqueness, and it governs our thinking in other fields besides that of cause and change. The causal relation is displayed in change, and involves time; an effect is always after

the action of its cause.[1] But the argument from identity of nature is used in generalizing where time and change do not enter, e.g. in geometry. And our understanding of connexion between one element and another in the being of things, where we have such understanding, though not the discovery that we must admit connexions which we do not understand, is independent of their repetition and consistent with their uniqueness.

With these explanations and qualifications we may say indifferently that the inductive sciences presuppose the Law of Universal Causation, or the Uniformity of Nature. But as it has been held by some[2] to be the task of induction to prove this principle about a world as to whose nature, prior to our observation of what happens in it, we must presuppose nothing, it may be worth while in conclusion to show that this is impossible. It is alleged upon the view now to be considered that our experience of the great extent to which like antecedents have like consequents is the ground upon which we believe that this is universally the case. Against this we may point out in the first place, that such an inference assumes the course of events in one time and place to be a guide to their course in other times and places: which is really the very principle that is to be proved. As Lotze has urged, if a reason can be given for the inference, it rests on some previous assumption; and if no reason can be given for it, what is its force?[3] Next it is to be noted that this view regards as of the same nature two arguments really very different. It is supposed that to infer an universal relation between two events a and x from the frequency with which one has been succeeded by the other, and to infer from the observed succession of like consequents upon like antecedents in divers pairs, a and x, b and y, &c., that every event is thus uniformly paired with some other, are arguments of the same form; and that since the former is allowed to have value, so must the latter. This however is not so. We infer from the frequency of their conjunction in a great variety of

[1] This is perfectly consistent with holding that there is no interval of time between them, just as one body being outside another is perfectly consistent with their being in contact. It is also consistent with things *interacting*. If A and B interact, the initial activity of A produces such a change in B that B then affects A otherwise than it did initially, and vice versa. The difficulty of dealing with or understanding the continuity of the operation is not in principle greater than if the action was not reciprocal. If A produced a change in B, and B did not react, its next effect on B would still be modified by the fact that B was no longer quite the same. In the first edition of this book, note 1 on p. 390 was wrong about this matter.

[2] Cf. e.g. Mill, *System of Logic*, III. xxi. [3] *Metaphysic*, Introd. § v.

circumstances a connexion between a and x, because *upon the assumption that there is some set of conditions upon which every change follows uniformly* the view that for x these conditions are a seems alone consistent with our experience. Now what is thus assumed is just the uniformity of nature; and without it the argument from the frequency of the succession a–x or b–y to their connexion could not be made. But the argument from the constancy of succession in divers pairs to the existence for every event of some set of conditions on which it follows uniformly can be made neither with this assumption nor without it. Not without it, any more than the other argument; and not with it, because no assumption can be used to prove itself. Again, the uniformities which are said to be the empirical basis of our generalization are not really matter of direct experience. We have said above, that the particular connexions which we believe to prevail in nature have been inferred with the help of the assumption that all changes occur in accordance with laws. But if any one likes to question this, he must at any rate agree that most of the uniformities in which we believe have been inferred somehow: very little has come directly under our observation. We believe that winds are caused by differences of atmospheric pressure: difference of atmospheric pressure is itself inferred rather than observed; but waiving that, for what proportion of winds have such differences been noted? We believe the sounds of a piano to be caused by the striking of strings: for what proportion of such sounds which we have heard have we first seen the strings struck by the hammer? It is needless to multiply such examples: but when it is alleged that we are justified in inferring the uniformity of nature to hold good universally because we have direct experience of it over vastly the larger portion of the field,[1] it is important to point out that our direct experience of it is singularly small, and that the vastly greater proportion of what we believe ourselves to have ascertained is matter not of experience but of inference. Now we may offer the empiricist his choice. If *this* inference is made by the help of the assumption of the uniformity of nature, its results cannot be used to prove that assumption. If it is made without that help, by his own admission it falls to the ground, for the inference of any particular uniformity is supposed to need that assumption; and so he is not left with experience sufficient to justify his generalization. We may present the argument against

[1] J. S. Mill, *System of Logic*, III. xxi. 3.

his position in yet one more light. The essence of his contention is, that we must come to the facts of experience without any preconceptions; we must have no antecedent view of what is conceivable or possible. For all that we can tell to the contrary until experience has instructed us, anything whatever is possible; and if it occurred with sufficient frequency, anything would be conceivable. Now, it will be admitted that if there are a number of independent alternatives all equally possible, an event that is inconsistent with only one of them leaves us quite unable to decide between the rest. But if, as the empiricist insists, all things are antecedently equally possible, then all proportions of regularity to irregularity in the world are equally possible antecedently. All events may occur in accordance with uniform principles: or there may be no event which ever has the same consequent twice; and between these two extremes of absolute regularity and absolute irregularity an infinity of intermediate alternatives may be conceived, among which we cannot select except upon the evidence of experience. The extent to which regularity, or uniformity, prevails may therefore be limited in any conceivable way, whether as regards place, or time, or subject. There is no reason why the succession of like consequents upon like antecedents, while exemplified at other times and places, should not fail in the hitherto unexplored parts of Central Asia, or on all Fridays subsequent to the Friday in next week. Nothing less than this is involved in the refusal to prejudge experience. But if that is so, past experience itself can never enable us to prejudge future experience. For why should any degree of uniformity observed till now in the succession of events induce us to expect such uniformity to continue? It was antecedently as probable that such uniformity should continue till to-day, and then terminate, as that it should continue till to-day and still continue. The fact that it has continued till to-day has disproved what until to-day was a possible hypothesis, viz. that it might terminate sooner; but between its terminating to-day, and still continuing—two independent and antecedently equally probable alternatives with which its continuing until to-day is equally consistent—it does not in the least enable us to decide. This argument will hold good, at whatever point in the series of time to-day may fall; it will hold good equally against an inference to the unobserved events of the present or the past; so that we never get any nearer being able to infer a degree of uniformity which goes beyond what has been actually observed.

It seems conclusive therefore against the view that the Uniformity of Nature can be an induction from experience, if by the term induction any legitimate process of inference is understood.[1]

With what right then do we assume it? The answer to this has been given in discussing what we mean by it. To deny it is to resolve the universe into items that have no intelligible connexion. If the universe and the events in it form a systematic whole, then any change must be determined by something in the nature of that whole; and for the same change to occur on different occasions except under the same conditions is not consistent with its having

[1] The last argument may be put in a way that will perhaps to some seem clearer as follows:

1. An event which is equally consistent with two hypotheses affords no ground for deciding between them.

e. g. if *A* and *B* keep a common stock of boots, and each uses every pair indifferently, footprints that fit one of these pairs afford no ground for deciding whether *A* or *B* has passed that way.

2. It is admitted by those who regard *uniformity* in nature as empirical, that antecedently to experience all issues, so far as regularity and irregularity in the succession of events are concerned, are equally probable.

By an *issue* is meant a certain *course* of events, however long.

3. These alternative issues must be regarded as perfectly detached alternatives: i. e., antecedently to experience, the rejection of one issue would not give any ground for or against the rejection of any other. To assume that it would is to assume, antecedently to experience, the existence of such degree of uniformity as enables you to say that if one specific issue happens, another must or cannot.

4. That events should occur with any specified degree of regularity down to the end of the year 2000 A. D., and with less or no regularity, or in apparent conformity to different rules, thenceforward, is one such issue; that they should occur with the same specific degree of regularity down to the end of the year 2001 A. D., and thence with less or none or other, is another such issue. And these issues are perfectly detached alternatives beforehand. Let them be called *X* and *Y*.

5. The empirical observation of that specified degree of regularity down to the end of 2000 A. D. is equally consistent with the hypothesis that *X*, or that *Y*, expresses the truth. Therefore it affords no ground for deciding between them.

6. It would therefore be equally likely at the end of 2000 A. D. that the events should thenceforward exhibit none or less of the regularity that they had hitherto exhibited, or conform to quite different rules, as that they should continue to exhibit the same regularity even for a year longer.

7. The dividing date might be taken anywhere; and one might take equally a dividing place, or department of fact.

8. Hence the actual issue never affords any ground for preferring the hypothesis of a continuance of the observed regularities to any hypothesis of their discontinuance, complete or partial, with or without the substitution of other regularities, in any period, region, or department of fact, in which they have not been empirically verified.

a determinate nature. It is not, of course, denied that changes partially the same may occur under conditions partially different; and the task of disentangling the identities in what is partially different is one of the tasks of the inductive sciences; but *ceteris paribus*—a proviso about which it is very difficult for us to know in individual cases how far it is fulfilled—the same conditions must produce the same effect, and the same effect must have been due to the same conditions. A changing universe is otherwise unintelligible or irrational. If any one likes to accept that alternative, it may be impossible to reason him out of it; for he has disallowed at the outset the appeal to reason. At least let him not maintain that, while the alternative is conceivable, experience proves that it is not actual.

CHAPTER XX

OF THE RULES BY WHICH TO JUDGE OF CAUSES AND EFFECTS

The world, as we have already insisted, is not a mere procession of events, but the events concern things; a cause is a thing acting; it produces a change in some thing. And the things exist before and after the action, sometimes apparently unchanged. A wall, for example, which changes the direction of motion of a ball striking it, exists before and after producing this effect, and the ball does so also. And if a bullet struck it, and were scattered in pieces, though we might say that the bullet no longer existed, the particles would still exist into which it was broken up, and we should say that the wall existed, even if scarred or fractured. It may be asked, if the wall repels a ball striking it, what was it doing until struck? Can it have been provoked by being struck, to an action which is momentary, as a man, we think, may be? Or is it acting continuously in a way whose effects vary with varying circumstances? If we are not to personify the wall, we must adopt the latter view; and such terms as *vis inertiae* and 'energy of position' are evidence of the attempt to reconcile the abrupt occurrence of noticeable changes with the continuous action of things. In the last resort, we seek to formulate laws of the action of things, from which we can deduce the changes that will occur under varying circumstances and in various periods of time. And having done this, we may disregard the troublesome questions connected with the nature of action, and treat our laws as enabling us to determine, from the state of things existing at one time, what state will exist, or has existed, at another. Even so, we still find ourselves assuming the existence of things; for a state of things is not an event that happens without happening to anything.

These laws, we saw, are principles of connexion between one change and another in a thing, or between change in one thing and

change in another. And all inference from experience rests on universal connexions in nature. If, for example, there are no circumstances material to the occurrence of a landslip, it would be foolish to expect that any examination of the circumstances under which landslips have been found to occur would enable us to determine under what circumstances they will occur in the future; but to say that certain circumstances are material to its occurrence means that in a like situation they would always produce such a landslip. If then we can detect these connexions, we can generalize. Our problem is, how to detect them.

Now a full account of these connexions requires us to pierce into the composition of things, and consider the operation even of their minutest parts. But at the ordinary level of enquiry and for many practical purposes we trace connexions between the changes that occur in such aggregates as we can sensibly distinguish and are interested in, like the land which slips and the rain which loosens it. And though we know that events must befall things, we take for granted in our formulation of connexions the things which they befall, and seek in one change the cause of another—in rainfall, for example, the cause of a landslip. The discovery of causes is indeed popularly regarded as the task of an inductive science. We may put its questions in the form 'What things how acting produce what changes in other things?' But as the action of the cause is displayed in a change in itself, or takes effect upon the occurrence of some other change, the form of question more commonly put is not that just given, but rather 'What change in one thing produces what change in another?' Whatever the philosophic imperfection of this formula, yet since such changes are connected through the causality of things, and our practical interest lies in discovering connexions of change, we come to speak of the changes, or events, causally connected as causes one of another, and of causal relations as lying between them. Often, as by J. S. Mill, the terms said to be connected as cause and effect are called *phenomena*. This word is convenient because it can be used either of an *event*, like the fall of a thunderbolt, of a *thing*, like the thunderbolt itself, of an *attribute*, like the velocity of its fall, or even of a *law*, like gravitation.[1] The difficulty of any description of

[1] Mill supposed that by *phenomenon* he meant not a thing, since he deemed things themselves unknowable by us, but the appearance or the state of consciousness which that produces in the mind, and whereby we know it. Such

inductive reasoning at once short and accurate arises largely from the fact that sometimes what we are concerned to discover is the things, whose agency is involved in the production of an effect, as in asking for the cause of a disease we might wish to know what microbe produces it: sometimes it is the attribute in things, which makes them capable of producing it, as if we asked why blankets keep us warmer than sheets do: sometimes it is the change in things, with the occurrence of which the effect is connected, as water freezes when it falls to a certain temperature: sometimes it is a law exemplified in the succession of one change upon another. If we include all these kinds of question together as enquiries into the causal connexions of phenomena, we must remember that we are sacrificing precision to brevity, and that our formula has different meanings on different occasions.

With these cautions we may proceed to consider how causal connexions are detected. They cannot themselves be perceived. Events occur and are observed; the lines of causation that connect them are not observed. It is here comes in the working importance of the uniformity which is involved in the conception of a causal relation. All manner of events are occurring simultaneously at every moment; and the events of one moment, taken in the lump, must be causally connected with those at the next.[1] But which is connected with which, the single experience of their succession will not determine. A man may run for an hour round his garden on a frosty night, and when he wakes up next morning may notice that his legs are stiff, and the dahlias in his garden blackened. If he had really no other experience of such events than in this succession, he might equally well conclude that the frost had made him stiff and his running blackened the dahlias, as vice versa. But it is involved

a meaning cannot be maintained in science or defended in philosophy. Nor does the word mean, as is sometimes stated, anything that can be perceived by the senses; it seems to be used to cover any thing, property, principle or event that can be a subject of scientific investigation, or used in a scientific explanation of what is investigated.

[1] It may be said that an event of to-day may be due partly to some event that occurred a long time ago: for example, a man may inherit a fortune on his twenty-first birthday in virtue of a will made before he was born. We shall see later that it is by no means always practically convenient to call the immediately preceding conditions the cause: and others remoter may without offence usurp the name. But the legatee becomes possessed of his fortune because he has just attained the age of twenty-one to-day; and the will may be regarded as having initiated a persistent legal position as regards the money; so that the statement in the text may be deemed sufficiently accurate in the context which it is intended to elucidate.

in the causal relation that if two things are really cause and effect, the one never exists without the other; and hence by comparison of that experience with others, he might conclude that running round the garden did not blacken dahlias, because at another time they had not gone black after he had been running round it; and that frosty nights did not make his legs stiff in the morning, because he had waked up after another frosty night without any stiffness in them. So far he would only have disproved the connexions to which his mind at first had jumped. To prove that frost does blacken dahlias, and that it was the running that made his legs stiff, is a more difficult matter; for the mere fact that one has been followed by the other many times constitutes no proof. Yet the repetition of the same event under different circumstances is constantly narrowing the field of possibilities; for no two events can be precisely cause and effect, of which one in any case occurs without the other; so that if we can show that out of all the circumstances under which the blackening of dahlias has been observed to occur, a frost is the only one that has not also on another occasion either occurred without such an effect befalling the dahlias, or failed to occur when it has befallen them, we may conclude that there is nothing except the frost to which their blackening can be attributed.

In this example we find the simple principle upon which the reasoning of induction rests: though the successful prosecution of inductive science requires very much besides such reasoning. The cause of any effect—in the strictest sense of that relation—is so related to it, as to occur whenever the effect occurs, and never when it does not; and to vary or be constant as the effect varies or is constant, when susceptible of variations in quantity or degree. From this it does not follow that because in a limited number of instances some particular two phenomena a and x have been observed to be present and absent, to vary and be constant together, they are related as cause and effect; since there may be another phenomenon b which also satisfies the conditions, and it is impossible so far to tell whether a or b or the combination of them is the cause of x. But it does follow that nothing is the cause of x which fails to satisfy the conditions; and it is upon that consideration that all discovery of causes from experience rests. In saying this we do indeed but repeat what was said in reference to the 'New Induction' of Bacon.

Thus inductive reasoning rests upon understanding what is involved in the causal relation [1]; for unless we know this, we cannot know that certain phenomena do not stand to each other in that relation. And from the nature of this relation proceed what may be called Topics of Cause, or rules whereby to judge whether two phenomena are thus related to each other or not: just as from the definition of Property proceeded what Aristotle called Topics of Property, or rules whereby to judge whether a given predicate was or was not a *proprium* of a given subject. But you can only prove that they are related as cause and effect by proving that there is nothing else with which either of them can be causally connected.

J. S. Mill formulated four 'Methods of Experimental Enquiry', or as he also called them, 'Inductive (or 'Experimental') Methods,' to which he attached considerable importance in his System of Logic.[2] He called them the Method of Agreement, the Method of Difference, the Method of Residues, and the Method of Concomitant Variations. Among other defects of his exposition, his treating these as so many separate methods darkens in a special degree the subject of induction.

We shall be able to appreciate the nature of this defect if we realize that the essence of inductive reasoning lies in the use of facts to disprove erroneous theories of causal connexion. It is, as Mill himself asserts, a process of *elimination*.[3] The facts will never show directly that a is the cause of x; you can only draw that conclusion, if they show that *nothing else is*. In order to show that nothing else is, it is of course in the first place necessary to know what other circumstances there are among which the cause might be sought; we cannot 'single out from among the circumstances which precede or follow a phenomenon those with which it is really connected by an invariable law' (to borrow an instructive phrase of Mill's [4]) unless we have ascertained what circumstances do precede or follow it on divers occasions. But as to do that is no part of the inductive *reasoning* which we are now considering, we may for the present neglect it, or assume it to have been done. The important thing to notice here is, that we do not discover what is the cause, except by eliminating the alternatives. Now it is very often impossible to do this completely; nevertheless the nature of

[1] Cf. Poste, *Sophistici Elenchi*, Appendix D, p. 221.
[2] *System of Logic*, III. viii. [3] e. g., ib. § 3 *init.* [4] Ib. § 1 *init.*

our reasoning is precisely the same, when we are left with the conclusion that the cause is either a or b or c, as if we had been able to eliminate b and c also, and so determine that the cause is a. Moreover, it makes no difference to the nature of our reasoning, as a process of advancing to the proof of the cause by the disproof of the alternatives, what the principle is to which we appeal in order to disprove them. We know that nothing is the cause of x which does not satisfy certain conditions—which is not present whenever x occurs and absent when it does not, which does not vary or remain constant as x does so. It is sufficient to be able to show that one of these conditions is not satisfied by a given circumstance p, in order to conclude that p is not the cause of x; and *which condition it is does not matter in the least.* It is unlikely that in any particular investigation every alternative hypothesis which we disprove as to the cause of the phenomenon that we are studying will be rejected because it fails to satisfy the same one of these conditions; the facts of our experience will probably show us one occurring where the phenomenon is absent, and the phenomenon occurring in the absence of another, a third unaffected in quantity or degree through all the variations of the phenomenon, and so on. All that is essential to the progress of our enquiry is that we should be able to show some fact inconsistent with supposing such and such an alternative to be the cause; then that alternative is eliminated, and the cause must lie among the rest.

The essence, then, of these inductive enquiries is the process of elimination. The reasoning is disjunctive. And the character of the reasoning is unaffected either by the completeness of the elimination (i.e. the fact that there are no *alternatives* left in the conclusion) or by the ground of elimination used. Yet Mill has so formulated his 'Methods' as to make it appear (*a*) that they are only used when the elimination is complete; (*b*) that they are different when the ground of elimination is different. From this it follows that very few inductive reasonings really conform to any of them; but the credit which this part of his work has obtained, and still more the currency given to the names of his 'Methods', in which his doctrine is enshrined, threaten us with a repetition of the same sort of mischief as arose from supposing that every argument could be put into the form of a syllogism. Just as arguments not syllogistic at all were forcibly tortured into the appearance of it, to the destruction of any proper understanding of what syllogism

really is, and how it differs from other forms of reasoning, so inductive arguments are now often forced into a pseudo-conformity with the canon of one of these 'Methods', to the utter confusion of the mind. For in the process, we are made to allege that some circumstance is (say) the only one in which a number of instances of a particular effect agree, in order to conclude in accordance with the canon of the 'Method of Agreement' that it is therefore its cause, when we know perfectly well that it is not the only such circumstance; and as we know that it is not by such assumptions that we really conclude that circumstance to be the cause, we are only confused by a Logic which makes it appear that it is.

There are passages in Mill's work (as is often the case with him) which implicitly correct his own error. In speaking of what he calls the 'Method of Agreement', he writes: 'The mode of discovering and proving laws of nature, which we have now examined, proceeds on the following axiom. Whatever circumstance can be excluded, without prejudice to the phenomenon, or can be absent notwithstanding its presence, is not connected with it in the way of causation. The casual circumstances being thus eliminated, if only one remains, that one is the cause which we are in search of: if more than one, they either are, or contain among them, the cause; and so, *mutatis mutandis*, of the effect.'[1] It is plain from this that I am not the less reasoning in accordance with this method, because I am only able to say in the conclusion that the cause of the phenomenon is one or other of several alternatives, than if I were able to offer a definite solution. Yet this is quite ignored in what immediately follows: 'As this method proceeds by comparing different instances to ascertain in what they agree, I have termed it the Method of Agreement; and we may adopt as its regulating principle the following canon:—

'*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.*'

Every one who has tried knows how difficult it is to find examples to which this canon can be applied; for it is seldom that instances of the subject under investigation have only one circumstance in common. Where such instances are forthcoming, they are peculiarly instructive to the investigator; and therefore Bacon placed

[1] *Logic*, III. viii. 1 *ad fin.*

them first in his list of *Prerogative Instances* (i.e. instances to be consulted first), under the name of *Instantiae Solitariae*.[1] But what if your instances have several circumstances in common? Are they therefore useless to the investigator? Throughout the organic world it is observed that species present a number of adaptive structures—that is, structures fitting them for the conditions under which they have to live. To the question how this has come about several answers have been suggested; one, the oldest, attributed them to special design on the part of the Creator: another to the inherited effects of use and disuse: another to the survival of those individuals who happened to be born with a body more suited in any respect than their neighbours' to the conditions of their life, combined with the elimination of the less fit. Now if it is pointed out that some adaptive structures, like the horny back of a tortoise or the shell of a mollusc, cannot be improved by use as a muscle can, one of these suggestions is overthrown, at least as a complete solution of the problem; but it remains doubtful so far whether we are to refer the structures in question to design or to natural selection: yet we have certainly made some way in our enquiry, and this argument is part of our inductive reasoning. Mill's canon, however, is inapplicable to such a case as that, because the tortoise with his horny back, and the elephant with his powerful trunk for seizing branches, though both possessing adaptive structures, which may in both have been established by natural selection, are not instances with only one circumstance in common. It is excellent advice to see in what the instances of your phenomenon agree; but the ground of the advice is that you may eliminate the circumstances in which they differ: and the principle at the foundation of the 'Method of Agreement' is not that 'the sole invariable antecedent of a phenomenon is probably its cause'[2], for the 'Method' is often employed when we can find no sole invariable antecedent; it is that *nothing is the cause of the phenomenon in the absence of which it occurs*.

The same defect appears in Mill's formulation of the 'Method of Difference'. In seeking to discover on what conditions an effect of a certain kind[3] depends by way of eliminating all that can be

[1] *Nov. Org.* II. 22, where instances such as are required by Mill's Method of Agreement and by his Method of Difference are described under this name. And this is the proper way to treat them—not as instances the use of which constitutes a distinct method of inductive reasoning.

[2] Jevons, *Elementary Lessons in Logic*, p. 241 (1880).

[3] And *mutatis mutandis*, if seeking to ascertain the effect of a given cause.

shown to be irrelevant, we may rule out not what is common to different instances in which the effect occurs, but what, among all the circumstances in which in some instance it occurs, can be shown equally to exist without the effect occurring. In technical phraseology, instead of comparing various *positive* instances of the effect, we may compare a *positive* and a *negative* instance: a negative instance being an instance of circumstances similar to those wherein the effect occurs, where the effect is nevertheless absent. Mill's canon for this procedure runs thus:—

'*If an instance in which the phenomenon occurs and an instance in which it does not occur agree in all circumstances but one, the circumstance in which alone the two instances differ is the cause, or effect, or an indispensable part of the cause, of the phenomenon.*'

He here implies that the use of this Method depends on finding a positive and negative instance agreeing in every circumstance but one. This is not indeed so far beyond achievement as it commonly is to find a number of positive instances agreeing in only one circumstance; for experiment, when we introduce a factor into or remove it from an existing situation, provides a positive and a negative instance; and we may be able to determine very exactly what factor we thus vary in a situation maintained otherwise the same. But even in experimenting the change introduced is often highly complex; and situations not artificially produced and maintained are subject to any number of simultaneous changes. Yet if in the course of them the effect under investigation should arise or disappear, we are not precluded by their number from arguing that those elements which have been the same before and after the emergence or disappearance of the effect do not account for it. We have one positive and one negative instance. They differ in many more circumstances than one; but still the phenomenon must be connected with something in the total difference, and still the circumstances present alike in the positive and the negative instance are thereby shown not fully to account for it.

Again, so obvious is the difficulty of finding such instances as these canons require, that Mill, having begun by mentioning four methods (of Agreement, of Difference, of Residues, and of Concomitant Variations), adds a fifth, which he calls the Joint Method of Agreement and Difference. Such instances as the 'Method of Agreement' and the 'Method of Difference' are supposed to require—positive instances agreeing, or a positive and a negative instance

differing, in one circumstance alone—may not be forthcoming ; and therefore, under the name of the Joint Method, Mill describes the case in which you look for a circumstance about which it can be said that it is the only one that is neither absent in any instance where the phenomenon occurs, nor present in any where it does not.[1] Here then both grounds of elimination are employed ; but there is no reason in the world, as a study of his account of his Methods would show, why he should not have had other Joint Methods, of Difference and Concomitant Variations, or of Agreement and Residues, and so forth. An enquiry into the cause of one phenomenon need not confine itself throughout to one ground of elimination.

For the above reasons it would be well to recognize that Mill has not formulated four (or five) but one 'Method of Experimental Enquiry'—as indeed Bacon might have shown him ; of which the essence is, that you establish a particular hypothesis about the cause of a phenomenon, by showing that, consistently with the nature of the relation of cause and effect, the facts do not permit you to regard it as the effect of anything else (and *mutatis mutandis* if you are enquiring into the effect of anything). It is this which makes the reasoning merely inductive. If you could show in accordance with known or accepted scientific principles that the alleged cause was of a nature to produce the effect ascribed to it, your reasoning would be deductive ; leaving aside the question how those scientific principles were ascertained, you would be reasoning from them to a conclusion which you see to be involved in their truth; and if we suppose the principles to be of such a nature that we can see they must be true, then the conclusion will appear

[1] Mill's canon for the 'Joint Method' is by no means carefully worded (*Logic*, III. viii. 4). It would be better if for 'the circumstance in which alone the two sets of instances differ' we read 'the circumstance in which alone the second set of instances agrees to differ from the first set'. Note that Mill represents it as necessary, under the terms of the Joint Method, to show of every other circumstance than that which is alleged as cause in the conclusion both that it is absent in some instance where the phenomenon occurs and that it is present in some instance where it does not. This is because he develops it as an answer to the objection, that although a circumstance b is absent in a particular instance of x there is no reason why it should not cause x on another occasion. The difficulties created by the so-called Plurality of Causes will be considered later. The point in the text here is, that it is quite possible, and very common, to show that one circumstance is not the cause on one ground—say that the phenomenon occurs without it, and another on another ground—say that it occurs without the phenomenon, and a third on a third ground—say that it is variable while the phenomenon is constant, all in the same investigation.

necessary, and a thing that could not conceivably be otherwise. Take, for example, the maxim that men hate those who have conferred a benefit on them.[1] We may regard that as, in the first place, discovered inductively from the consideration of many instances of ill will, which are unaccountable otherwise than on that principle; yet so far it remains a thing obscure and unintelligible, a relation which the facts forbid us to dispute, but in which we see no necessity. Now if a man were to say that men hate those who cause them what they dislike, that they dislike to feel themselves in a position of inferiority, and that they do feel themselves in a position of inferiority to those from whom they have received a benefit, the maxim follows deductively; and these principles are not only, like the original maxim, capable of being inductively supported by an appeal to experience, but they are also intelligible to us in a way in which that was not; it is mercifully untrue to say that they appear necessary, but they do appear more or less natural, and we see that such men must hate their benefactors. Where, however, we have to rely purely on induction, there is none of this 'naturalness': I stand on my conclusion because 'I can no other', and not because I see any *intrinsic necessity* in it. Necessity I do see, if I am right about my facts, and am to reason in this case consistently with what I know to be involved in the causal relation; but that necessity is not intrinsic; had the facts been otherwise, and for all I can see they might have been, I should have concluded otherwise; and then I should have been just as content to accept that as I now am to accept this conclusion.

There is an enormous number of general propositions, which we accept for no better reason than that the facts are inconsistent with our denying them, and not because we see anything in what they state which could have led us to suppose them true, antecedently to our experience. When it is said that we ought always to follow experience, it is meant that we ought not to trust our notions of what seems antecedently fit to be true, or mere guesses as to the connexions that subsist in nature, but accept only those connexions which our experience forces us to accept because it is inconsistent with any alternative. Such reasoning is called *a posteriori*, because it starts from the facts, which are conceived as logically dependent

[1] Of course this, like most maxims with regard to human nature, is not an universal truth: *what kind of men* hate those who have conferred a benefit on them would be the next subject for enquiry.

on, or posterior to, their principles, and thence infers the principles on which they are dependent. Conversely, deductive reasoning is often called *a priori*, because it starts from general principles, which are conceived as logically prior to the particular facts that accord with them.[1] When *a priori* reasoning is condemned, it is not meant that we are never to reason deductively, but only that we are not to reason from principles that are not warranted by experience; at any rate this is the only sense in which the condemnation can be justified. But it is an error to suppose that all general principles are arrived at *a posteriori*, or by process merely of showing that facts are not consistent with any other; the Law of the Uniformity of Nature itself, as we have seen, is not arrived at in that way, since if we once doubt it, it is impossible to show that the facts are any more inconsistent with its falsity than with its truth; neither are mathematical principles so arrived at: we do not believe that three times three is nine, because we show successively that it is not five or ten or any other number except nine. Still it is true that in the inductive sciences the vast majority of our generalizations are reached either in this *a posteriori* manner, or by the help of deduction from other generalizations so reached. And it may be well to show by one or two examples how generalizations that rest merely on induction present as it were a blank wall to our intelligence, as something at which we cannot help arriving, but which we can in no way see through or find intrinsically plausible. Facts show that the excision of the thyroid gland dulls the intelligence: could any one see that this must be so? Some explanation may be afforded by showing that on a contribution which the gland, when properly functioning, makes to the circulating blood depends the health of the brain; but that comes later than the discovery of the effects of excision; and even so, can we understand the connexion, which facts establish, between the state of the mind and the health of the brain? Or take a thing more frequent and familiar. It seems perhaps the most natural thing in the world, that we should see with our eyes, hear with our ears, taste with our palate, and so forth. Yet for all that we can see *a priori*, it might just as well have been that we should see with our ears and hear with our eyes, smell with our

[1] Or, in another sense, illustrated in most mathematical reasoning, because the premisses, without being more general than the conclusion, or giving the cause why it is true, are not based upon an appeal to facts which might conceivably have been otherwise: cf. *supra*, p. 210, n. 2; *infra*, p. 545, n. 2.

palate and taste with our fingers. Doubtless if we tasted with our fingers, we should not have to eat in order to taste; there might be some advantages in that, and at any rate it is not antecedently inconceivable. It may be said that the mechanism of the eye, by which light is focused from many points at once upon the extended surface of the retina, and the eye is readily turned in any direction, makes it *a priori* a more suitable organ of sight than the ear could be; and it is true that upon the assumptions that light-sensations are produced by the stimulation of a nerve, that this stimulation is supplied by wave-motions in the ether, that distinguishable colours are produced by differences in the wave-length, and that the arrangement of coloured points in the visual field corresponds to that of the nerve-fibres appropriately stimulated in the retina, we can find in the eye an excellent apparatus for securing clear vision. There is nothing, however, in those assumptions (which have only been proved inductively) that is any more intelligible to us than if the wave-motions of the ether stimulated the fibres of the ear, and those of the air the fibres of the retina; though doubtless our vision would be less serviceable in the latter case. There is in fact no psycho-physical correspondence that is at present intelligible to us, although particular correspondences may be intelligible in the sense of conforming to the more general principles which we have found to prevail. The same may be said with regard to the properties of chemical compounds, which are not for the most part intelligible from a consideration of the properties of their elements; hence in saying that they depend upon the composition of the substance we rely merely upon this, that no other view consists with the facts which we have observed in our experiments. The largeness of these two classes of inductive generalizations may perhaps make it unnecessary to illustrate further what Bacon would call the 'surd and positive'[1] character of conclusions resting only on induction; but, as showing how the mind desiderates something better, we may notice the attempt continually made to conceive chemical as at bottom only physical processes. In the physical process, the successive stages do to some extent at least appear to follow necessarily one out of another; on their mathematical side, the principles that connect them are not mere matter of fact, but matter of necessity which we cannot conceive otherwise. Hence the attraction of reducing chemical processes to physical terms. It

[1] *De Principiis atque Originibus*, Ellis and Spedding's ed., III. p. 80.

is true that the appearance of new sensible properties in bodies in virtue of their physico-chemical composition is not hereby explained; but it is supposed that they only possess these *for us*: that the appearance is subjective, or in other words that while the processes in bodies themselves are purely physical, *we* are determined to receive qualitatively different sensations by different physical stimuli. There is not much prospect at present of rendering psycho-physical correspondences really intelligible; thus there is a temptation to regard the emergence in a chemical compound of properties which cannot be seen to have any necessary connexion with the properties of its elements as merely a fresh case of that psycho-physical correspondence which we already admit that we can ascertain and not understand: in order that we may if possible find in the principles of chemistry itself something intelligible, and not merely necessary to be admitted. The gain is more apparent than real; but the procedure betrays a sense that though it may lead us far and win us much, induction turns out at last to be the blind alley of the reason.

We must return, however, from these general considerations upon the nature of induction to the particular inductive reasoning which rests upon our knowledge of the requirements of the causal relation. By and by we shall find that reasoning which is really inductive enters into processes of a more complex and partially deductive kind. What we are at present considering is in principle quite simple. The cause of a phenomenon [1] is to be sought among those circumstances under which it occurs in the instances that we take. The causal circumstances are found by a process of exhaustive elimination. Those which are not causal can be eliminated because the facts show that in regard to this phenomenon they do not satisfy the conditions of a cause. Now the grounds on which we may eliminate are these; and each points to some particular requirement of the causal relation, failure to satisfy which disproves that relation as between two given phenomena:

1. Nothing is the cause of a phenomenon in the absence of which it nevertheless occurs.
2. Nothing is the cause of a phenomenon in the presence of which it nevertheless fails to occur.

[1] Or *mutatis mutandis* the effect. I shall not complicate the exposition by always adding this.

3. Nothing is the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it.

To these may be added a fourth ground :

4. Nothing is the cause of one phenomenon which is known to be the cause of a different phenomenon.

This last principle is also, like the others, involved in the general conception of a reciprocal causal relation ; but in applying it we appeal not merely to what we observe in the instances of the phenomenon under investigation, or in the instances where under more or less similar circumstances the phenomenon does not occur; we appeal also to previous generalizations regarding the connexion of phenomena. These generalizations, however, are used not to account for the connexion which we are now establishing—it is not deduced from them ; but merely to exclude alternative explanations of the present phenomenon, and so force us upon the one which we finally accept ; and so far the reasoning which appeals to such a ground of elimination is still inductive.[1] But it belongs especially

[1] On these grounds of elimination Mill's 'Inductive Methods' severally repose. The first is the foundation of his 'Method of Agreement', the second of his 'Method of Difference', the first and second jointly of his 'Joint Method of Agreement and Difference', the third of his 'Method of Concomitant Variations', and the fourth of his 'Method of Residues'. All of them are quite general, and have been stated above in a way which only holds if in the cause we include everything necessary and nothing superfluous to the production of the phenomenon in question. The illustrations in the present chapter are not confined to that, the strictest, sense of cause ; but the important point involved will be considered later in Chapter XXII. on Non-reciprocating Causal Relations. Where the cause sought is a non-reciprocating cause, other principles call to be applied : e. g. we may wish to ascertain whether some condition which cannot by itself produce an effect is indispensable to its production ; and if such *sine qua non* be called a cause, that is a cause (in this sense) whose removal from a situation is followed by the cessation of the effect, though its restoration when the situation is otherwise changed is not followed by the recurrence of it. Lotze, in Bk. II. c. vii. of his *Logic*, headed *Universal Inductions from Perception*, has paid some attention in § 261 to the formulation of principles of this kind, stating what degree of connexion between two elements *C* and *E* can be inferred from what kind of observations with regard to the circumstances of their occurrence. The section is eminently worth consulting in reference to the nature of inductive reasoning ; and the principles in question might all be called Topics of Cause, though some of them are doubtful ; just as Aristotle recognized Topics which hold true in application only for the most part. Hume too in Part III. § XV. of his *Treatise of Human Nature, Of the Understanding* (already, like this chapter in Lotze, referred to), gives a number of Rules by which to judge of Causes and Effects which are derivative, but highly important, as for example that 'where several different objects produce the same effect, it must be by means of some quality,

to the later stages of a science, because it presupposes the discovery of other causal connexions, as a means of prosecuting some present enquiry.

It is plain that we cannot get to work in the application of these principles, until we have at least provisionally conceived and learnt to recognize the phenomenon we are studying, and ascertained and distinguished the circumstances under which it occurs (or fails to occur) from one another. And if all this were done, their application would be an easy matter, as Bacon imagined he could make it. All symbolic representation of such inductive arguments by letters of the alphabet, where one letter stands for the phenomenon investigated, and others for the circumstances among which its cause is sought, presumes these tasks to have been achieved; and thus it is apt to convey a totally false impression of the degree of difficulty attaching to inductive enquiries.[1] The truth is, that inductive *reasoning* is in form very simple; but the discovery of the proper premisses is very hard. As Hume well observes of the rules he

which we discover to be common amongst them'. But those in the text seem to be really the ultimate principles, if a reciprocating cause is meant.

[1] On the artificial simplification which letters of the alphabet also imply, cf. Venn's *Empirical Logic*, c. xvii. pp. 406, 407. If they are to be used at all, to which I see no objection so long as their limitations are understood, it is important how we use them. In Mill's use of them, which has been followed by Jevons, *Elementary Lessons in Logic*, and by Fowler, *Inductive Logic*, and I dare say by others, there are two defects. He uses big letters to symbolize 'antecedents' or causes, and the corresponding small letters to symbolize 'consequents' or effects. Now in the first place he has thus always an equal number of big and small letters; but when we are looking for the cause of some phenomenon x, and seek it among a number of alternatives $A\ B\ C\ D\ldots$, we have not also before us effects as many as the alternatives among which the cause of this phenomenon is sought. Only in symbolizing his 'Method of Residues' is this feature appropriate; there certain circumstances collectively are supposed to be known to be the cause of a *number* of effects (or of an effect of a certain quantity or degree), and out of these we reject, as not the cause of one among the effects, those which we know to produce the others (or if the question is one of quantity or degree, we reject those whose total effect we know to differ from what we have to account for, as not accounting for the remaining component). Hence separate symbols for the effects (or components of the effect) of the various circumstances among which the cause of one effect (or component) is sought, as well as separate symbols for the causes, are required. The second objection is, that he uses *corresponding* big and small letters (*A B C* followed by *a b c*, &c.). Now, as Mr. F. H. Bradley points out (*Principles of Logic*, p. 339, note *), the letters are intended to symbolize the phenomena as presented to us *before* we apply our inductive canons; and therefore they ought not to imply, as by this correspondence they do, that the phenomena themselves, as distinct from the facts of their joint or separate occurrence, have anything about them that proclaims which is the cause of which. Cf. also Professor Bosanquet's *Logic*[2], II. iv. vol. ii. p. 122.

gives ' by which to judge of causes and effects ', ' All the rules of this nature are very easy in their invention, but extremely difficult in their application.' [1] It is easy enough to see that if out of so many alternatives *a b c d . . . z*, the cause of *x* is not *b c d . . .* or *z*, it must be *a* ; and it is easy enough to see that if *c* occurs without *x*, it is not its cause. But to show that *c* occurs without *x*, and to show some reason for rejecting *b d . . . z*, as well, and to discover *a b c d . . . z*, and show that no other alternatives are possible—all these things are extremely difficult. Something will be said of these operations in the next chapter. Here we are concerned with the form of the reasoning, which is of a disjunctive kind, and may be symbolized thus :—

The cause of *x* is either *a* or *b* or *c* or *d* . . . or *z*
It is not *b* or *c* or *d* . . . or *z*
∴ It is *a*.

In this argument the minor premiss is proved piecemeal by hypothetical arguments that rest upon one or other of the above grounds of elimination, or ' rules by which to judge of causes and effects '.

If *b* were the cause of *x*, it would be present whenever *x* is present
But (in this instance) it is not.
If *c* were the cause of *x*, it would be absent whenever *x* is absent
But (in that instance) it is not :

and so forth. Or if any one prefers it, he may represent this part of the argument as a syllogism :

Nothing is the cause of *x*, in the absence of which *x* occurs
b is a thing in the absence of which *x* occurs ∴ &c.
Nothing is the cause of *x*, which varies without relation to it
d varies without relation to *x*. ∴ &c.

It is of course possible that *b c d . . . z* may all be eliminated, or shown not to be the cause of *x*, by the application of the same principle or major premiss ; in this case the minor of the above disjunctive argument might be proved *en bloc*, and not piecemeal ; but this is by no means necessary, and in fact unusual, and does not affect the nature of the argument. It is, however, the only case contemplated in Mill's formulation of inductive reasoning. It is also possible (and this Mill's formulation does not recognize at

[1] *Treatise of Human Nature, Of the Understanding*, loc. cit.

all) that we may not be able to prove the whole of the above minor premiss ; and then our argument will take the form

The cause of x is either a or b or c or d . . . or z
It is not c or d . . . or z
$\therefore$ It is a or b

or It is not d . . . or z
$\therefore$ It is a or b or c

where the degree of uncertainty symbolized as remaining at the end of our enquiry is greater.

It appears plainly enough in this analysis how all induction rests on the Uniformity of Nature ; for in proving the minor of the disjunctive argument a principle is always appealed to, that would fall to the ground if the Uniformity of Nature were denied. It is not indeed necessary, in a particular investigation, to assume this uniformity to extend beyond the department of facts with which we are dealing ; if I am looking for the cause of cancer, it is enough that cancer should be subject to uniform conditions in its occurrence ; and I should not be impeded in my research by the fact that thunderstorms occurred quite capriciously. There is, however, no ground for assuming cancer to be subject to uniform conditions in its occurrence which does not apply equally to thunderstorms, or to anything else that could be mentioned ; if I assume the principle of Uniformity at all, I must logically assume it altogether ; and so, though I may be said to appeal to it in any particular inductive argument only so far as concerns the department of nature to which my investigation belongs, I really assume it universally.[1] Nevertheless it is not correct to say that it is the ultimate major premiss of all inductions[2] ; for that implies that an inductive argument is, formally considered, a syllogism, and we have seen that it is not. It is indeed impossible to see how this principle can be made the major premiss of any inductive argument as a whole, though its particular applications may afford the major premiss of an argument by which we prove any part of the minor in our disjunctive argument. Let us say that ' Nature is uniform ', or (since we can hardly make a middle term of ' Nature ', which in the sense of nature as a whole is not predicable of any particular

[1] Cf. what Aristotle says of the assumption of the Law of Contradiction implied in all syllogisms, *An. Post. a.* xi. 77ª 22–24.
[2] Mill, *System of Logic*, III. iii. § 1 *med.*

subject) that 'All events in nature take place in accordance with uniform laws'; we may then proceed to argue that 'Cancer is an event in nature', and therefore that it takes place in accordance with uniform laws; but we are thus no further advanced than we were at the beginning, since so much is assumed in looking for a cause of it at all. Or if we put our major premiss in the form 'Every relation of cause and effect that is observed in any instance between one phenomenon and another holds good universally', and then used as our minor 'The relation between a and x is a relation of cause and effect between one phenomenon and another observed in certain instances', we might indeed take the formal step of concluding that it holds good universally (though that is already implied in calling it a relation of cause and effect), but the whole question at issue is here begged in the minor premiss; for what we want to prove is just that a is related to x as a cause, and not in time only and accidentally. For the formulation of the reasoning by which that is proved—which is the inductive reasoning—nothing therefore has been done. And any other attempt to reduce inductive reasoning to syllogism with the principle of the Uniformity of Nature as ultimate major premiss will be found equally unsuccessful.

It should be noted that the above account of inductive reasoning holds good generally, whether by the cause of an event we mean a thing to whose action it is due, or an attribute which things must have which are to produce it, or an event with whose occurrence it is connected by a law, or the law exemplified in its succession upon another event. We cannot indeed speak of a law being present or absent in particular instances. But whichever form our problem takes, we ask ourselves what should happen if the cause were thus or thus; and if the effect is absent in any instance where it should be present, or present where it should be absent, or is constant or variable as it should not be, or different from what we know that the cause suggested would involve, the suggestion is wrong. Nor can any suggestion be inductively established, unless it successfully, and it alone successfully, runs the gauntlet of such questions.

It remains to illustrate by a few examples the truth of the contention that inductive conclusions are established disjunctively by the disproof of alternatives.

1. The power of the chameleon to change colour in accordance with the colour of its surroundings is well known. But this power

is not confined to the chameleon; it occurs, for example, also in certain frogs.[1] The question raised is as to the cause of this change. We have first indeed to show that the change is due in some way to the colour of the surroundings; that implies a previous inductive argument; for so long as it was only noticed that the frog changed colour from time to time, it would be quite uncertain with what that change was connected. We may disregard such suggestions as might occur to a collector of portents; Livy gravely records as portents of disaster some facts quite on a par with the statement that 'a frog changed its colour in broad daylight', but it would be easy to show that the change had occurred at a time of no disaster. But of the suggestions that might occur to a biologist we may conceive the nature of the animal's food to be one: time of day or season of year to be another: intensity of sunlight to be a third, and so on; but when it was shown that the frog might variously change its diet, and still be of the same colour, and that the change of colour might take place at any time of the day or year, and in various degrees of sunlight, these suggestions would be discarded, and so on until the only reasonable suggestion left was that which connected the change of colour with the colour of the surroundings. Of course this conclusion would acquire great strength so soon as any one noticed the frog in the process of changing colour upon removal to new surroundings; for if the change of colour is to be connected with some other change that has just occurred, the range of alternatives is thereby much limited. The preliminary induction implied in saying that it changes colour *according to* the colour of the ground on which it rests need not, however, be further considered; we wish to know more precisely what produces the change. Now differently coloured grounds may vary in temperature as well as in colour; but it can be shown experimentally that the colour-reaction is independent of temperature. Granting then, in the absence of any other alternative, that it depends on the colour as such, we may ask in what way the differently coloured rays[2] affect the animal. Lord Lister showed that they affected it through the eyes; for a specimen of *Rana temporaria* whose eyes had been removed was no longer affected by any change in the colour of the

[1] This example is taken from Dr. Vernon's *Variation in Animals and Plants* (Internat. Scient. Series), pp. 255 seq.

[2] To speak strictly, rays are not differently coloured, but of different wavelengths.

surroundings in which it was placed; thus the alternative, otherwise not unreasonable, is excluded, that the reaction is somehow determined through the skin, the principle applied being that no circumstance in the presence of which the phenomenon fails to occur is its cause. This conclusion is further confirmed by the fact that in other species that normally exhibit a similar colour-reaction individuals have been found, in whom the power of adjustment to the colour of their surroundings is absent, and that these individuals on examination have been ascertained to be blind; but it may still be asked how the stimulation of the eye by different kinds of light effects the colour-change. Perhaps there are two alternatives here; it might be necessary for the frog to be aware of the colour of its surroundings, or there might be a reflex mechanism. The latter is supported by the fact that a blinded frog, after a violent struggle to escape, changed from dark to light, but in half an hour, though placed in a bright light, became almost coal-black again. Here it is shown that a colour-reaction can take place without awareness of colour; so that awareness of colour is eliminated from among the conditions necessary to the production of the reaction, on the principle that a circumstance in the absence of which the phenomenon nevertheless occurs is not its cause. We must look then for some circumstance common to a blind frog changing colour after a violent struggle, and a normal frog changing colour with a change of surroundings; and we may find this in nervous excitation, for that may be produced by the action of light upon the eye, and also by the struggle. Until some other feature common to the two cases was suggested, we should accept this on the principle just cited: but it is also supported by the known physiological function of the nervous system in the building up of reflexes; it consists too with the fact that when the excitement subsided the blind frog returned to a colour not adapted to its environment. Yet how can the animal's colour be affected by different kinds of nerve-stimulation? There have been found in the skin of the frog pigment granules of divers colours, so arranged that different surface effects can be produced by different degrees of concentration in the granules. The final connexion of colour-reaction in the frog with these pigment granules is indeed rather deductive than inductive; for the part which efferent currents from the nerve-centres play in provoking muscular contractions and relaxations is already known, and so is the fact that an afferent nerve-current discharges into an efferent nerve; and we

have just shown that the colour-reaction is connected with afferent nerve-stimulations.

2. Let us take next a simpler example, and one in which there is little or no generalization: for inductive reasoning may be applied to discover the cause of a single event, as well as of events of a certain kind; and it is not necessary to carry the analysis (of which more in the next chapter) so far as to make a general conclusion possible. Let a novice notice that his bicycle makes an unpleasant noise in running, and try to ascertain the cause. We are to suppose a novice, because a rider of any experience may be presumed already to have arrived by induction at the knowledge that one kind of noise is made in the chain, and another kind in the bearings; and the application of this previously acquired knowledge to a particular case would be deductive. In this problem the determination of the alternatives among which the cause is to be sought is tolerably simple; for the noise must originate in one or other (or it may be several) of the non-rigid parts. Say that these are, on the machine in question, the axle-bearings of either wheel and of the cranks, the bearings of the head, the pedal-bearings, the clutch, the back-pedalling brake, and the saddle-springs. All that the rider has to do is to ascertain which of these parts may be at rest while the noise occurs, and which may be in motion without the noise. If the noise ceases in free-wheeling, it is not produced in the axle-bearings of either wheel, for they are still running, and that is not the cause, in the presence of which the phenomenon fails to occur; for the same reason it is not in the bearings of the clutch, which is now running. If it is not produced in 'wobbling' the head, or turning sharp corners, he may acquit the bearings of the head on the same principle. If it occurs in driving with each pedal singly, it does not arise in either pedal-bearings, because it occurs with each pedal in turn undriven, and that is not the cause in the absence of which the phenomenon occurs. Similarly if it occurs without putting on the back-pedalling brake, or when he removes his weight from the saddle, it does not originate in either of those quarters. Two alternatives remain: it may be in the crank axle-bearings, or in some looseness of the clutch when that is caught and driving. Between these alternatives a decision might be made if he dismounted, and listened while he whirled the hind wheel round by the pedals; here however he would be reasoning deductively from the principle that sounds are more distinct when you are nearer to their

point of origin. The difficulty of *generalizing* in such a case arises from the difficulty of distinguishing the phenomenon investigated from others that may be like it but have different causes. If the noise which each part of his bicycle could make were of a distinctive kind easily recognized, a man might very soon determine that such and such a noise (at least in his bicycle) only originated in such and such a part; and further experience, argued from on similar lines, might show him that a particular character in a noise was due to want of oil in a bearing, and another character to a broken ball. But so long as the phenomenon studied is submitted to no such scrutiny, it is liable to be confused with others that are not really the same, and error would obviously arise if we generalized together about this noise and others like it but differently caused. Hence one may have to be content with a conclusion that assigns the cause of it in the particular case. It is, however, instructive to observe that the same process of elimination among the members of a disjunction is employed here, as if one were establishing a general conclusion. For *ex hypothesi* the novice recognizes in the noise no intrinsic character which he knows to be connected according to any principle with a particular origin; he has therefore to fall back upon ascertaining its origin by the indirect method of showing that among the possible origins to which it can be ascribed there is none but one to which the facts permit him to ascribe it consistently with the principles of causation.

3. Professor Weismann's theory of the 'Continuity of the Germ-Plasm' is well known. The reproductive cells, whether of a plant or animal, are different in certain important respects from those which compose other parts and tissues (called somatic or body-cells); in particular, whereas the latter, in the process of increase and division, produce only cells of one kind, such as compose the part or tissue to which they belong, the former produce cells of every kind that occurs in the organism, and, in fact, are capable of reproducing the whole organism and not merely a special part of it.[1] In so doing they must, of course, reproduce the reproductive cells also, in order to provide for the following generation. Now Weismann held that in the division of the reproductive cell, or *germ-plasm*, a part is set aside from the outset to serve the purpose of reproduction once more, and that this, which is still germ-plasm, remains as it were

[1] Sometimes however body-tissue will reproduce a complete organ, or even (as the leaf of a begonia will) the whole organism.

isolated in the developing organism, and unaffected by the other and heterogeneous parts, or *somatoplasm*, which also arise in the division of the reproductive cell; and as this happens in each generation, there is an absolute continuity of the germ-plasm; from which it follows in his view that no characters acquired by the individual in the course of its lifetime and not congenital can be transmitted to its offspring; for a character so acquired arises in the somatoplasm, and the germ-plasm is from the first secluded from the possibility of being affected by the somatoplasm. Influences which reach the germ-plasm can alone modify subsequent generations; of which (at least in metazoa) the most important is the fusion of two reproductive cells that takes place in sexual propagation; here the germ-plasm of the ovum blends with another germ-plasm conveying more or less different heritable tendencies, and a sort of shuffling takes place, as a result of which there arises a new individual resembling precisely neither parent, but exhibiting those 'spontaneous variations', as Darwin called them, which form the material for Natural Selection to work upon. Darwin himself, on the other hand, believed that 'acquired characters' may in certain cases be inherited, and that it is very difficult to account entirely for the progressive modification of species in adaptation to their environment, without allowing the influence of this so-called 'Lamarckian' factor.[1] The question has formed a subject of protracted controversy among biologists, and it is not an easy one to settle conclusively on inductive principles by appeal to evidence, because most facts admit of being interpreted in either way. One of the most important investigations into the subject[2] is a series of experiments on guinea-pigs, conducted during thirty years by Brown-Séquard and extended by two or three other naturalists; and it is claimed that in the course of these experiments certain modifications appeared in some of the guinea-pigs, the cause of which lay in injuries done to the nervous system of their parents.

It was found that epilepsy sometimes appeared in animals born of parents which had been rendered epileptic by an injury to the spinal cord or a section of the sciatic nerve. Here was a fact to be accounted for, and the cause must be sought among the circum-

[1] Because Lamarck (1744–1829) had propounded a theory which ascribed the gradual modification of species largely to the inherited and accumulated effects of use and disuse of organs.

[2] The following argument is taken from G. J. Romanes' *Darwin and after Darwin*, vol. II. ch. iv.

stances to which the epileptic offspring were directly or indirectly exposed. Brown-Séquard attributed it to the injury done to the parent; but nobody professes to see *how* that could produce the effect, so that one can only be forced to accept that explanation by default of anything else to which to attribute it. It might be said that the epilepsy was due to some congenital defect that had no relation to the experiment performed on the parents; but epilepsy is not otherwise known to occur spontaneously in guinea-pigs, and apart from any improbability in the coincidence, we should expect that if some congenital modification of the germ-plasm produced epilepsy in these cases, it would have occurred and produced it in others. Weismann suggested that it was due not to the injury to the parent, but to 'some unknown microbe' which, entering at the incision whereby the injury was made, both produced the epilepsy in the parent, and by invading the ova or spermatozoa, produced it also in the offspring. But against this suggestion we may urge that, though there may be microbes enough unknown to us, yet if this microbe of epilepsy in guinea-pigs exist, it would be likely to seize other opportunities of entering; the disease, however, as already mentioned, is not otherwise known to attack them. And it was also found that the epilepsy might be produced (and apparently transmitted) without incision, by a blow on the head with a hammer, in circumstances that preclude the entry of microbes. To this Weismann rejoined that the shock of the blow might have 'caused morphological and functional changes in the centre of the pons and medulla oblongata, identical with those produced by microbes in other cases', and so set up the epilepsy; but these changes would not penetrate, as microbes may be conceived to do, to the ova or spermatozoa, and so the disease in the offspring occurs without the presence of the cause alleged. Moreover, there are cases (though the facts of them are not so clear or well confirmed) in which other diseases produced by other traumatic injuries to the parent have reappeared in the offspring; these diseases were not such as could have been produced by microbes; and to suppose, with Weismann, that the shock of the injury caused a general weakness of the nervous system, in consequence of which the animals would be likely to bear 'weak descendants, and such as are readily affected by disease', does not account for the diseases in the offspring being of the same sort as those respectively produced in the parents. So far, therefore, the alternative hypotheses

to that which attributes the disease in the offspring to the injury done the parent seem to be excluded; but Weismann has a final argument to urge against the 'Lamarckian' hypothesis. If the epilepsy was produced in the parent by the injury inflicted, it ought not to occur in the offspring in the absence of that injury in the offspring; and it would therefore be necessary to show that the nervous lesion which is the alleged cause of the epilepsy, and not merely the epilepsy itself, is transmitted. To this Romanes replies, that it very well may be transmitted; since even if adequate examination had been made (which is not the case), there may be structural injuries in a nerve which are not discernible. Nevertheless, he admits that the result of the whole debate is to leave 'the Lamarckian interpretation of Brown-Séquard's results' rather unassailed than proved. The facts alleged are 'highly peculiar', and hardly sufficient by themselves to furnish 'positive proof of the transmission of acquired characters'.

This example has been chosen because it illustrates very well how the inductive proof of a conclusion rests on excluding alternative explanations. The whole chapter in Romanes' work, from which it is taken, may be profitably studied from that point of view.[1] A further knowledge of facts might enable a biologist to suggest a cause for the appearance of epilepsy in the second (or later) generations of guinea-pigs, consistent at once with the facts and with Weismann's theory of the continuity of the germ-plasm. But this does not detract from the value of the example as an illustration of the method of inductive reasoning, which may be sound, though the conclusion is false, if there is error in the premisses. Note, however, that in the process of excluding alternative suggestions as to the cause, it was sometimes necessary to do more than merely appeal to one of the grounds of elimination set down earlier in this chapter; some deduction of the consequences of accepting such alternative was needed, more elaborate than is involved in saying that, if such were the cause, the epilepsy would appear where it did not, or not appear where it did. Thus it was argued that the epilepsy was not to be attributed to a microbe, because other diseases equally appeared

[1] Cf. Romanes' own words with reference to another experiment on guinea-pigs: 'Naturally, therefore, the hypothesis of heredity seems less probable than that of mere coincidence on the one hand, or of transmitted microbes on the other. *But I hope to have fairly excluded both these alternative explanations.*' *Darwin and after Darwin*, p. 119. (The italics are mine.)

to be transmitted, which a microbe could not have originated; we cannot be said to be here applying the simple principle, that that is not the cause of a phenomenon, in the absence of which it occurs, for these other diseases are not the same phenomenon as the epilepsy. To make the evidence of these other diseases serviceable, it had to be shown that there was no tenable alternative to the Lamarckian interpretation put forward (in lieu of microbes) in their case; and the principle involved in the use of their evidence was this, that if it is necessary to attribute the reappearance of one kind of disease in offspring to its production in the parents, it is more reasonable to attribute the reappearance of another kind of disease (epilepsy) in offspring to *its* artificial production in the parents, than to a different sort of cause of whose presence and operation there is no evidence. This principle may in turn be said to rest upon the principle that like effects have causes correspondingly like; and all rests ultimately on our understanding of the causal relation; but in order to see that facts are inconsistent with the ascription of a given phenomenon to some particular cause, a more or less extensive hypothetical deduction of the consequences that ought to follow if that were the cause is often necessary—more extensive, as Dr. Bosanquet points out,[1] in proportion as with growing knowledge we grasp more of system in nature. It may be noted, too, in this example, that some of the steps of the argument are only *probable*; if the entry of a microbe at the incision were the cause of the epilepsy, it would probably occur in cases of natural injury where, so far as we can see, the microbe might equally well enter: according to the principle that that is not likely to be the cause of the phenomenon, which is probably present on some occasion when the phenomenon fails to occur.[2] And lastly, Romanes cautiously concludes that the attribution of epilepsy in the offspring to its artificial production in the parent is not proved, because the cause may lie in something hitherto undetected; and this illustrates what was maintained earlier in the chapter, that the getting of a positive conclusion, but

[1] *v.* his paper 'On a defect in the customary formulation of Inductive Reasoning', *Proc. London Aristotelian Society*, N. S. xi. 1910–11, p. 29.

[2] In the *Prior Analytics* Aristotle discusses at great length *modal* syllogisms, i. e. syllogisms where one or both premisses are problematic or apodeictic; showing under what conditions the conclusion will be problematic or apodeictic. We have here an example of what might be called a modal induction; the parallelism may be commended to the notice of any who think, with Mill, that an inductive argument which can be represented in symbols (like his 'Inductive Methods') is the less formal because it is inductive.

not the inductive character of the argument, depends on the completeness of the elimination.

4. Adam Smith, in the *Wealth of Nations*,[1] discussing the inferences which can be drawn from the low money prices of goods in ancient times, and wishing to show that from the low prices of goods in general nothing can be inferred as to the wealth of a country, though much can be inferred from the relative prices of different kinds of goods, such as corn and meat, mentions that it was commonly supposed that the said low money prices of goods in ancient times were a proof of the poverty and barbarism of the countries where they prevailed. He uses the following argument to show that this is not so, but that they prove only the barrenness of the mines which then supplied the commercial world. First, he says that China is a richer country than any part of Europe, yet the value (i.e. purchasing power) of the precious metals is higher there than anywhere in Europe: now on the principle that that is not the cause of a phenomenon which does not vary proportionately with it, we cannot attribute low money prices to poverty in the face of lower prices where poverty is less. Next, he admits that since the discovery of America the wealth of Europe had increased, and the value of gold and silver diminished; but he urges that the two events have scarcely any connexion; the first being due to the fall of the feudal system and the growth of public security, the second to the discovery of more fertile mines. In support of this way of connecting the facts he points to Poland. Poland was the most beggarly country in Europe, as beggarly as before the discovery of America; yet the money price of corn (the most important single commodity) had risen equally there: if poverty were the cause of low money prices, it ought not to be found where prices were high. On the other hand, Poland was still feudal, so that her beggarly state was consistent with the connexion of facts alleged by Adam Smith. Again, Spain and Portugal were the next most beggarly countries in Europe to Poland, and prices ought therefore to be low there, if there were the connexion between low money prices and poverty that was supposed; but it was not so; prices were high; as might be expected if they depend on the facility with which the precious metals are obtained, for, owing to their control of the American mines, gold and silver were brought more cheaply to Spain and Portugal than to any other country in Europe.

[1] Bk. I. c. xi vol. i. p. 365, 7th ed., 1793.

The cause of low money prices in general, therefore, is not poverty and barbarism, and may be the barrenness of the mines supplying the commercial world with gold and silver; and this has been shown by inductive reasoning. Adam Smith also offers deductive arguments to show that it is the latter, and is not the former. It is not the former, because a poor could not afford to pay as much as a rich country, in labour and means of subsistence, for such comparative superfluities as gold and silver; it is the latter, because the purchasing power of gold and silver, or the amount of goods for which they will exchange, depends on what has to be given in order to get them; and where the mines are fertile, a less amount of labour and means of subsistence needs to be supplied in the work of getting them, than where they are more barren. The logician may distinguish an inductive from a deductive argument; but investigators will gladly use arguments of both kinds to support the same conclusion.

5. We may conclude with an example drawn from the Poor Law Commissioners' Report of 1834, with regard to the cause of the appalling increase of pauperism in England during the early part of the last century.[1] The Commissioners who were appointed to find the cause and to suggest a remedy attributed the evil to one principal fact in the situation, viz. that the condition of those receiving parochial relief had been allowed to become not less eligible than the lowest condition of men maintaining themselves by independent labour. In proof of this finding, they pointed out in the first place that the cause alleged was present in all instances of the phenomenon to be accounted for. The great increase of pauperism had dated from 1796. In that year, an Act of 1723, providing that no one should be entitled to relief who would not enter the workhouse, had been repealed; and it had become customary for the parish to assure to all labourers, in their own homes, a certain weekly sum, varying with the numbers in the family and the price of bread. This sum was made up in various ways; sometimes grants were given in supplementation of wages (which naturally tended to make farmers and other employers give a lesser wage, and so interested them in the support of a system from which they saw more clearly their immediately resulting benefit than the remoter but far greater evils); sometimes the parish found work, generally lighter than what was exacted for the same price by private employers (and this led men to prefer to work for

[1] *v.* the Blue-book, esp. pp. 186–216.

the parish); sometimes a money-grant without any return of labour was made to men out of work (who were not, therefore, the more likely to look for work); but in any case, it was made possible for a man to count upon parish pay, sufficient to maintain him as well as many independent labourers were maintained, whether or not he endeavoured to support himself.

The cause alleged, then, was present where the pauperism was present; but that was not enough to show that it was the cause. It might indeed be plausibly argued, from familiar principles of human nature, that such a method of administering poor-relief would be likely to increase pauperism faster than it relieved it: but this deductive reasoning was not, and still is not, sufficiently convincing to men who, from one motive or another, are attached to such a policy—whether from compassion for the immediate suffering of those applying for relief, or from desire to get relief on the easiest terms, or from fear, if relief is less readily given, that it will become necessary to give higher wages to the labourer. To bring conviction, it was necessary to show that there was nothing else to account for the phenomenon. Now several other causes had been suggested to account for this growth of pauperism. One was the great rise in the price of corn, which had occurred during, and partly in consequence of, the French war: another was the increase of population: and another was the introduction of machinery—a highly unpopular thing at the time, because its first and most obvious effect was to displace labour; and there had been agricultural riots directed against the use of machinery in 1830.

It would not be possible to show that none of these causes had ever made a man a pauper. But it was possible to show that in the main the pauperism so widely prevailing (which was so great a national evil because it prevailed so widely) could not be due to them. The Commissioners were able to point to numerous instances of three kinds, in which the pauperism so prevalent elsewhere was absent; in all of them, the cause they alleged was absent too; but the alternatives which they wished to disprove were present.

The first class of instances consisted of certain parishes where what was called a Select Vestry had adopted the plan (still then lawful, though not since 1796 compulsory) of refusing relief to any able-bodied labourer except in a workhouse where a full task of work was exacted. It was their experience that pauperism immediately

and greatly diminished. And naturally; for when men who had hitherto been content to take parish pay found they had to work as hard all the same, they preferred to work for themselves; with a motive for independent industry and thrift, they became more industrious and thrifty; becoming more industrious, they were better worth employing; and the farmer besides, knowing that the parish would no longer supplement the inadequate wages by which he had obtained labourers upon his farm, was compelled, if he would still have labourers, to give a better wage.

The second class of instances was furnished not by parishes which, removing the cause alleged, had also removed the pauperism of which it was alleged to be the cause; but in the parishes themselves where the pauperism existed. It was furnished by the so-called non-settled labourers, who in all parishes were found to be more industrious, thrifty, and prosperous, and less pauperized, than the settled labourers. As the circumstances of two sets of labourers in one parish are likely to be more similar than those of labourers in distinct parishes, these constituted what Bacon calls a prerogative instance; for all the conditions equally affecting settled and non-settled labourers may be excluded, in looking for the cause of this difference between them, on the principle of rejecting the circumstances present when the phenomenon is absent. By a non-settled labourer is meant a labourer living in another parish than that which is legally bound to support him. If he becomes a pauper, such a person can be removed to the parish to which he is legally chargeable; and to save their own rates, overseers were always anxious to remove any one they could. To the labourer, on the other hand, removal was as a rule by no means welcome; such labourers, therefore, found that they had to choose between removal, which they did not want, and an effort to maintain themselves by their own labour; for if the parish relieved them at all, they would only get from it—unlike their settled neighbours—little relief on hard terms.

The third class of instances was afforded by parishes which had never adopted the practice, so common since the Act of 1796, of relieving able-bodied men out of the workhouse; i.e. they had never consented to make the condition of the pauper as eligible as that of independent labourers; and in them the same extensive pauperization and increase in the rates, which had occurred elsewhere, had never happened.

Now in all these three classes of case, the Commissioners' theory held good; for when the effect was absent, so was the cause to which they attributed it. But the same could not be said for the alternative theories put forward. If it were alleged that non-settled labourers had smaller families, which is doubtful, yet the increase of population was not confined to parishes which had adopted, or banished from those which had abandoned, the practice rendered permissive by the Act of 1796. The price of corn had risen, and the introduction of machinery must have had its effects —whatever they were—in the parishes which had abandoned or never adopted that practice as much as in the rest, and among the non-settled as much as among the settled labourers of any parish. In short, looking to the mass of pauperism, there was no other circumstance which might be suggested as its cause, that could not, upon one or other of the plain grounds of elimination so often referred to, be rejected; and the Commissioners' cause was left in possession of the field; with the additional support derived from the deductive reasoning that might not have been thought of—even if it would have carried conviction—by itself. For it often happens that we can subsequently show that a cause, to which an effect has been attributed on the ground that there is nothing else to which the facts permit us to ascribe it, must, in accordance with some accepted principles prevailing in the subject-matter to which the enquiry belongs,[1] produce that effect: although, but for the help which the inductive argument had given us in finding the cause, the deductive argument would never have occurred to us.

[1] i.e. special principles, or ἴδιαι ἀρχαί. Cf. *supra*, p. 387. Cf. the account of his very successful administration of famine relief in the North-Western, now the United, Provinces of India in 1896-97, by the Lieutenant-Governor, Sir A. P. (since Lord) MacDonnell, *North-Western Provinces Government Gazette*, Nov. 27, 1897, quoted by Sir Theodore Morison, *The Industrial Organisation of an Indian Province*[2], c. xi, pp. 272-283. 'This result was obtained', says Sir Theodore Morison, p. 281 n., 'by steadily keeping the pay upon relief works below the "standard wage" which could be earned in any ordinary labour market.'

CHAPTER XXI

OF OPERATIONS PRELIMINARY TO THE APPLICATION OF THE FOREGOING RULES

It was allowed in the last chapter that it is impossible to apply the kind of reasoning there analysed until a good deal of work has already been performed upon the material which experience offers us. That work is really much harder than the reasoning that succeeds it; indeed so simple does the reasoning look when thrown into symbolic form, that it would not be surprising if any one mistrusted the foregoing account on the mere ground that induction must be a harder business. A consideration of the present chapter may reassure him on this point.[1]

The operations that have to be performed in order that the foregoing rules, or any other more special rules of the same kind, may be applied, are difficult to classify in a perfectly satisfactory manner. Different writers have called attention, and have given different names, to processes some of which are really more or less the same. Moreover, we should make our list shorter or longer according to the extent to which we considered what may be called the Methodology of the several sciences. By this is meant an attempt to give special directions, based partly on general logical considerations and partly on the nature of the facts with which it deals, for mastering the special difficulties which a particular science presents; for example, a mythologist might be enjoined to adopt the comparative method, and collect, with all the precautions which the experience of those who know the difficulty of rightly interpreting the savage mind can suggest, the myths and customs of many different lands: in biology again we should probably be told of the importance of obtaining statistics of a trustworthy kind regarding the mode in which divergences were distributed on either side of the average or normal in respect of divers measurable

[1] Mill deals with the subject of this chapter for the most part in his Fourth Book, *Of Operations subsidiary to Induction*. In the sense that the reasoning described in the Third Book cannot be profitably performed till they have taken place, they may be called subsidiary; but Induction is perhaps rather the whole process of eliciting from facts the principles that account for them than merely the form of reasoning involved therein; and these operations certainly hold no subordinate place in that process, as indeed Mill recognizes.

characters in animals and plants: and so forth. The particular preliminaries, without which inductive reasoning in each science may have little prospect of success, could of course only be determined by some one well acquainted with that science; though it is quite possible that a man of logical training, coming fresh to the study of what others have done, may be the better able for that training to make contributions to the work of scientific investigation; still, here as elsewhere, Logic learns by reflection on the immediate operations of thought about things. A methodology of the several sciences lies however beyond the scope of this volume, and would require far greater knowledge of their detail than the writer possesses. The list of operations therefore which follows makes no pretence to go as far as it might, or to embody the only possible division.

First of all may be placed what has been called the **Analysis of the Given**[1]: and this is requisite in two ways,

1. *in determining precisely the phenomenon to be studied;*

2. *in distinguishing and detecting the various circumstances under which it occurs, or under which it fails to occur when perhaps it might have been expected.*

Long before we consciously seek 'rerum cognoscere causas', a beginning has been made in the performance of this analysis: and the results are embodied in the general names by which men group and distinguish different things, attributes, or events. But there are many distinctions which ordinary language ignores, and it often gives different names to things which are in some important respect identical. For ordinary purposes the identity may be of no account, and yet in a scientific enquiry it may prove fundamental. For example, to the lawyer hares and rabbits are vermin, to the sportsman they are game, and to the zoologist they are rodents; each of these men for his own purposes is interested in characters that unite them respectively with quite a different group of other animals; but there is nothing in their specific names to indicate their affinities with any one of these groups. Or again breathing, burning, and rusting are three processes occurring in such different connexions and of importance to us in such very different ways, that they naturally have obtained distinct names; yet one of the greatest steps in the history of chemistry was connected with the discovery that they are, chemically speaking, all processes of the same kind, viz. the combination in the first two cases of carbon and in the third

[1] Professor Welton's *Inductive Logic*, c. v.

of iron with the oxygen of the air.[1] These examples illustrate the way in which it may be necessary to ignore our customary classification of things, and bring together, upon the strength of some identity which an analysis may have discovered in them, things that we have habitually kept quite apart in thought. It is equally necessary at times to distinguish things which we have habitually classed together, if we are to make any progress in the investigation of them. Rent furnishes a good instance. The name is given equally to the sum which a man pays for the occupation of land, and to that which he pays for the occupation of a building; as these are very commonly paid to the same person, as a lump sum is then charged for the two, and as the ordinary tenant in search of a dwelling is prepared to pay so much for accommodation, but indifferent to the question whether the owner considers his charge to be based on the value of the house or of the site it stands on, it follows that most of us find no inconvenience in this double use of the word. The farmer who has to consider separately what the land he farms is worth to him per acre, and what the value of the homestead is to him, is more or less aware of the ambiguity; but the political economist, when he comes to consider the causes that determine rents, is bound to distinguish house-rent and ground-rent by name. Indeed until that is done, his investigation will make no progress; for the two depend upon quite different conditions. The rent of a house, apart from any special history or sentiment, depends chiefly on the cost of building another like it, and the current rate of interest on money in the country at the time; but land cannot be produced as it is wanted, and this natural limitation of supply may give to a particular piece of land, in virtue of its fertility or its situation, a rentable value that depends mainly on its superiority in those respects over other land which cannot be dispensed with for cultivation or for building, and only very slightly and remotely, if at all, upon the circumstances which regulate house-rent.

The process of discovering identities between things in which we commonly ignore them, and that of discovering differences between things which we commonly take for the same, very generally involve one another.[2] We perform as it were a mental re-grouping; and in

[1] Cf. pp. 471-473, *infra*. Of course the oxygen *need* not be atmospheric oxygen.

[2] Thus economists have followed up the above distinction between house-rent and ground-rent (or economic rent) by grouping with the latter, under the name of *quasi-rents*, various other differential advantages, not superficially recognizable as of the same kind, such as what the abler *entrepreneurs*

the act of bringing together what we had hitherto only distinguished we most probably break up or find distinctions in the groups from which members are brought together. But in a given case one aspect may be much more prominent than the other; and Bacon has observed[1] that some men have a greater capacity for the one kind of work than for the other, insisting (like Plato before him) on the necessity of noting, in the investigation of nature, both the resemblances and the differences that are ordinarily overlooked. Analysis is at the bottom of each process, for until we have distinguished the various characters of things, we have not discovered the bases on which to compare them. It must be added however that analysis may be of great importance, yet without leading to any act of fresh classification, when we want primarily to know the circumstances under which a phenomenon occurs.

These remarks will indicate generally the nature of the work involved in the performance of the two tasks above mentioned: namely, in determining precisely the phenomenon we have to study, and in distinguishing and detecting the various circumstances under which it occurs, or under which it fails to occur when perhaps we should have expected it. It is sufficiently obvious that without performing them we should hope in vain to discover causal connexions by way of induction. If we have no precise or exact conception of what is to be studied, or have not (as one might say) duly determined it, we may examine instances that we ought to ignore, and ignore instances that we ought to examine. The result of the former error will be that we shall try to make our theory as to the cause of x consistent with the facts of the occurrence of a different phenomenon y: and the result of the latter, that we may be ignorant of facts which might throw great light upon the cause of x. The necessity of making a correct enumeration of the circumstances under which a phenomenon occurs, before asking with which of them it is causally connected, needs no comment; nor is it less plain that, if the question is to be answered, we need equally to recognize the circumstances, where they occur also in the absence of the phenomenon.

But though this work is so necessary, it is impossible to give any rules for the efficient dispatch of it. Familiarity with a science may help a man to perform it in the investigations of that science,

enjoy over their competitors, or some buyers and sellers over others forced to do business at the same price.

[1] *Nov. Org.* I. 55.

teaching him the sort of thing to look for, and the sort of way in which to look for it. Yet the sagacity upon which the discovery of new truth depends does not come to most men even by such familiarity. The logician's business at any rate, since he cannot teach men to do it, is to make them realize the part which it plays; and one or two further examples may be given with that object.

A research which has been so frequently cited in works on Induction as to become almost a stock instance will serve this purpose —Wells's *Theory of Dew*. Dew, as is now pretty generally known, does not rise but falls: the atmosphere can hold water in suspension in the form of vapour, but the amount depends upon the temperature of the atmosphere, and increases with it. If the atmosphere is suddenly chilled, it precipitates such a portion of the moisture which it holds as exceeds the saturation-point, or maximum it can hold at the temperature to which it is reduced. It may be chilled in various ways. One is the contact of a colder surface, on which the moisture is then precipitated; another way is by the inrush of a heavier and colder current: another is by radiation to the sky, and the degree to which that takes place depends partly on the amount of cloud about, partly on the substance, surface-form, &c., of the body itself; and a sheet or other covering stretched over the ground acts in the same sort of way over a small area, though with more effect over that area, as the clouds spread out over the earth. This precipitation of moisture held in suspension in the air is seen not only when dew falls; when warmer weather comes after a frost, particularly if accompanied by rain, the cold surface of a stone wall, if painted or otherwise not porous, drips with the water it has extracted from the air which its contact chills. In the same way cold spring water poured into a glass in summer will chill the outside of the glass, so that water is deposited on it from the air without: and when hot water is poured into a glass without filling it, and sends its vapour into the air above, some of this vapour bedews the interior surface of the glass above the water-level, until this portion of the glass has acquired by conduction the temperature of that below it. Now our present business is not with the reasoning by which Wells showed the deposition of dew to depend upon a relation between the temperature of the atmosphere and of the body on which the dew falls, taken in conjunction with the degree of saturation of the atmosphere at the time. But it is plain that he could never have done this, if he had not taken note of all the above

points, the material and texture of bodies, as affecting their surface-temperature, the clearness or cloudiness of the nights on which he looked for dew, the conditions of air and wall when the latter drips with moisture, and so forth. It would have been vain to observe that one body collected more dew and another less, unless their roughness and smoothness were noted, as well as their substance: or that on some nights there was heavy dew and none on others, unless the saturation of the atmosphere were considered as well as its temperature. And similarly, it was necessary that he should get a right conception of the thing called dew that he proposed investigating. There are days when everything grows damp from a moist fog hanging in the air. It would not have been unnatural to take this for a phenomenon of the same nature as dew-fall, and to overlook such things as dripping walls and moisture-frosted tumblers. Yet the mistake would have put the enquirer altogether off the scent.

Curative effects of different kinds are exhibited by certain waters. To the eye many of the waters are indistinguishable; and if the palate detects a difference, yet it would not be found possible to connect efficacy in particular complaints with particular flavours according to any explicit and invariable rule. It is plain that no progress can be made unless the various diseases are described not merely by their more obvious symptoms but by reference to the physiological character involved: and the water chemically analysed, so that one may know each separate ingredient, and the different proportions in which they are present in different cases. Again, the bacteriological theory of disease would never have been formulated, until the bacteria themselves were found—bodies so small that before the construction of powerful microscopes their presence was of necessity overlooked; and when one hears of pathologists endeavouring to isolate the microbe of some particular disease, one realizes how impossible it is, without the preliminary work of distinguishing the circumstances, to apply the 'canons of induction' to any purpose. Or suppose that an enquiry is undertaken not into the physiological cause of a disease, but into the causes of its dissemination, either generally or on some particular occasion: let the disease, for example, be malaria. Malaria was long supposed to be contracted from the exhalations of the ground; and it was true that many malarious districts were marshy, and that persons who avoided the swamps at dusk and dawn seemed less liable to be infected; but it

was not until it was noticed that such districts were infested with mosquitoes of a particular species, and it occurred to some one to connect this circumstance with the communication of the disease, that false beliefs were exposed and the true law of the matter established.

The last remark suggests a transition to the next preliminary operation that we may notice—**the formation of hypotheses.** Much has been written upon the question whether Logic can lay down any rules by which the formation of hypotheses should be controlled; but beyond offering the somewhat obvious and quite general considerations that an hypothesis must contain nothing inconsistent with principles which thought finds necessary, and that it must be such from which we can reason to consequences that should be found in one set of circumstances or another if it were true, it does not seem that Logic can be of any more service here than in the performance of the work of analysis. It would be an illegitimate hypothesis on the part of a bank clerk confronted with a small discrepancy in his books, to suppose that on this occasion two and two made three; but a petty theft on the part of the Principal Manager, though very likely a foolish hypothesis, would not be logically illegitimate. On the other hand, the hypothesis of angelic intervention, though there is nothing inconceivable in the existence of angels, would not be a legitimate way of proposing to account for the event; for there is no use in attributing phenomena to causes whose existence, and mode of action if they exist, we have no means of ascertaining; since such hypotheses can never be brought to the test of facts. It is obviously more reasonable to go on trying to account for them by ascertainable natural causes in the hope of being able to connect them by general principles with other observable phenomena, than to abandon that hope at the outset and invoke the agency of beings whose existence cannot be empirically verified; so that although we can hardly pronounce it logically inconceivable (however repugnant to scientific hopes) for the physical order so to depend on something beyond itself as to make it impossible to account for a particular natural event by reference solely to other natural events preceding it, yet we *may* on logical grounds pronounce it unscientific[1]: i.e. it is seen to be unscientific[1] not in virtue of any special knowledge of the particular science to which such hypothesis belongs, but in virtue of our general appreciation of the aim of any science, and of the logical conditions under which that aim can be

[1] Or at any rate non-scientific.

realized. And this is perhaps what Mill really had in his mind when he said[1] that 'It appears, then, to be a condition of the most genuinely scientific hypothesis, that it be not destined always to remain an hypothesis, but be of such a nature as to be either proved or disproved by comparison with observed facts'. It should be of such a nature that observable facts, *if we could find them*, might prove or disprove it[2]: i.e. it should not appeal to the agency of causes (like the intervention of an angel[3], or the influence of the organic type as a whole upon the growth of the individual organism) of whose presence we can have no independent evidence, and whose nature we are not able so to ascertain as to determine deductively how they must act if they are present; for with the agency of such causes as these any facts are equally compatible; and thus they furnish no explanation why the facts are so and not otherwise. For this reason, as Bacon said, in looking for the causes of things in nature *Deum semper excipimus*[4]: and Laplace, when Napoleon observed to him that there was no mention of God in his *Mécanique céleste*, replied that he had no need of that hypothesis. But that an hypothesis should be of such a nature that observed facts *will* ultimately either prove or disprove it, and not merely *might* ultimately do so, seems a condition quite impossible to lay down. We cannot tell the future in these matters; how long may an hypothesis be destined to remain an hypothesis without prejudice to its genuinely scientific character? The ultimate destruction of life on the earth is assumed by science; for human minds, an hypothesis which is not proved or disproved before that date will always remain an hypothesis. We cannot suppose that its scientific character, when it is made, is to be estimated by the prospect of its truth being definitely ascertained a few years, or even a few myriads of years, earlier or later. Darwin, in the *Origin of Species*,[5] writes as follows: 'As the embryo often shows more or less plainly the structure of the less modified and ancient progenitor of the group, we can see why ancient and extinct forms so often resemble in their adult state the embryos of existing species of the same class. Agassiz believes

[1] *System of Logic*, III. xiv. 4.

[2] Facts, as we have seen, cannot prove an hypothesis by their agreement with it, except so far as at the same time they disprove its rivals by their disagreement.

[3] Cf. Newman's *Parochial and Plain Sermons*, vol. ii, Sermon xxix, on *The Feast of St. Michael and all Angels.*

[4] *De Principiis atque Originibus.* Ellis and Spedding, III. p. 80.

[5] *Origin of Species*, c. xiv, 6th ed. p. 396. The italics are mine.

this to be a universal law of nature ; and we may hope hereafter to see the law proved true. It can, however, be proved true only in those cases in which the ancient state of the progenitor of the group has not been wholly obliterated, either by successive variations having supervened at a very early period of growth, or by such variations having been inherited at an earlier stage than that at which they first appeared. It should also be borne in mind, that the law may be true, but yet, owing to the geological record not extending far enough back in time, may remain for a long time, *or for ever*, incapable of demonstration '. But that the rule in question is an universal law is a scientific hypothesis.

An hypothesis then must be thinkable[1], consistently with the fundamental assumptions of the science which makes it, and it must be one whose consequences, if it were true, we can determine by reasoning : but we cannot restrict, within these limits, the freedom of scientific hypothesis. What is important is that men should be cautious not in framing but in testing hypotheses. The publication of every wild conjecture is undesirable ; but it would be equally undesirable that a man should never entertain an hypothesis which contemporary opinion could pronounce wild. Darwin said that he had framed and abandoned many an hypothesis which he would be ashamed to avow : he does not imply that he was ashamed to have framed them. The best control over the licence of the imagination is exercised by special knowledge. The man who knows most about any department of nature will see most readily what hypotheses are foolish in that department, just as in such practical matters as legislation the best critics of a bill are those who have experience of the affairs with which it deals.

[1] Lotze would explain this by saying that our hypotheses must conform to our postulates. He draws a distinction (*Logic*, § 273) between a *postulate* as 'an absolutely necessary assumption, without which the content of the observation with which we are dealing would contradict the laws of our thought ', and an *hypothesis* as ' a conjecture, which seeks to fill up the postulate thus abstractly stated by specifying the concrete causes, forces, or processes, out of which the given phenomenon really arose in this particular case, while in other cases maybe the same postulate is to be satisfied by utterly different though equivalent combinations of forces or active elements '. One should add, that in saying that an hypothesis must be thinkable *consistently with the fundamental assumptions of the science which makes it* we are enlarging as well as restricting the liberty of the mind in framing them. We restrict it to something which the facts of experience might test ; but the fundamental assumptions of a science may be metaphysically untenable, and we enlarge it to extend to all which these assumptions cover, however it may be ultimately impossible to think the facts in terms of them.

It is clear that every causal connexion presents itself at the outset in the light of an hypothesis, to the mind to which it first occurs. The framing of the hypothesis may sometimes be very simple, though the proof of it may be very difficult. If we know exactly what persons were cognizant of a secret which has been betrayed, it is easy to say that one of them must have betrayed it; and so far there is no hypothesis; hypothesis begins so soon as we ascribe the offence tentatively to any one of them, and in this there is not the least difficulty; but a proper test of it may be impossible. Whereas here, however, all the alternatives are before us, and in the abstract any one of them would equally fit the facts, because it is simply a question of connecting an event x with one of a number of conditions a b c, about which we do not know enough to say that it might not be connected with any one of them: yet commonly it happens that the facts which an hypothesis has to fit are more or less elaborate; and then the framing of it is not such a simple matter as the pairing off of two terms a and x. Take for example the question of the authorship of the Acts of the Apostles; if that book must have been written as it stands by one of the recorded companions of St. Paul's journeys, it is a simple thing to say that the author may be Luke, or may be Silas: although it need be by no means a simple thing to decide between them. But if that is not necessary, if the book may be of late date, and contain the work of several hands, very complicated and elaborate hypotheses will be possible. We have a large number of facts to co-ordinate; and the assumptions by which we connect them must all be mutually coherent. Historical criticism presents many problems, where no hypothesis is free from difficulty; and though doubtless a problem must have a solution, yet an ignorance of some details, and very likely the erroneous accounts that we have received of others, may leave us permanently unable to find it. And the penetration and ingenuity of the historian are shown in such cases in devising as well as in testing hypotheses; indeed the two operations cannot be kept altogether distinct: for when our knowledge of the concrete detail of events is considerable, the process of framing an hypothesis to fit them all is itself a process of testing. Now what is true in history, where upon the whole[1] our business is rather to determine events

[1] Upon the whole, because the historian has often to rediscover principles—constitutional, legal, social, or economic; and history advances by changes in men's way of conceiving the relations of past facts to one another as well as by changes in their view of what the facts were. We no longer believe

in conformity with acknowledged principles than to determine principles in accordance with empirically ascertained events, is true also in science, of whose business the latter would be the more accurate description. Scientific hypotheses consist for the most part not in the mere coupling in the mind, as cause and effect, of two insulated phenomena (if the epithet may be allowed) : but in the weaving of a large number of phenomena into a coherent system by means of principles that fit the facts. In the framing of hypotheses therefore we are called upon to regard facts in new ways : and to suggest not simply that certain facts are connected, but how, or in accordance with what principle, they are connected. And this often involves a radical transformation in our way of looking at the facts themselves ; for a fact is not such an easily ascertainable thing as the language we sometimes use might seem to imply. In a sense facts are stubborn : in another sense they are pliant to our thought. They are stubborn so far as we have rightly apprehended them ; but what we call fact is largely matter of inference and interpretation, performed often unconsciously, and often erroneously ; there is room here for re-interpretation, in accordance with the requirements of the rest of our knowledge, and so far as what are called facts lend themselves to this they may fairly be called pliant. It would have been called a fact, for example, in the days before Copernicus (though some of the Greeks had questioned it) that the sun went round the earth ; but this was only an interpretation of observations which we now see to be equally compatible with the earth's revolution round the sun. It would have been called a fact that species are fixed and immutable ; and it is the case that they breed so true upon the whole in any one generation as to make that a fairly accurate statement for practical purposes. Yet we have learnt to see that this comparative stability is consistent with any degree of modification over long enough periods of time. These instances will be enough to show how the familiar facts take on a new appearance in the light of new theories.

Now some new theories or hypotheses are, as we all know, more far-reaching in their effects than others ; for some are much more general, and apply to a much larger number and variety of facts. Their introduction marks an epoch in the progress of science ; and Whewell attached more importance to the framing of such hypo-

in William Tell ; but the Patriarchal Theory has also changed our views as to the relations between the individual and the State in some ancient societies.

theses than to any other of the operations connected with inductive reasoning. Indeed he held that this step *was* the induction; and that the history of the inductive sciences could be represented as the preparation, elaboration, and diffusion of successive hypotheses each more adequate to all the facts of a science than its predecessors. He did not use the word *hypothesis* very prominently in this connexion; he preferred to speak of *conceptions*: and what he called the *colligation of facts by means of appropriate conceptions*[1] was in his view the essence of induction. The new conception, however, is always put forward at first as an hypothesis, and only accepted as correct for its superior success in co-ordinating facts. This work of 'colligation' therefore must not be regarded as something distinct in its nature from the framing of hypotheses: it is rather a special and important case of it, where the hypothesis, instead of merely connecting facts in a more or less familiar way that leaves our view of the system to which they belong very much what it was before, involves a profound and far-reaching change in our view of the system and so also of the facts themselves. Thus the suggestion that malaria is communicated by the bite of the Anopheles mosquito neither altered seriously our notion of the nature of that insect (though it altered our practical attitude towards it in a way by no means favourable to the numbers of Anopheles) nor introduced any new way of conceiving disease; for the bacteriological theory of disease had already been applied to many other fevers. But the first suggestion that a disease depended on or consisted in the presence and multiplication of some specific noxious bacillus in the blood altered profoundly men's view of what many a disease is, of how it was communicable or curable, and of the whole economy of living nature. In the relation of this 'colligation' to the general framing of hypotheses we have an instance of the difficulty of distinguishing sharply the different operations of thought which logicians have enumerated as preliminary (though by no means subordinate) to such application of the rules on which inductive reasoning rests as we examined in the last chapter.

A somewhat unprofitable controversy arose between Whewell and Mill as to the part which the 'colligation of facts' should be regarded as playing in induction. While Whewell said it was the induction, Mill said that it was improperly so called. Mill seems to

[1] v. *Novum Organum Renovatum*, Bk. II. c. iv: *Philosophy of Discovery*, c. xxii. §§ 1–37.

have been influenced in part by thinking that an induction must end in establishing a general proposition, whereas it is possible to bind facts together by a new conception and so place them in a different light and reinterpret them, without apparently generalizing; he seems too to have considered that nothing in the whole process of thought, by which general conclusions were reached from the examination of particular facts, ought to be called induction, except what could be reduced to the form of inference or reasoning: the rest was all subsidiary to induction. But the operations of thought preliminary to the application of such rules as inductive reasoning rests on are not subsidiary in the sense of being of secondary importance; and it would perhaps also be better to distinguish induction as the whole process from the reasoning employed in it. We might then agree with Whewell that in induction, i. e. the whole process of the 'interpretation of nature', what he called the 'colligation of facts' is an operation of the very first importance, demanding higher and more uncommon powers of mind than inductive reasoning; while we agree with Mill that it is not the inferential operation. But if by induction we mean the inferential operation, then we shall have to say that this 'colligation of facts' is more momentous in the history of science than induction; for most of us, as Bacon rightly said,[1] would light upon the use of the methods of inference to which Mill would restrict the name of induction, by our ordinary intelligence, without their being formulated for us; but few can originate the new conceptions that show order and intelligibility in a mass of facts.

The instance which served to illustrate the dispute will help to show what this 'colligation' is. The Greek astronomer Eudoxus supposed the planets to move round the earth as if fixed in concentric spheres, with the 'fixed stars' in the outermost sphere. When further observation showed that this was not so, because the planets were not always at the same distance from the earth, circles were substituted for spheres, and the centre of the circle in which a planet moved was supposed to travel on the circumference of another circle; these circles were conceived not as mere imaginary paths, but as physical entities actually revolving; and it was possible to assign such a radius and rate of revolution to them as would account for the planet fixed upon the outer circle describing the path it does. This hypothesis had grown more and more complicated, as the mass

[1] *Nov. Org.* I. 130.

of observations upon the movements of the planets had increased; and though it was capable of application to the heliocentric no less than the geocentric theory, Kepler sought for one more satisfactory. After trying a large number of other curves, and rejecting them on the ground that they did not agree with the observations, he at last discovered that the planet Mars—the primary subject of his investigations—moved in an elliptical orbit round the sun, which stood in one of the foci. Now the ellipse is here the appropriate conception which binds together into an unity the successive observed positions of the planet Mars. Any position taken singly must of course necessarily be on the circumference of that or any other curve; for any curve can pass through any point. But he sought for a curve which would pass through all the positions; and he found that in an ellipse. There was indeed nothing disjunctive in his argument. Other curves were rejected because disproved by the observations; but the ellipse was accepted because the observations agreed with it, and not because no other curve would satisfy them. If it had suggested itself sooner, the others would not all have been tried. There are curves, of higher degree, that will equally satisfy a limited number of observations, and had they occurred to Kepler, he could perhaps have given no other reason for preferring to accept the ellipse than an *a priori* preference for the simplest curve that would do so. It is to be noted, however, that even here the critical matter was the thinking of an ellipse, and not the testing its agreement with the facts: any one with the necessary mathematical training could have done that, whenever the ellipse had been thought of. And so it often is, though not always, when the appropriate conception is a conception of causal relation: not always, because sometimes there may be as much difficulty or more in testing the conception than in thinking of it. To test it, we may have to deduce its consequences by some intricate mathematical calculus, as happened with the Newtonian theory of gravitation; or to devise an experiment in which we may see whether the theoretical consequences of our conception occur. Great mathematical power or great ingenuity may be wanted here; but the reasoning will be deductive. Yet even so, to introduce the appropriate conception is much; new theories are scarce; inductive reasoning, if the material were given all ready prepared, is easy.

An excellent example of the part which a new hypothesis may play in inductive enquiry is furnished by the *Oxygen theory*. It is

borrowed from Whewell,[1] whose works afford many more. It was for a time supposed that combustible bodies were combustible because of the presence in them of a peculiar substance, that escaped in the process of burning. This hypothetical substance was called phlogiston; and it was very natural to think that one could see it escaping into the air wherever a fire was burning. When it was found that there was one air (or, as we should now say, gas) in which bodies burnt readily, and another in which they would not burn at all, it was conceived that air could only absorb a limited quantity of phlogiston in proportion to its volume; in the former it was supposed that there was no phlogiston, and it was called dephlogisticated air; the latter was supposed to be already saturated, and was called phlogisticated air accordingly. The phlogiston theory received a shock when it was discovered that if a body were calcined, or reduced to ashes, in a closed vessel, the weight of the ashes was greater than that of the body before it was burnt. This, however, was explained by supposing phlogiston to be a substance naturally light, whose escape therefore left a body heavier—a view plausible, perhaps, when we remember how the sparks fly upward, yet really presenting great difficulties in relation to the theory of gravitation. The great French chemist Lavoisier, however, conceived the facts in a new way: he conceived that, when a body burned, what happened was not that a substance naturally light escaped from it into the air, and so left it heavier; but that a substance naturally heavy was withdrawn from the air and combined with the burning body; burning in fact was a process of what we should call chemical combination; and Lavoisier supported his theory by showing that after the calcination of a body in a closed vessel the air in the vessel was lighter by the same amount by which the ashes were heavier; this observation perhaps was not conclusive, if the phlogiston had carried its natural levity into the air; but the new way of conceiving the facts accorded far better with the general theory of gravitation. The substance thus withdrawn from the air in burning he called oxygen; and oxygen now took the place of dephlogisticated air; while phlogisticated air, instead of being conceived as the same air saturated with phlogiston, was conceived to be a different substance from oxygen, incapable of entering into those chemical combinations which constituted burning. This substance was now named azote, and afterwards nitrogen. Lavoisier further showed that oxygen

[1] Whewell, *Hist. Ind. Sci.*, vol. iii. Bk. XIV. cc. 4–7.

was withdrawn from the air and chemically combined with other substances not only in burning but also in the familiar process of breathing, and in the rusting or oxidation of iron, which could rust in water also because oxygen was present there as well; and thus his new conception, that burning was really a process of chemical combination between a substance in the atmosphere, which he called oxygen, and the substance of the body burnt, served to throw light equally on processes at first sight quite remote from burning. In this example, therefore, we have as it were a 'colligation' of two kinds: primarily, in so far as a large number of facts about burning were all rendered consistent with one another and bound together by the help of this new conception of what goes on when a body burns; secondarily, in so far as that conception was shown to be applicable to other phenomena as well as burning, and they are therefore brought under the same explanation with it. It may be worth while to give one more example of the transforming and connecting power exercised by a new and appropriate conception upon a multitude of facts, in the biological theory of Evolution, or the modification of species through natural descent. We are not for the moment concerned with the question whether the only agency in determining such modification is Natural Selection. The theory of Natural Selection, as a theory of the way in which modifications have, not indeed originated, but been established when they had once arisen, teaches that in each generation individuals vary more or less in colour, size, structure, &c., from their parents; that some of these variations are useful to their possessors under the circumstances in which they live; who will therefore, in the constant struggle for existence going on in the world, have an advantage over less fortunate competitors; so that those individuals who happen to possess 'adaptive' variations will survive and propagate, while the less well-adapted will perish; and thus species are brought into and kept in conformity with the conditions under which they have to live. Now there is not complete agreement among biologists either as to the extent to which the peculiarities of different species of plant or animal are adaptive, or as to the extent to which those that are adaptive can be accounted for by the theory of Natural Selection alone; though there is no doubt that the doctrine of Evolution won its way through the success of the principle of Natural Selection in accounting for at any rate a vast number of adaptive structures, instincts, and colourings. But the doctrine of

the Evolution of Species, or their modification by descent, as opposed to their special creation in immutable form, does not stand or fall with the view that Natural Selection is its exclusive *modus operandi*. This doctrine has brought into intelligible connexion with one another whole departments of fact. It explains the various and intricate relations of likeness and unlikeness between different species of the same genus, different genera of the same family, different families of the same order, &c.; it explains why the same structural plan is observed in many cases where the function of some part of the structure has been lost or altogether altered: and why it is that where their life requires the performance of the same function in groups otherwise very remote morphologically from one another, we find the function fulfilled by such very different means as are, for example, the wing of an insect, of a bird, of a bat, and of a flying-fish. Again, it explains the divers series of fossil forms: and accords with the facts of embryology, such as that the embryo of a given vertebrate only gradually develops the more distinctive specific features, and at an earlier stage is very little distinguishable from the embryo belonging to a different genus or family; for the characters which appeared later in the course of evolution and supervened as it were upon a simpler structure appear later in the growth of each subsequent individual of the same more complex type, and supervene upon the simpler structure there.[1] Again, it explains the facts of geographical distribution, such as that the degree of affinity between species is much greater when they inhabit a continuous area, than on either side of a geographical barrier; and that the barriers on either side of which the difference is most marked are not the same for every kind of organism, but are for each kind those which would offer the most effective obstacle to the migration of that kind—high mountain ranges in the case of land animals or fresh-water fish, wide stretches of open sea in the case of certain salt-water fish, and so forth: or such facts again as this, that 'wherever there is evidence of land areas having been for a long time separated from other land areas, there we meet with a more or less extraordinary profusion of unique species, often running up into unique genera'.[2] All these facts, and many others, for which upon the old hypothesis of the special creation

[1] Cf. on this the interesting appendix by Professor H. H. Turner in *The Laws of Heredity*, by Dr. Archdall Reid.

[2] Romanes, *Darwin and after Darwin*, i. 235 *et al.*

of immutable species it is impossible to suggest a reason or a motive, fall into line upon the hypothesis of modification by descent, and are bound together by that conception as common consequences.

We have now considered some of the most important operations, without which inductive reasoning would be powerless to advance inductive science. One or two others may be noticed. It may seem unnecessary to mention the *observation and registration of facts*; yet that is no small part of the work that has to be performed before we are in a position to tell how phenomena may be supposed to stand related in the way of cause and effect. Every lawyer knows how hard it is to make an uneducated witness distinguish rigidly between what he has observed, and what he has been led thereby to suppose was happening. And scientific observers have to be trained to be accurate in thus distinguishing, alert in noticing, quick in selecting what is new and instructive, and, where observation is of something confused or faint (as often with a microscope) intelligent in disentangling or interpreting. In such matters practice and instruction will do little without natural aptitude. But whatever the aptitude, it is found that there are certain constant errors of observation to which different men are differently liable. One man will regard as synchronous two sounds between which another will detect a slight interval of time; one man watching to record the moment when the image of a star touches a line will make the record just before, another just after contact, and so forth. The experience of these tendencies to error has led to the establishment for different observers of what is called their 'personal equation'; i. e. their observations are corrected by a co-efficient which is based on examination of the direction and amount of error to which they are severally liable. As no extension of self-recording apparatus will do away with the necessity for men to observe and record at certain points, and even the records of the apparatus need observing, it is clear that in the last resort the average error to which one man is subject in observing is to be ascertained through the observations of another man with a personal equation of his own, and hence the problem is intricate, and the theory of error has become a difficult branch of largely mathematical reasoning. Again, the registration of facts, where they are many and their relations complex, is not a simple matter. We must not ignore the value of the mechanical aids that can be given in tabulating, cataloguing,

indexing, &c.; but more important are formulae which enable us to record in brief collective statements what is relevant to a particular enquiry in vast numbers of observations. A simple and familiar instance of this is furnished by an *average*; for certain purposes to know the average of many observations is valuable, where an enumeration of each detailed observation would only confuse. But we have to consider carefully where an average is enough, and where it is not. Thus the average age of women at marriage is an important figure in relation to a country's birth-rate; but the average rainfall of a country is very uninstructive unless we know how it is distributed in places and years. There are however more elaborate and difficult devices than averages for reducing to manageable formulae what is relevant in a mass of observations, such as the 'co-efficient of correlation' which attempts to measure how closely the changes in one variable accompany the changes in another—e.g. in the corresponding right and left parts of animals bilaterally symmetrical, in the size of parents and off-spring, in the price of corn and the birth-rate. Here too mathematical problems arise for solution. And geometrical methods of registration may also be useful. In many enquiries where the collecting and tabulating of statistics is a necessary preliminary to the application of the rule that nothing can be the cause of a varying phenomenon which does not vary proportionately with it, the most helpful way of exhibiting the facts is by plotting curves or 'graphs'.[1] What was incidentally referred to on p. 471[2] is also of importance—the *devising of experiments* by which to test whether a phenomenon is present or absent, variable or constant, as it should be if its cause were what we take it to be. If it be supposed, for example, that spirit-rapping is really produced by 'cracking' the joints, it will be necessary not only to show that a man can produce such noises that way, but to devise conditions under which one may be certain that the joints cannot be 'cracked' without its being detected, and see whether the 'spirits' still continue to rap.[3] This is comparatively simple; but all the resources of mathematical reasoning and mechanical ingenuity are sometimes needed to determine and con-

[1] Cf. *infra*, p. 562.

[2] The other process, of mathematical calculation, there referred to, falls rather to be considered later: as belonging to a stage of science in which deductive reasoning plays a larger part than in the application of the rules discussed in the last chapter.

[3] *v.* Podmore's *History of Modern Spiritualism*, i. 184, 185.

struct the apparatus required for the conduct of an experiment that shall put a theory to the test.

This is perhaps enough to say upon the present subject. There are other tasks set to our thought in science, which are of great importance to its development; but we have been concerned especially with those that are presupposed in inductive reasoning. The help afforded to the 'interpretation of nature' by a *well-chosen armoury of technical terms*, great as it is, is not confined to the use of inductive reasoning. And account has been taken of *abstraction* in what was said of analysis and hypothesis and the formation of conceptions. By abstraction we mean considering some special feature of the concrete fact, in mental separation from all with which it is combined in its existence. It is between feature and feature that we strive to trace connexion. The tangle of things changes from moment to moment. Not until we pick it to pieces are we able to see what in one state of it determines what in another. Every common term involves some degree of abstraction; but in science we have to break up what in daily life we treat as a single matter, and to consider by itself, or in abstraction, that which had not hitherto been specially noted and distinguished in the totality of some comparatively complex nature.

CHAPTER XXII

OF NON-RECIPROCATING CAUSAL RELATIONS

In what has been so far said with regard to the process of inductively determining the cause of a phenomenon, it has been for the most part assumed that the cause, whatever it is, reciprocates with the phenomenon studied: i. e. that not only does the phenomenon occur whenever the cause is present, but that the cause must be present whenever the phenomenon occurs; so that you may safely argue from either to the other, as in geometry you may equally infer that a triangle is equilateral from the fact that it is equiangular, and that it is equiangular from the fact that it is equilateral.

But we often speak of one thing as being the cause of another, where this reciprocal relation by no means obtains. We say that drunkenness causes crime, although many people get drunk without committing crime, and many people commit crime without getting drunk. And in some of the examples of inductive reasoning given in previous chapters, the cause found was not a reciprocating cause. The appearance of congenital epilepsy in guinea-pigs was shown to be due possibly to a wound producing epilepsy in the parent; yet it was not alleged that the production of epilepsy by these means in the parent was always followed by the appearance of epilepsy in the offspring.

It was said that the inductive proof of the cause of a phenomenon rested on an understanding of the causal relation: for nothing that does not stand to the phenomenon in such relation as a cause should can be the cause of it; and it is by eliminating all alternatives that its cause is inductively established. Our account of cause assumed that it reciprocated with its effect. But if it does not, we clearly have no right to eliminate whatever fails to reciprocate. The admission that there are non-reciprocating causal relations may seem therefore to invalidate reasoning that starts with the assumption that cause and effect reciprocate.

This difficulty has been postponed till now, partly that the exposition of the subject might not be unduly complicated: but also,

because the causal relation is really, and in its strict sense, reciprocal, and without understanding that first, we could never render non-reciprocating causal relations intelligible to ourselves. Properly speaking, to give the cause of anything is to give everything necessary, and nothing superfluous, to its existence. Nevertheless we should often defeat our ends, if we gave precisely this; if our object in seeking the cause of a thing is that we may be able to produce or prevent it, and if something is necessary to its existence which is a property of a thing otherwise superfluous, it would be of no use to specify the property necessary unless we also specified the otherwise superfluous thing in which it was found.[1] Even though we have no such practical purpose, so long as we do not know what thing contributes, in the property which it possesses, the factor necessary to the effect, we can hardly be said to understand completely the production of the effect. Hearing at a distance, for example, depends on the transmission of certain vibrations through an elastic medium; the necessary elasticity is a property of the air; and therefore we can hear at a distance in the air, while if there is a vacuum interposed between the sounding (i.e. the vibrating) body and the ear, the transmission of the sound is prevented. It is true that, except in respect of its elasticity, air is quite superfluous so far as hearing at a distance is concerned; not air in the concrete, but that property in abstraction, is one of the conditions that make up the reciprocating cause of hearing at a distance. But an elastic medium cannot be just elastic and nothing else besides.[2] We want to know what possessed of the necessary elasticity is present when we hear at a distance; nor could any one, without knowing that, prevent the transmission of sound by removing the elastic medium; for he would not know what to remove.

We may pursue this illustration a little further. It might be shown inductively that the intervening air was the cause of the transmission of sound; indeed it was shown inductively, by the

[1] e. g. it may be the texture of pumice-stone that fits it to remove ink-stains from the skin; but it would be of more use to tell a man with inky fingers to get a piece of pumice-stone, than to give him a description of the fineness of texture which would render a body capable of making his fingers clean.

[2] It is just the fact that we know no more about the ether than its form of elasticity which makes the conception of it somewhat unsatisfactory; and led the late Lord Salisbury, in his Presidential Address to the British Association at Oxford in 1894, to say of it that it merely 'furnishes a nominative case to the verb *to undulate*'.

help of a well-known experiment. And speaking loosely, it is true that from the presence of air it can be inferred that sound will be transmitted, and reciprocally, from the transmission of sound, that air intervenes. Yet neither inference is quite safe. The first is only true with qualifications: the distance must not be too great in proportion to the loudness of the sound, and so forth. The second may be altogether false; for sound can be transmitted through water, or (with the help of a telephone [1]) through a vacuum. And in this case the reason is that the elasticity is provided in some other way than by means of a continuum of air. We saw that, except in respect of its elasticity, air was superfluous: but we could not get the elasticity alone. Now we find that there are other elastic media which will serve, and the elasticity may be provided by them. An elastic medium is what is wanted; but divers things will supply the want. They are alternatives, and none of them exclusively reciprocates with the effect; for the effect may be produced by the help of any one of them, so that the occurrence of the effect does not prove that any one more than another is producing it. But their common property of providing an elastic medium does reciprocate; sound cannot be transmitted without that.

There is, then, always a reciprocating cause; but it is not always most instructive to state only that. And very often that is not what we want to know. There are several reasons for this.

In the first place, though the object of a science is to discover strictly universal propositions, and though in most sciences [2] these involve relations of cause and effect, yet as a science advances, its problems often take a different form than that of an enquiry after the cause of a given phenomenon. We may start with something that seems comparatively simple; and, as we proceed, may find that it depends upon a number of conditions being combined together, each of which can be fulfilled in a number of ways, but none of them without much that is superfluous or irrelevant to the production of the precise effect in question; each is an incident of some complex event, or involves a property of some concrete thing, like the elasticity of air in the transmission of sound. To state in abstract form the conditions that must be satisfied, without indicating

[1] The elasticity of the air is employed also in the telephone: but not continuously. It is hardly necessary for the present purpose to go into the detail of the apparatus.

[2] Not in any branch of purely mathematical study; nor elsewhere where we are not concerned with change.

the kind of thing or event in which such conditions can be realized, is uninstructive ; for it fails to explain by what the phenomenon is produced ; yet to mention every thing or event in which the conditions might be realized would be an endless and unprofitable task. Hence we alter the form of our problem. Looking upon the phenomenon as the complex result of many conditions, we attempt to determine not what assemblages of things will produce the result, for many will do so, nor on what properties or incidents therein it depends ; but what is the principle of action in different things, in virtue of which any of them will serve equally among the conditions necessary to the production of the phenomenon. For the reciprocating cause of a complex phenomenon we substitute as the object of our search the principle in accordance with which a certain *kind* of thing acts. Our problem is better expressed as that of discovering laws of nature, than causes. For example, we may ask what is the cause of the monsoons—that is, of the regular and periodic winds that blow steadily in certain regions for one part of the year in one and for another in the opposite direction. If we said that they were due to periodic alternations in the distribution of atmospheric pressure, it would not be very instructive ; for we really want to know what events, happening in those regions, produce these differences. Yet the events which contribute to determine the deviation and direction of the monsoons are numerous and variable : the exact combination of them differs from year to year and from place to place, and produces corresponding differences in the result. It is better therefore to take the things concerned in these events, by their kinds, singly : to point out the difference in power of the sun at any place according to the varying directness of its rays ; that the sea gives off vapour ; that vapour absorbs part of the heat of the sun's rays ; how the heated water circulates with the colder ; that the earth absorbs and retains the heat of the sun ; that air is expanded by heat ; how the principle of atmospheric pressure acts under conditions of different expansion ; and so forth. Then we can see that if a certain combination of events occurs, a particular complex result must arise ; if the sun travels from over the sea to over the interior of a continent, we shall find monsoons ; for the difference between summer and winter temperature will in the interior be very great, but on the sea, owing to the way in which the moisture of the air absorbs part of the heat, and the currents in the water carry away part, it is not so great ; hence as summer

is ending, the air inland will be hotter and have expanded more than out at sea, as winter is ending it will be colder and have contracted more; so that at one time the current of air sets inland in accordance with the laws of atmospheric pressure, and at another time it sets shoreward. The principles, or ways of acting, on the part of the sun according to its altitude, of the earth and sea respectively under the influence of heat, of air when unequally expanded, &c., are not exhibited solely in the phenomena of monsoons; while the details of those phenomena display the influence of other principles of action on the part of other things (e.g. the action of a mountain-wall on a moisture-laden wind). To give the cause of monsoons, without deficiency or superfluity, would mean that we must not mention the sun (because only the heat of its rays is material) nor the sea (because only its fluidity and its power of giving off vapour concern us, and a lake, if it was big enough, would do as well) nor any other of the concrete things which act in the way required, but only their requisite actions. If we do not go to this length of abstraction, we shall have to include in our statement of the cause elements at least theoretically superfluous; and even so, we shall have to choose some particular monsoon, supposing we are to state everything that goes to produce it. It is clearly simpler to break up the problem, and look for the principles in accordance with which things of a certain kind act under certain circumstances; then we can show that the monsoon is only the complex result of the action of a number of things under the particular circumstances of its occurrence, and in accordance with the principles of action which our 'laws' express.

This then is one reason why what we want to know is not by any means always the reciprocating cause of a determinate phenomenon: the phenomenon under investigation is often highly complex, and subject to all sorts of variation on the different occasions of its occurrence, through variation in the things or events contributing to its production; not the whole nature of the things or events under whose influence it occurs is relevant to its occurrence, but only certain particular properties or modes of action; and it is possible to formulate severally the principles of action involved, from which the joint result may be seen to follow, where it would not be possible to assign to the phenomenon any group of concrete things or events as cause, about which we could say not only that, given them, the phenomenon must be given, but also that, given

the phenomenon, they must have been given too. These laws or principles of action may of course be proved inductively in just the same way as may a causal connexion between two particular phenomena a and x. Just as we may argue that a cannot be the cause of x, if it occurs in the absence of x, or is absent when x occurs, so we may argue that a law or principle of action cannot be rightly stated, if consequences should follow from it as thus stated which do not actually arise, or should not follow, which do arise. Here, as there, we may have no other reason for accepting a theory than that the facts are inconsistent with any other that we can devise; and then our argument is inductive.

Another reason why we do not always look for a reciprocating cause is that for practical purposes it is generally more important to know what means will produce a certain result, than by what it has been produced. We cannot alter the past; we may control the future. The means prescribed for the production of a certain result may contain much that is not relevant precisely to the production of that result; and as this irrelevant matter may be different on different occasions, there may be a choice of means. To have a choice of means is undoubtedly useful; but if any of these means is called the cause of the result in question, the term 'cause' is clearly not used in the strict sense; for we may be able to argue forward from the means as cause to the result as effect; but we cannot argue backward from the result as effect to this particular means as cause. Yet this will be of comparatively little consequence, if our interest lies less in being able to determine by which means the result in question was produced on a past occasion, than whether certain means will produce it in the future. About a variety of advertised rat-poisons, all that we should care to know would be that they would rid us of rats; and we might endeavour to determine inductively whether a particular poison was efficacious. But we should be indifferent to the fact that other poisons might be equally efficacious, and that rats who died off need not have been killed by this particular poison; in other words, we shall not want to learn the reciprocating cause of the dying off of rats. Indeed as long as the effect is stated in such a general way, a reciprocating cause cannot be given. There are, as Mill observed, many causes of death; and though he was referring to men, it is also true of rats. But death is not altogether the same thing whenever it occurs; and the doctor or the coroner knows this. The many different causes

of death do not have altogether the same effects; if you shoot a man and if you behead him, the difference in the result is visible; if you poleaxe an ox and if you poison him, he is not equally edible. As soon as we begin to be interested in the particular variety of death produced, we find the number of causes that produce the result in which we are interested diminishing rapidly; if we carried our interest far enough into detail, we might say that for death of a particular kind there was only one cause possible. But since much of this detail is quite unimportant, we treat as instances of the same kind events which in some respects are different, and then say that the same event has divers causes: forgetting that the differences between these several causes consist partly in circumstances irrelevant to the kind of the event, which are included in our statement because indissolubly bound up with what is relevant, but otherwise superfluous to the production of it: and partly in circumstances that are represented by differences in the resulting event, but differences which we ignore. Here then, in the fact that our search is often for means to the production of a phenomenon of a certain general character, to the precise form of which we may be indifferent, is a second reason why the causal relations which we seek to establish are often non-reciprocating.

On the other hand, thirdly, there are cases where it concerns us more to be able to argue from one phenomenon to another as its cause, than from the latter to the presence of the former as effect. For example, there may be alternative symptoms of the same disease: for the effects of the disease may differ to some extent in patients of different age, or sex, or race. Here it may be important to show, that if a certain symptom occurs, that disease must be present to produce it; while the fact that the disease may exist without giving rise to that symptom is a minor matter, and one which, if we could be certain that some other equally conspicuous and unambiguous symptom would occur instead, might be called altogether unimportant. In such a case we shall be anxious to show a causal connexion between the disease and the symptom in question, though again the relation will be non-reciprocating; but it will fail to reciprocate this time, because the so-called cause may exist without the so-called effect, although the so-called effect cannot exist without the so-called cause; whereas in such cases as were considered in the last paragraph, the so-called cause always produced the so-called effect, but the so-called effect might exist without the so-called cause.

Fourthly, our enquiries are often directed to the discovery of the cause or effect of some *singular* event—singular, not in the sense of unusual, but of individual: we ask, for example, what has been the effect of the repeal of the corn laws in 1846, or what was the cause of a particular railway accident, or epidemic. It is plain that the relation we wish to establish in such cases as these is a non-reciprocating relation. The repeal of the corn laws was a measure introduced into a highly complex social and economic state, and whatever results we can point to depend on much else besides that measure; no one would pretend that the same measure would have produced the same results in other circumstances. It might be possible here to substitute for the question, What effect has their repeal produced in the United Kingdom? the more scientific question, In what way do corn laws act? The answer to the latter question might be given in the form of one or more universal propositions: but the answer to the former will be a singular judgement. For it is practically impossible to specify all the conditions which have combined with repeal to produce the results in which the influence of repeal is exhibited; so that we cannot hope to establish an universal proposition of the form that repeal of corn laws produces always under such and such conditions the result which we ascribe to their repeal in 1846 in the United Kingdom. If a man says therefore that the repeal of the corn laws has increased the population, or depopulated the rural districts, or crippled the ancient Universities, or made inevitable a celibate clergy, he is not to be understood to mean either that it would always produce any one of these effects, or that they must always be due to a repeal of corn laws: but only that in the history of the United Kingdom, had the corn laws remained in force, other things being equal, these effects would not have occurred in the same degree. So also when we enquire the cause of a singular effect: it may be known that the reciprocating cause of small-pox is the presence of a certain microbe in sufficient strength in the blood; but if we ask for the cause of a definite outbreak, something else than that is wanted. We want to know what particular precaution has been omitted, by taking which this outbreak might have been prevented; or in what particular way the infection was conveyed to the neighbourhood. Thus we might say that the outbreak was due to a tramp sleeping in a common lodging-house, or to insufficient vaccination; but it is not imagined that a tramp suffering from small-pox cannot sleep in any common lodging-house

without an outbreak of small-pox following in the place; or that no such outbreak ever occurs unless from that reason; while insufficient vaccination, even if no serious outbreak ever occurred where it could not be alleged, may prevail without an outbreak following, so long as nothing brings the infection. Similarly about a railway accident the question is, what particular act or omission that some one is responsible for, or what other preventable event, can be alleged, without which *on this occasion* there would have been no accident: did a signalman give the wrong signal, or pull the wrong points? did an engine-driver disregard a signal? had a flood washed out the ballast of the line, or an axle failed? These and many more are the 'causes' of railway accidents, though railway accidents occur without them, and they may occur without accidents following.

In previous chapters we have represented the phenomena between which it is sought to establish causal relations by letters of the alphabet. Each of these letters is quite distinct from the rest, insulated as it were, and discontinuous both with those grouped with it to indicate contemporaneous phenomena, and with those placed apart to indicate phenomena preceding or succeeding it; and the use of them as symbols tends to suggest that the course of events is a succession of discontinuous phenomena, which produce each the next in a number of parallel or contemporaneous series. Nothing could be further from the truth: it is impossible to conceive the matter thus.[1] We have already noted the ambiguity —the convenient ambiguity—of the term phenomenon; some 'phenomena' which we isolate and individualize by a name do succeed one another; but others do not precede or succeed at all, but

[1] Let nobody object that in such a matter we must ask what experience teaches, and not what it is possible to conceive. Experience can teach nothing inconceivable. All thinking is an attempt to make experience more intelligible, and so far as it is not intelligible, we assume our account of it to be untrue. It is for this reason that we are always recasting in thought the account of what appears in our experience. The very search for causal connexions is an example of this operation. It rests on the principle that change is only intelligible if it displays necessary principles of change: but these principles are not presented to our observation. Therefore we believe that events occurred, which have not fallen within our experience: as Robinson Crusoe, seeing footprints, concluded that men must have been to the island whom he had not seen. And if we deny that the events 'experienced' are all that occur, on the ground that their succession would then be without principle and unintelligible, we may equally deny that history can consist of streams of discontinuous events, even though these succeeded one another according to the most constant rules, on the ground that such a succession would be unintelligible.

endure or persist. Kant said that 'only the permanent can change'[1]: we look on events as occurring to things; permanent things change their states; and the permanent thing enters into the earlier and the later state alike, or persists through them. What that is which remains unchanged, how we are to conceive it, and how we are to conceive the junction between its abiding nature and its changing states—these are very difficult questions which do not belong to Inductive Science. But it is clear that our alphabetic symbols fail in the first place to represent the persistence of anything through change: they are discontinuous in their series where they symbolize a change which is continuous. And secondly they are discontinuous within the group that represents contemporaneous phenomena; whereas the contemporaneous phenomena they represent are not similarly insulated from one another. What we commonly speak of as single phenomena are bound together not in independent series unit to successive unit, but by all sorts of cross ramifications, so that each is what it is in consequence of conditions which are at the same time conditioning many others in the most complicated way. To this complication the letters of the alphabet do no justice. Doubtless if we carry our analysis far enough, we may find the a which is the reciprocating cause of x: but a will not in that case as a rule be anything for which we have any single name; a long and carefully guarded statement of conditions will be what it must symbolize.

The fact is that in most cases the reciprocating cause of anything, if we push our enquiries far enough, emerges as the conditions that constitute it, and not those that precede it and bring it about. The reciprocating cause of small-pox is that activity of a specific bacillus in the blood in which small-pox consists: the reciprocating cause of malarial fever is the corresponding activity of another bacillus. But in the procession of events by which that state is brought about there may be one, which—for one reason or another—it concerns us to single out, and call the cause: and that will often be non-reciprocating. It need not be so; it is possible to find an event, whose happening in a given set of conditions or to a given subject always gives rise to some definite new event or state of that subject, and without whose happening such new event or state of that subject never arises. It is supposed for example that malaria is always communicated to man by the bite of the Anopheles

[1] *Kritik of Pure Reason*, section 'On the First Analogy of Experience'.

mosquito ; there are persons immune to the bacillus, and therefore the bite of Anopheles is still a non-reciprocating cause ; but if we knew what state of a subject precluded immunity, then we could say that the bite of Anopheles caused malarial fever in any man in that state, and we should have stated a reciprocating relation ; for no man in that state could be bitten without getting malaria, nor get malaria without being bitten. If with Aristotle we call the conditions which constitute anything the *formal* cause, and that whose activity brings those conditions into being when they had previously not all of them existed, the *efficient* cause,[1] we may say that the formal cause reciprocates or is commensurate with the phenomenon (as indeed anything must which can in any sense be called the definition of it : and the conditions into which it can be analysed may be called its definition) ; while the efficient cause seldom reciprocates. The concrete thing or complex event which includes the conditions, or part of the conditions, constituting the phenomenon, may also be called, in a metaphor of Bacon's using, the *vehicle* of the formal cause, or of part of it ; the biting Anopheles is the vehicle of, or conveys, the bacillus in whose activity malaria fever consists ; the headsman's axe, or the bullets of the firing party, convey, or are the vehicle of, that bodily state which we call death. The expression is not equally metaphorical in both these cases, for the mosquito really carries the bacillus into the blood of the patient, as a vehicle carries its occupants, and bullets or axes do not thus carry death ; but what is meant is that events occur, involving things, whose existence and activity is irrelevant to the effect in question except just so far as they contribute to the constitution of that total state which is the effect.

There are indeed many cases where our ignorance of the conditions constitutive of a certain phenomenon compels us to seek instead for some event indispensable to its occurrence, even though our scientific interest would be better satisfied by discovering the constitutive conditions. And there is one most extensive and important class of cases where the reciprocating conditions cannot really be called constitutive of the phenomenon ; it is this class of cases which made it necessary at the beginning of the last paragraph to write 'most' and not 'all'. The former sort may be readily

[1] Besides the *formal* and the *efficient*, Aristotle distinguished the *material* cause, or matter of which a thing is made, and the *final* cause, or purpose of its being. These were all causes in the sense of being necessary to the existence of what they are the cause of. Cf. e. g. *Phys.* β. iii. 194^{b} 16–195^{a} 3.

exemplified in the biological sciences. 'That form of barrenness', writes an authority quoted by Romanes,[1] 'very common in some districts, which makes heifers become what are called "bullers" —i. e. irregularly in season, wild, and failing to conceive—is certainly produced by excess of iron in their drinking water, and I suspect also by a deficiency of potash in the soil.' Here we have one and perhaps two causes alleged for an effect, whose nature we do not understand sufficiently to see how the causes bring it about, though the facts may prove the connexion. Such a relation may be called *discontinuous*—i.e. we do not see how the alleged cause, by any intelligible procession of events, passes into the effect, or helps to set up the conditions constitutive of it. We connect one phenomenon as cause with another as effect, where from our ignorance of the intimate nature of the effect, and of the subject in which it is produced, and from the fact that the intervening process of change is withdrawn from view, the two seem quite heterogeneous. In Chicago, one is told, there are machines into which you place a pig at one end, and receive sausages at the other. The pig and the sausages, to any one who has no conception of the nature of the machine and what befalls the pig in it, appear in a relation of sequence without continuity: first the pig exists, and then instead of it, the sausages; but we do not see how the one becomes the other. This somewhat mythical machine may serve to illustrate how our ignorance of the nature of the process of change connecting one event with another may produce apparently discontinuous causal relations; and such relations are often all that we can at present hope to discover; and they are generally, as may easily be understood, non-reciprocating relations. This case is different from that mentioned previously on p. 483; for there it was our practical ends which interested us in causes that were non-reciprocating; here it is due to the limitation of our scientific knowledge that we have to acquiesce in them.

But in the extensive and important class of cases to which attention must be called next, we find discontinuity even where the causal relation reciprocates: viz. when the cause is physical and the effect psychical, or vice versa. It has already been stated that such connexions furnish one of the best kinds of example of purely inductive reasoning, because there is nothing in the nature of a particular physical process which would lead us to anticipate the particular

[1] J. W. Crompton: *v. Darwin and after Darwin*, iii. 170.

psychical state that we find ourselves led by the facts to connect with it. What may be the true interpretation of this apparent dependence of psychical states on physical processes, and physical movements on psychical states, is the hardest question in metaphysics. Meanwhile, at the standpoint at which many sciences and all of us in our ordinary thought are content to stop, we attribute many psychical events to physical causes, and vice versa. In science indeed the attribution of physical effects to psychical causes is less common than that of psychical effects to physical causes; just because between the successive events in the physical order there are prospects of establishing that continuity, which there seems less hope of establishing in the psychical series, and none of establishing between members of one series and members of the other, between a motion of matter in the brain and a sensation or thought or feeling or emotion. The series therefore whose members do appear capable of continuous and coherent connexion is often treated as independent, and psychical states regarded as by-products of particular terms in the physical series; although further reflection can easily show that such a statement of the case, when thought out into its consequences, involves us in hopeless contradiction.[1] We are however at present only concerned with the interdependence of physical and psychical states as it appears to exist, and is at least for many practical purposes rightly treated as existing.

It is supposed that to every distinct state of consciousness there corresponds some distinct state of the body; and this bodily state is not separated from the state of consciousness by any intervening process, the discovery of which might help us to see how one gives rise to the other (as drinking water with an excess of iron in it is separated from the supervening barrenness in a heifer). There is perhaps no interval of time between them, but the completion of the conditions in which the bodily state consists is *eo ipso* the production of the corresponding state of consciousness; so that some writers have been led to speak as if the state of consciousness could be analysed into these bodily conditions, and they really constituted it. That however, when examined, proves to be nonsense.

Yet though in this field we may hope to find relations that reciprocate in spite of the discontinuity between the so-called cause and its effect, there are instances here too where the causal relations are non-reciprocating; and of this perhaps the most notable instance

[1] Cf. *supra*, p. 411.

is death. It was explained above, how the many alternative causes of death are not all of them causes of the *same* effect; because they do not put the body into the same state, although the differences may not concern us. But if we look not to what befalls the body, but to the result on consciousness—whether we suppose it to be that the soul is separated from the body, or that it is destroyed—we can see no difference in that main result corresponding to the difference of the means by which it is produced. If the soul, or individual consciousness, be destroyed at death, there is of course nothing any longer in which a corresponding difference can be displayed; if it be not, we may conceive that as the manner of a man's death, if it be not absolutely sudden, affects him while he yet lives—one death being more painful, for example, than another—so the differences between one death and another are represented by some difference that persists in the experience of the soul after death, and therefore the effect is not really the same upon the soul when the physical 'cause' is different. But such a suggestion is quite unverifiable; and however that may be, it is well to realize the peculiarity of the relations which we try to establish between physical causes and psychical effects; owing to the heterogeneity of the two terms, we cannot hope to find an intelligible cause of the psychical state in the conditions constitutive of the physical state with which it is connected; at this point there is discontinuity; and so there may arise an appearance of different causes producing the same effect which we cannot explain as we explained it in a purely physical sequence. There we saw that different series of events having partially the same nature might, in their course and as a part of their result, agree in establishing the same complex of conditions constitutive of some particular phenomenon, although the difference in the events occasioned in the rest of the result differences which we ignored. Here, inasmuch as we cannot see that the different causes establish conditions constitutive of the effect at all, the appearance of the same effect when the causes are different cannot be exhibited as a case where effects different as a whole (in a way corresponding to the difference of the causes) agree so far as concerns the conditions constitutive of the phenomenon we are investigating.

The term *Plurality of Causes*[1] has been used to indicate the fact

[1] The term was introduced by Mill, who sometimes speaks as if he thought the Plurality of Causes more than apparent: as if he thought that, in the

that the same phenomenon may have different causes on different occasions. We have seen that the fact is more apparent than real: that the alternative 'causes' of a phenomenon, which make up the plurality, are none of them causes in the strictest sense, but rather events which agree so far as the production of the phenomenon requires, though taken as a whole they are very different. It would perhaps be well if there was a term to indicate the corresponding fact, that the same phenomenon may produce different effects on different occasions: a fact also more apparent than real, for such phenomenon cannot be the cause, in the strictest sense, of any of the alternative effects which it produces. We might speak in this sense of the *Diversity of Effects*. In neither case do cause and effect reciprocate.

Where the cause or effect sought is non-reciprocating, it is obvious that the rules on which the elimination involved in inductive reasoning rests are no longer to be trusted. If the same effect may have divers causes, we cannot say that nothing in the absence of which a phenomenon occurs can be the cause of it; it cannot be its cause in the particular instance in which it is absent; but it may be on another occasion. If a small group of plants be geographically isolated from the main stock, it will diverge, and in course of time probably give rise to a new species; but there are other ways in which a particular group may be prevented from interbreeding with the main stock (e. g. by flowering at a different season), so that new species may arise in the absence of geographical isolation; it would clearly be unsafe to conclude, from the fact that new species had arisen without geographical isolation, that geographical isolation was not a cause of new species arising.

No doubt such an argument would betray insufficient analysis: it would overlook the fact that geographical isolation was not a single factor, but highly complex; and that one feature about it —viz. that it prevented interbreeding with the rest of the stock— characterized also such very different phenomena as difference of flowering-season, or selective sterility.[1] However, our analysis is

strictest sense of the term *cause*, the same phenomenon may have different causes on different occasions. The Plurality of Causes must be distinguished from the Composition of Causes: which means that a complex phenomenon, which we call one, may be due to a number of causes acting together on one occasion. Clearly none of these is the cause in the full sense, but only part of the cause.

[1] Or 'physiological isolation'—i.e. that certain members of a species x which happen to exhibit some modification m are more fertile with one

very commonly incomplete ; and then it is possible, that by applying the above rule, of eliminating whatever fails to occur in any instance of the effect, we have eliminated the cause altogether : and that if some circumstance is left uneliminated, because it occurs in all the instances of the phenomenon, we take it to be the cause of that with which it has really nothing to do. If a child were given the same medicine in a variety of jams, and always had a particular biscuit afterwards, it might very likely attribute the effects of the medicine to the biscuit. Suppose my apple-crop fails four years in succession, and that each year it was ' overlooked ' by a woman reputed to have the evil eye : were I to argue that the failure was not due to insufficient rain, since in the first year there was plenty—nor to late frosts, for in the last year there were none—nor to blight, which only occurred once—nor to high winds, since the third year was singularly quiet, I might at last attribute the failure of the crop to the ' witch-woman ' overlooking it.

In such a situation it is well to test one's results by the second rule, that nothing is the cause of a phenomenon, in the presence of which the phenomenon fails to occur. If the child were frequently given the same biscuit when it had not been dosed, it would learn to disconnect the biscuit from the effects properly connected with the medicine ; and if the witch-woman were observed to overlook my orchard in several years when I subsequently obtained an excellent crop, I might be cured of my superstition. It is however possible that I might still hold her responsible for the bad crops, and apply the doctrine of the Diversity of Effects to explain why her action had failed of its previous result on other occasions. Perhaps I might have had the crop blessed by a priest, and attribute to that an effect counteracting the influence of the evil eye ; or merely say, that the evil eye cannot be expected always to produce the same results, when there must be many contributory conditions that are varying.

There is no remedy against such errors except a wider acquaintance with facts, and a closer analysis of them, and a better way of conceiving them and their connexions. To this end however very special help is given by *experiment*. The results of an experiment are of the same kind with the data of observation—facts, namely,

another than with the rest of the species in which this modification has not appeared. This would prevent swamping by intercrossing, and so, for breeding purposes, isolate the new variety.

with which we have to make our theories consistent; and the inductive reasoning to which the facts contribute premisses is not altered in character because the facts are obtained experimentally. But where we can experiment, we can commonly discover facts which observation would never reveal to us. We can introduce a factor into conditions carefully prepared, so that we know more or less accurately what change we make, and in what we make it; and then, when we watch the effect, the work of elimination has more grounds to proceed on. If we are in doubt whether to refer some phenomenon to a plurality of causes, or to a single circumstance which, as present in all our instances, they have not so far enabled us to eliminate, we might resolve the doubt by producing this circumstance experimentally: should the phenomenon not follow, we have then shown that, at least in the conditions into which we introduced the factor in question, that factor will not produce it. We may then try one and another out of the plurality of alleged alternative causes: and if we find that on the introduction of each the phenomenon follows, we shall conclude that they are causes of it. We shall still be far from having discovered its precise cause, without deficiency or superfluity; but we shall have advanced our enquiry. The child who attributed to the biscuit the effects of the medicine could correct its error by experimenting with the biscuit separately, and the medicated jams separately. And if I could bring myself to experiment with the evil eye, I might convince myself that it was innocuous to orchards.

It should be noted that though the Plurality of Causes and the Diversity of Effects render precarious, when our analysis is imperfect, the application of both the grounds of elimination just cited—viz. that nothing is the cause of a phenomenon in the absence of which it occurs, and nothing also, in the presence of which it fails to occur—yet the amount of error in which we may be involved is not the same in each case. Should we reject in turn everything, among the circumstances under which a phenomenon occurs, without which also it is found to occur in other instances, we might reject all its several causes, and fall back on something whose presence in the instances we have examined is quite accidental: something altogether immaterial to the phenomenon. On the other hand, should we reject everything, among the circumstances under which it occurs, which at another time is found without it, though we might be wrong in concluding that

what is left is the whole cause of the phenomenon, or that other things might not serve as well as it, yet we should be right in concluding that it was not altogether irrelevant to the production of the phenomenon on this occasion. I give a dog cyanide of potassium, and it dies ; assuming this to be the only fresh circumstance in the case, I cannot conclude that dogs do not die without taking cyanide of potassium ; but I can conclude that taking cyanide of potassium contributed something to the death of this dog, and that the conjunction of the two events was not merely accidental, as eating the biscuit was accidental to the child's subsequent experience, or as being ' overlooked ' by a witch-woman was accidental to the failure of my apple-crop. In the former case, where I reject everything in whose absence the phenomenon occurs, I reject too much : the essential factor lurks undetected each time in a different ' vehicle ' : each of these ' vehicles ' is rejected in turn, and the essential facts rejected with them. In the latter case, where I reject everything in whose presence the phenomenon fails to occur, I may reject both too much and too little—perhaps too much, for what I reject, though insufficient of itself to produce the phenomenon, may contain conditions without which it cannot be produced : perhaps also too little, for what is left, while I take it all to be essential to the phenomenon, may be in part superfluous, though containing the essential factor within it ; so that other things, in which the same essential factor is contained, may equally serve to produce the phenomenon ; yet still I retain something essential, and do not reject everything which I need to retain.

A reciprocating cause would be at once necessary and sufficient to the production of the effect ascribed to it. What is called the cause of anything in a looser and commoner sense may fail to reciprocate with its so-called effect either because it is not sufficient to its production, although necessary : or because it is not necessary, although sufficient : or because it is neither sufficient nor necessary. But what is neither sufficient nor necessary to the production of an effect x would not be called its cause at all, unless it included something that was necessary. Now when we seek the cause of an effect by comparison of instances in which the effect occurs, rejecting those circumstances which are not common to them all, we proceed on the principle that what is absent where it occurs is not its cause. But we mean by this that such circumstances are not necessary to the production of the effect, since the effect occurs without them ;

we cannot conclude that they are none of them sufficient. There may be several of them sufficient to produce it; each might contain what is really necessary, but none be necessary as a whole; or if the effect is one dependent on the maintenance of a complex variety of conditions, the removal of each might remove some one of those conditions, and so destroy the effect: in this way asphyxiation, decapitation, heart failure, are each sufficient to destroy life. The circumstances which we reject may of course be unnecessary because wholly irrelevant, and the cause must be sought outside what is unnecessary in this sense. But if they are severally unnecessary only because though different they are each sufficient, so that, in the absence of one, another will serve, we are wrong in looking outside them for any part of the cause. In such cases our principle of rejection misleads us altogether. With the other principle, that what is present where the effect fails to occur is not its cause, our risk is less. This principle we use, when seeking to determine the cause by comparison of an instance in which the effect occurs with an instance of like circumstances without it. And what we mean by saying that the circumstances common to both the positive and negative instances are not the cause of the effect that occurs in the positive instance is, not that such circumstances are not necessary to the effect—this they, or some of them, may or may not be—but that they are not sufficient. And if they are not sufficient, we must look outside them for something which, though perhaps also insufficient by itself, is still necessary for the occurrence of the effect.

What we thus find in the positive instance before us may indeed be only sufficient for the occurrence of the effect *in this situation*, not necessary in all situations; it may be one of several alternatives, one or other of which, but none in particular, is necessary. But though we are liable to error in overlooking this, we are still justified in the use of our principle to this extent, that what we reject, though it may contain factors necessary to the occurrence of the effect, does not contain all the necessary factors, and therefore not what is sufficient: whereas in using the other principle, we were not justified correspondingly; for what we rejected may have contained more than was sufficient and all that was necessary to the occurrence of the effect. For the sufficient includes the necessary; if it reciprocates with that to which it is sufficient, it includes no more, and is the precise aggregate of the factors necessary; but the necessary

does not include the sufficient ; only the aggregate of the necessary factors and not each of them is sufficient.

[J. S. Mill, who spoke of what he called the Plurality of Causes as the 'characteristic imperfection of the Method of Agreement', said that the Method of Difference was unaffected by it. Clearly he was wrong. The above argument endeavours to bring out the truth underlying the exaggeration of his statement. That he was wrong may be seen further by help of the following considerations. If x occurs under the circumstances abc, and not under the circumstances bc, I can infer that bc is not sufficient to produce x, and that a contributed to its production on this occasion ; but I cannot infer that x could not have been produced without a : pbc might equally produce it. That a and p can equally produce x (or equally produce it in bc) is an instance of the Plurality of Causes ; and it is the Plurality of Causes therefore which prevents my inferring universally that x is produced by a, or requires a for its production, and limits me to the inference that a produces x, at least in bc. It will be said that a and p must have some common property r, which is the really essential factor. No doubt : or else they must have this in common, that each removes one, though perhaps a different one, of a number of factors collectively necessary to the existence of x. But, as we have seen, this is equally so in any instance of Plurality of Causes ; if I refuse to infer, in accordance with the 'Method of Agreement', from the fact that x occurs under the circumstances abc, ade, afg, that a is its cause, urging that for aught I know the cause may be c in one case, e in the next, and g in the third, I must believe that c, e, and g contain a common r which is the really essential factor ; and then a is not the 'only circumstance in common', for r is another : just as in the other case a was not the 'only circumstance of difference', where x occurred and where it did not, but really r contained and overlooked in a was a circumstance of difference as well.

The distinction which Mill draws between the two 'Methods' then is not altogether sound ; for the appearance of Plurality of Causes affects the inference which can be drawn in each. But there is this much truth in it, as was pointed out in the text : that in the 'Method of Agreement', where I am eliminating that in the absence of which the phenomenon occurs, I may unwittingly eliminate the essential factor : I throw away the baby with the bath, and am left supposing that a is the cause of x, when a may really have nothing to do with it, and its presence in each of my instances be a mere accident ; in the 'Method of Difference', where I eliminate that in the presence of which the phenomenon fails to occur, though a large part of a may be superfluous to the occurrence of x, yet it is not altogether superfluous ; I do not this time connect x with something that has

[nothing to do with it. But I am unable to infer a reciprocating relation between a and x for the same reason that in the former case I was unable to infer any relation at all—viz. the Plurality of Causes. And let it not be said that this difficulty would not arise, if the conditions of the 'Method of Difference' were fulfilled, and a were the only circumstance of difference where x occurred and where it did not. For I should still be unable to infer a reciprocating relation: I could only conclude that a was necessary to the production of x in bc: how much of bc was also necessary I should not yet have discovered. In both cases, if the analysis of the circumstances were complete, the Plurality of Causes would disappear; in neither, while it is incomplete, is it without effect on our liberty of conclusion.

Mill seems unconsciously to assume that this analysis is more complete when we employ his 'Method of Difference' than when we employ his 'Method of Agreement'. The reason of his doing so is probably that the experimenter uses the 'Method of Difference' (or the principle of elimination which it involves), and a completer analysis is generally obtainable when we can experiment than when we are confined to the observation of events as they occur in nature: the experimenter uses the 'Method of Difference', because in experimenting we introduce or remove some particular factor—and that under circumstances which we have endeavoured to ascertain as precisely as possible—and watch the result; and if we are right in assuming these circumstances to remain otherwise unchanged, we do approximate to having only the 'one circumstance of difference' which Mill's canon requires; in other words, we are really eliminating at once and by appeal to a single principle all except this factor removed or introduced by us; though it must not be forgotten that what we eliminate is only shown to be insufficient to the production of the phenomenon, and may still contain conditions that are essential though not sufficient. We may note here the reason why the 'Method of Difference' seemed to Mill, and in a sense rightly seemed, to be of superior cogency. The reasoning is clearly no better in it; but facts such as are required in order that the reasoning may lead to a conclusion of value are far more often available for this 'Method', because we can contribute to their production by experiment, and this 'Method' is practically a formulation of one of the commonest ways in which we reason from the results of experiment. We may indeed say that the error into which reasoning from an incomplete analysis of the facts may lead us is greater when our ground of elimination is that underlying the 'Method of Agreement' than when it is that underlying the 'Method of Difference': because in the former case we may reject what is essential, and end by attributing the phenomenon under investigation to something whose presence is quite accidental; while in the latter case, we may

[rather end by supposing that more is essential to it than really is so. Yet there may be error in both cases, and for the same reason, viz. our incomplete acquaintance with the facts. What Mill however saw was, that where you can experiment with precision, your acquaintance with the facts is most complete, and hence the conclusions to be drawn most cogent. It is just in these cases that the 'Method of Difference' as he formulates it is specially applicable; for it requires instances where the phenomenon occurs and where it does not occur with 'only one circumstance of difference'. He overlooked the fact that the Method is just the same, where this condition is not fulfilled, so long as our ground of elimination is the same—viz. that nothing in the presence of which the phenomenon fails to occur is its cause; and so he attributed universally to the 'Method of Difference' a superior cogency which really belongs to the 'prerogative' nature of the instances in connexion with which chiefly he considered its use.]

It has been the object of the present chapter in the first place to acknowledge that the 'Rules by which to judge of causes and effects', whereon inductive reasoning depends, are not infallible where we are dealing with non-reciprocating causal relations; for they rest on the assumption that one effect has only one cause, and conversely that the same cause has never any but the same effect; and so they furnish no safe guide to the discovery of 'causes' which are not the only causes of the effect assigned to them, or of effects which are not the only effects that the alleged cause may have. Its second object has been to show that such non-reciprocating causal relations arise from the fact of our including in the cause more than is necessary, and perhaps also less than is necessary, to the production of the effect: or including in the effect less or more than the cause assigned produces; i. e. our analysis is not perfect: we combine with the matters strictly relevant to one another others irrelevant, but closely bound up in their occurrence with what is relevant: so that there appears to be a Plurality of Causes for the same effect, or a Diversity of Effects for the same cause, while really, if we could 'purify' our statements of the cause and of the effect sufficiently, we should see this not to be the case. But we admitted that for many purposes, practical and even scientific, it is causes in the looser sense that we need to discover—the sense in which the cause includes more than is material to the production of the effect in question, but a more from which what is material cannot be dissevered, and so forth. And we saw that science, when pushing its

investigation beyond such a level as that, tends to substitute for the search for the determinate cause of some concrete effect the search for laws or principles in accordance with which things of a certain kind act on one another under specified conditions.

In illustrating these points, the rules whose guidance was shown to become unsafe when non-reciprocating relations were in question were the first two of the rules laid down in the Twentieth Chapter. But the last two are also liable to mislead us in such cases. These are, that nothing which is constant when the phenomenon varies, or varies when it is constant, or varies independently of it, is its cause: and that nothing which produces a different effect is its cause. In particular I cannot, because elimination based upon these rules reveals that x is not independent of a in the instances before me, infer that x never occurs without a; for p might do as well. If I find that the faster I run, the hotter I get, and if I know that the temperature of the atmosphere has not altered, and so forth, I may infer that running makes me hot; but not that no one gets hot without running. If I experiment over a series of years with a particular manure, and take care to ascertain by 'controlling' experiments the average crop that I might have expected without its use, I may be led to attribute the excess to the use of the manure; but I cannot conclude that a similarly large crop is always due to the use of it. Errors of that sort would be similar to those which I might commit in applying the rule that nothing is the cause of a phenomenon, in the presence of which it fails to occur: then too I have no right to assume that what I fail to eliminate is altogether necessary, and that nothing else would serve equally instead of it. But the danger of eliminating too much, which besets the application of the rule that nothing is the cause of a phenomenon, in the absence of which it occurs, does not equally beset the application of the two rules we are now considering. It is true that in investigating the cause of a phenomenon that may vary in quantity or degree, and is due as a whole to a number of contributory factors, this danger is theoretically possible. The quantity or degree of the phenomenon might remain constant, owing to divers complementary variations in the factors, some increasing as others decreased; and because the variations masked one another, I might reject each varying factor in turn, until I had rejected all the contributory factors, as capable of varying with no corresponding variation in the phenomenon. But this is not a probable error. And the fact that

the phenomena, to which these rules are applicable, are chiefly *measurable* phenomena, is of great importance in the use of them. Peculiar difficulties no doubt often beset us in tracing the influence of some particular factor upon a phenomenon which varies in magnitude dependently upon the joint action of a large number of conditions independently variable; it is for example exceedingly hard to determine inductively whether the corn duty of 1902 influenced the price of bread in Great Britain. But these difficulties would obviously be altogether insurmountable if no measurement of the conditions and of their result were possible. The introduction of the element of quantity enables us to determine laws which connect a definite amount of change in one phenomenon with some corresponding amount in another. Where we can do this, we are already getting clear of the errors lurking in non-reciprocating causal relations. It still remains true that we cannot, in virtue of a law which connects with a change in the condition a a corresponding change in the result x, argue backwards from a change in x to the action of a. But that point has been sufficiently exemplified already; and inasmuch as some special attention will have to be paid in another connexion[1], when we are dealing with the importance of quantitative methods in induction, to the two rules or principles of elimination last mentioned, it is perhaps unnecessary to say anything further here upon the care that must be used in arguing from them when the causal relations which we have it in mind to establish are non-reciprocating.

[1] Cf. *infra*, c. XXVI, pp. 557–562.

CHAPTER XXIII

OF EXPLANATION

We are said to **explain**, when a conjunction of elements or features in the real, whose connexion is not intelligible from a consideration of themselves, is made clear through connexions shown between them and others. The connexion explained is said to be the consequence of the connexions which explain it, but this expression must not be taken as implying sequence in time[1]; it is also said to be deduced from them. What is explained may be either a particular fact, or a general principle; there is no fundamental difference between explanation of the one and of the other. If it be a particular fact, the detail must be accounted for by corresponding details in the facts referred to in the premisses. If it be a general principle, we shall omit such detail in the premisses. There are many conjunctions repeated frequently in our experience with variations of individualizing detail or concomitant circumstance; if we recognize under this their identity of kind, we can explain them together by reference to relations of elements similarly identical in

[1] That which is explained may of course be a sequence in time, and then time-relations enter into the explanation. Thus if I were to explain the beneficial effect of root-pruning on fruit-trees, I should point out that roots whose ends are cut off proceed to throw out fibrous rootlets in greater abundance, and these extract from the soil more of the nourishment which the tree requires than did the parts removed. But the facts stated in my explanation do not precede the sequence explained; I am merely showing what relations are really involved in the sequence I am explaining. We do indeed speak both of one event being explained by another which precedes it, and of the sequence being explained by a principle of sequence displayed in it. Here two cases need distinguishing. (i) The particular sequence a_1–x_1 may be explained as an instance of the general principle of sequence a–x, although that principle is only inductively established and not intelligible; and to accept this explanation means simply that we are content with finding in the particular event an instance of a principle of connexion which we have reason elsewhere to accept; the principle stands in no time-relation to its instances. Thus, granted that belladonna dilates the pupil of the eye, we might be said to have explained the unusual size of the pupils in X's eyes by the fact that belladonna had been injected into them. (ii) The sequence a_1–x_1 may be intelligible from the nature of the terms a and x; thus the fact that M was angry with N might be said to be explained, if I learnt that N had insulted him. Human nature and the nature of an insult are such that when a man insults another he angers him. This we realize in the case of M and N, and it again has no time-relation to the sequence of M's anger upon N's insult.

kind, ignoring such detail as does not affect the truth of our general statement of connexions. But in thus explaining a general principle we are always explaining at the same time, up to a point, the particular facts in which it is manifested.

In all scientific explanations, our premisses are 'special' or 'proper' or scientific principles. General logical considerations, such as direct us in the inductive search for causal relations, account for nothing in particular [1]; every explanation must be consistent with them, but they will not themselves explain anything. The explanation of the facts or derivative laws of any science rests therefore on a scientific knowledge of the subject-matter of that science.

The first or fundamental principles of science are themselves insusceptible of scientific explanation. It does not follow from this that the principles which at any given time are the most ultimate to which a science appeals should be insusceptible of explanation; the Law of Gravitation, for example, is and has long been a fundamental physical principle, but various mathematicians have attempted to show that the behaviour of matter expressed in that law follows necessarily from some more general principles exhibited also in activities whose principles we commonly regard as different, like electricity and light. But the process of explaining must come somewhere to an end, with principles deducible from nothing prior to themselves.

These principles, as has been pointed out [2], may possibly appear self-evident when we have reached them; the First Law of Motion has often been thought to be a self-evident or necessary truth. But in most cases, they do not; and then all that we can say about them is that nothing so well explains those facts, the study of which has led us to their enunciation. This however is a *pis aller*.

It has not infrequently been said that scientific *certainty* is unattainable. Jevons urges that the conclusions of Induction are only probable at the best. The reason is that the principles which we arrive at as those which explain things are not—at least as a rule—seen to be necessary; and that we cannot absolutely prove that *no* other principles will explain the facts: just as in simpler inductive enquiries our confidence in the cause which we assign to a phenomenon is qualified by the difficulty of being sure that we have

[1] With the help of these considerations we may be led by the observation of certain facts to believe some general proposition about their connexion, but we do not thereby explain the connexion. That we have observed the facts explains our believing the connexion; they do not explain it.

[2] *Supra*, pp. 382–386, 414.

overlooked nothing which might equally, upon the facts examined, be allowed to be the cause.

Jevons indeed suggests[1] that the true though impracticable road to certainty would lie in Complete Enumeration. 'Perfect Induction' rests on complete enumeration, the 'Imperfect Induction' of actual scientific procedure does not; and in this he sees the source of the 'imperfection' which conclusions only approximately certain possess. But though we may agree with him that many of the conclusions accepted in science fall short of certainty, we cannot agree that they would rank higher if they were reached by complete enumeration; for in that case they would not be universal truths at all, in the proper sense, but only truths about the whole of a limited number of particular facts. Indeed the antithesis of Perfect and Imperfect Induction is an unfortunate one. It belongs to a different sense of the term Induction from that which, in the phrase Imperfect Induction, the term now bears. It is drawn from the completeness and incompleteness of the enumeration of the particulars on which the Induction rests, and to which its conclusion refers; we have seen that if a generalization rests merely on citation of particular facts, without any attempt to establish connexions of a causal character by analysis and elimination, the citation should be complete; though in such cases, the conclusion has not the true character of an universal proposition. But the reasoning which infers general truths from the analysis of a limited number of particulars does not rely on enumeration, and is not an operation of the same kind as that which proceeds by complete enumeration. Though the one therefore may cite every instance, and the other not, yet they are not to be contrasted as if they were operations of the same kind differing only in that respect. They are operations of different kinds; and their other differences are more fundamental than the difference in the completeness or incompleteness of the enumeration they involve. If the one is called perfect because its enumeration is complete, it must be remembered that it requires a complete enumeration; but since the other does not require it, it is misleading to call it imperfect for not employing it. The imperfection attaching to the conclusions of inductive science—conclusions which are said to be reached by 'Imperfect Induction'—springs from the defective analysis of the instances cited, not from

[1] *Elementary Lessons in Logic*, XXV, 'New Edition', p. 213: *Principles of Science*, 2nd ed. pp. 146–152.

failure to cite every instance; and it is a mistake to suppose that 'Perfect Induction', if it could be employed—as it is acknowledged it cannot—would remove the defect of certainty attaching to scientific generalizations. For science seeks after the necessary and the universal, not after the merely exceptionless.

However, our present concern is less with the reason for the want of absolute certainty in the principles of scientific explanation, than with the fact itself. It cannot be denied that the first principles of science rest for the most part on no better foundation than this, that no others have been suggested which explain the facts equally well; and this is not the same as saying that no others can be suggested which will do so. And even if we were satisfied that no others could be suggested, i.e. if we could be certain that *nothing* so well explains the facts as the principles to which we appeal in our explanation, yet if we cannot see why these principles need be as we find them, we are still left with something that at once demands to be and cannot be accounted for.

We shall be wise therefore to recognize these two things about scientific explanation at the outset, viz. (i) that it often starts with principles, or truths, or laws, which are neither accounted for nor in themselves self-evident, but only warranted by the success with which they account for the facts of our experience: and (ii) that these principles are not absolutely and irrefragably proved, so long as any others which might equally well account for the facts remain conceivable. But it would be foolish to let these considerations engage us in a general and indiscriminate distrust of scientific principles. Such principles may lack that demonstrable character which we should like them to have; and Logic would abandon its function, if it hesitated, out of respect for the greatness of scientific achievement, to point this out. But they hold the field; we are not entitled to treat them as dogma, which cannot be questioned; but we are entitled to say that so long as they remain unshaken, they should be treated as true.

It may be objected that they are not unshaken; that some of the fundamental assumptions of science are unable to resist metaphysical criticism: the independent existence of matter, the action of one thing on another, the production of a conscious state by a process in a physical organism, are all unintelligible. And it must be allowed that the scientific account of reality cannot be the ultimate truth. But if the provisional nature of certain of its metaphysical

assumptions be borne in mind (for science does not really discard, though it sometimes professes contempt for, metaphysics), we may then admit the explanations which it offers within their limits.

If however we are to accept those principles which best explain the facts of our experience, we must have some antecedent notion of what a good explanation is. Now it can certainly be required of an explanation that it should be self-consistent. But we are not content with this. There are a number of maxims, which do actually guide us in theorizing about the laws of nature, pointing to some more positive ideal than self-consistency. The influence of these maxims shows that there operates upon scientific minds some notion of what a rational universe should be, as well as a belief that the universe is rational, not derived from experience, but controlling the interpretation of experience. 'The common notion that he who would search out the secrets of nature must humbly wait on experience, obedient to its slightest hint, is,' it has been said [1], 'but partly true. This may be his ordinary attitude; but now and again it happens that observation and experience are not treated as guides to be meekly followed, but as witnesses to be broken down in cross-examination. Their plain message is disbelieved, and the investigating judge does not pause until a confession in harmony with his preconceived idea has, if possible, been wrung from their reluctant evidence.' What these preconceived ideas are, it would be difficult to say precisely; nor is the question of their justification an easy one. They have formed the subject of considerable discussion on the part of philosophical writers since the time at least of Leibniz, who perhaps did most to call attention to them. But one of the most famous has a much higher antiquity. 'Occam's razor' [2]—*entia non sunt multiplicanda praeter necessitatem*—is a maxim to which science constantly appeals. It is felt that there is a presumption in favour of theories which require the smallest number of ultimate principles: that there is a presumption in favour of the derivation of the chemical elements from some common source, or of the reduction of the laws of gravitation, electricity, light, and heat to a common basis. Again, we are inclined to believe that the

[1] *Presidential Address at the British Association, Cambridge*, 1904, by the Rt. Hon. A. J. Balfour (*Times* of Aug. 18). He illustrates his statement by reference to two cases, the persistent belief that the chemical elements will be found to have a common origin, and the persistent refusal to believe in action at a distance. It may however be doubted whether this refusal is as well justified as that belief by the maxims in question.

[2] William of Occam, ob. 1347.

ultimate laws of nature are not only few but simple. The law of gravitation states that the attraction between any two bodies varies inversely as the square of the distance. But it is conceivable that the true relation of the force of attraction to the distance of the bodies between which it acts is not so simple; provided it diverged from the ratio of the inverse square so slightly that the difference would be less than our observation, with the margin of error to which it is liable, could detect, such less simple relation would have as much to be said for it, so far as the facts go, as the simple relation that Newton established. Yet few would seriously consider its claims. It may be said, and truly, that there are sound practical reasons for accepting the simple relation, in preference to any other that has no better claims, because it renders our calculations much easier; yet it may be doubted whether we really regard it as only a more convenient hypothesis. We are more disposed to think it true because such a simple relation satisfies better our ideal of explanation. J. S. Mill's definition of Laws of Nature has been already quoted[1]—'the fewest and simplest assumptions, which being granted, the whole existing order of nature would result'. In the words 'fewest and simplest' are contained perhaps the most important of the preconceived ideas which we have about the explanation of the facts of nature.

It is impossible to reduce explanation to any definite formulae. When nothing but a middle term is wanted, to connect with a subject a predicate empirically found to characterize it, there it will fall into the form of syllogism.[2] But comparatively few explanations can be expressed in a single syllogism. Where, as is commonly the case, they trace the complex result of several principles in some particular combination of circumstances, the building up of this result in thought is not a syllogistic process.

As has been said above, there is no fundamental difference between explanation of a particular fact and of a general principle. In the latter case, more abstraction has been performed; we are explaining something exemplified in facts that constantly occur, which has been extricated in thought from varying and irrelevant detail. In the former also, some amount of abstraction must have taken place; but the fact we have thus isolated still retains details that make it unique. An oculist may explain the common fact that short-sighted persons grow longer-sighted as they grow older, by

[1] *Supra*, p. 386, n. 3.

[2] But cf. *infra*, p. 524, n. 2.

showing how clear vision depends on focusing all the rays proceeding to the eye from each several point precisely upon the surface of the retina ; in short-sighted persons, the curvature of the lens of the eye is excessive, and therefore objects have to be nearer than would normally be necessary, in order that the rays proceeding from any point in them may be focused on the retina and not in front of it ; but the curvature of the lens is maintained by certain muscles, which relax with age, and therefore, as years advance, clear vision of objects is possible at a greater distance. If he were called upon to explain some unique peculiarity of vision in a particular patient, the task would still be of the same kind ; but the facts to be taken into account would partly be facts peculiar to this case, and though their consequences would be traced according to general principles, their special combination would make the complex result unique : unique however not necessarily, for the same combination might conceivably recur, but only as a fact within medical experience.

Historical explanation is largely concerned with events in this sense unique. History has generalizations that admit of explanation also ; but human affairs are so complex, and our interest in them extends into so much detail, that the unique occupies a quite peculiar share of attention in its investigations. And its task consists largely in making facts intelligible by tracing their development. For an institution or event, when we come upon it as it were abruptly, may surprise us : whereas if we know the past, we may see that its existence or occurrence connects itself with other facts about the same folk or period in accordance with accepted principles. The institution of primogeniture for example, according to which land descends upon the eldest son, is a peculiar institution, unknown, according to Sir Henry Maine, to the Hellenic, to the Roman, and apparently to the whole Semitic world ; neither did the Teutonic races when they spread over Western Europe bring it with them as their ordinary rule of succession. Whence then did it originate ? for such institutions do not occur at haphazard. Maine accounts for it as 'a product of tribal leadership in its decay'. Chieftaincy is not the same thing as being a landowner ; but some of the tribal lands were generally the appanage of chieftaincy. So long as times were warlike, the chieftaincy seems not necessarily to have gone to the eldest son of the deceased chief ; but 'wherever some degree of internal peace was maintained during tolerably long periods of time, wherever an approach was made to the formation

of societies of the distinctive modern type, wherever military and civil institutions began to group themselves round the central authority of a king, the value of strategical capacity in the humbler chiefs would diminish, and in the smaller brotherhoods the respect for purity of blood would have unchecked play. The most natural object of this respect is he who most directly derives his blood from the last ruler, and thus the eldest son, even though a minor, comes to be preferred in the succession to his uncle; and, in default of sons, the succession may even devolve on a woman. There are not a few indications that the transformation of ideas was gradual'. The custom, Maine thinks, was greatly fixed by Edward I's decision in the controversy between Bruce and Baliol; where the celebrity of the dispute gave force to the precedent. The rule of primogeniture was extended from succession to the lord's demesne to succession to all the estates of the holder of the signory however acquired, and ultimately applied to all the privileged classes throughout feudalized Europe.[1] In a case like this, a knowledge of past facts enables us to see how a new custom might emerge conformably to known principles of human nature. There are motives for allowing the chieftaincy to devolve upon the eldest son, and motives for conferring it upon the strongest of the near kindred; when the latter are weakened by change of circumstance, the former are likely to prevail. The influence of precedent upon the human mind is also a familiar principle; and though it is impossible to show that in such cases nothing else could at any point have happened (Edward I for example might have decided differently), yet in the light of what we know of men's passions and purposes and of the physical conditions under which they live, we are able to understand many of the connexions which link events together.

Sciences like Geology or Biology set themselves for the most part to solve more generalized problems of development: though to them too some particular fact, apparently in conflict with a theory, may offer occasion for a detailed historical enquiry. But the explanation of the occurrence of crystallized rock, commonly as that occurs, is not logically different from what it would be if it occurred once only; and if we set about accounting for that local and temporal affinity of species which is expressed in Mr. A. R. Wallace's principle that 'Every species has come into

[1] *v.* Maine's *Early Institutions*, pp. 197–205, from which the above example is abridged.

existence coincident both in space and time with a pre-existing and closely allied species'[1], we shall not proceed otherwise than if the affinities of one particular historical group of species were to be accounted for.

There are other sciences (e.g. Political Economy or Kinematics) which do not concern themselves with tracing any particular historical development, yet have to explain the laws manifested in a succession of events. Here too it may be of the essence of the explanation to show how one change determines another, and the new fact thus introduced determines a third, and so forth. The laws involved may be various, and the sequence be explained by resolution into stages, each of which exhibits a general principle, while the special circumstances in which such a principle is exhibited furnish the occasion for a further change that exemplifies another.

There are cases where the element of time is one of the most important of the facts. Many effects depend upon the juxtaposition of bodies in space, and their juxtaposition depends on time-conditions. The fortune of a campaign may be decided by the rapidity of a march, bringing troops upon the field at a critical moment; the troops may fight upon the same principles and with the same degrees of courage all through, but the result is determined by their being there at the time. The working of a machine would be thrown out by anything that delayed or hastened the movement of a part with which other moving parts had to engage; and the same is of course true as regards the articulated movements of an animal. The disintegration of mountains is largely produced by frost succeeding rain; if rain only came just after frost, it would not take place in the same way. Professor Marshall has called attention, in his *Principles of Economics*, to the great importance of the element of time in the working of economic laws.[2]

There are however also many results that are to be accounted for through the concurrent operation of several principles: or rather —for principles cannot in strictness be said themselves to operate— through the concurrent operation of several causes, each according to its own principle. The path of a projectile at any moment is determined by its own inertia, the pull of the earth, and the resistance of the atmosphere. It is true that at every moment these forces are producing a new direction and velocity in the projectile,

[1] Quoted Romanes, *Darwin and after Darwin*, i. 243.
[2] e.g. Bk. III. c. iv. § 5, 4th ed. p. 184.

which forms the basis for an immediate further change; and that it is by following the continuous series of these successive changes that its path is ascertained—a task which the notation of the calculus alone renders possible. The consideration of any term in the series of changes as the resultant of simultaneously operating causes is however different from the consideration of the succession of one resultant change upon another in the series. And the explanation of many problems lies in showing the concurrent operation of different causes, each acting continuously according to its own law; as opposed to the case just considered, where one cause may produce an effect that, by virtue of the conditions with which its production coincides, then produces a fresh effect in accordance with a different law. The column of mercury in the barometer is maintained according to laws that are all continuously exemplified, and not first one and then another of them; the atmosphere is always exerting pressure, and in the mercury the pressure is always equalized in virtue of its nature as a fluid. Economists are familiar with 'Gresham's Law' that bad money drives out good, i. e. that if in any country the circulating medium is not of uniform quality, the best is always exported and the worst left behind. By 'best' is meant that whose intrinsic value bears the highest proportion to its nominal value; a sovereign which contains the proper weight of fine gold being better than one containing less, and so forth. The explanation of the Law is simple. Government can make the bad money legal tender for the payment of debts at home; it cannot compel the foreigner to receive it. For discharging debts abroad the better money is therefore more valuable, for discharging debts at home it is no more valuable than the worse; it is therefore more profitable to export the good, and keep the bad money for home purposes; and the desire of wealth being one of the strongest and most uniform motives in mankind, what is most profitable is naturally done. Nothing turns here upon the resolution of a sequence into stages exhibiting different laws; the derivative law is shown to follow from more general laws, under the special assemblage of circumstances described in saying that the circulating medium in a country is not of uniform quality; but these general laws are exhibited simultaneously and not successively. That the power of any government extends to its own subjects only, and that men desire wealth, are principles more general than Gresham's Law; and both apply to money, which is at once, as legal tender, a matter to

which the power of government applies, and, as medium of exchange, the equivalent of wealth.

No logical importance attaches to the distinction between explanations that derive a complex law from simpler laws exemplified together, and those that derive it from simpler laws exemplified successively. Many explanations involve both features. But there is a difference of more importance between either of these, and that form of explanation which consists in showing that laws, hitherto regarded as distinct, are really one and the same. Newton showed that the familiar fact that heavy bodies fall to the earth, and the equally familiar fact that the planets are retained in their orbits, were really instances of the same principle, the general Law of Attraction. Something of the same sort is done when Romanes points out that Natural Selection, and Sexual Selection, and Physiological Selection, and Geographical Isolation are in their operation so many forms of Isolation preventing free intercrossing among all the members of a species, and thereby leading to modification of type.[1] In cases like these, we do not derive a derivative law from *several* more general laws exemplified together or successively in complex circumstances of a particular kind; but a single more general law or kind of process is shown to be exemplified in a diversity of circumstances which have hitherto concealed its identity. This operation is sometimes called *subsumption*, as bringing several concepts under one, in the character of instances, or of subjects of which it can be predicated in common. Yet even here it is plain that the operation, of tracing the distinctive peculiarities of the laws or processes explained or subsumed to the special character of the circumstances in which the same more general principle is exhibited, is of the same kind as occurs in all other forms of explanation: only the further synthesis, in which the complex consequences of the concurrent or successive laws or kinds of process are traced, is lacking.

Explanation, as was said at the beginning of the chapter, is deductive—deductive, that is, in respect of the reasoning involved in it. Yet it has a close relation with the work of Induction, and the consideration of this will form the subject of the remainder of the chapter.

Explanation starts, as we have seen, from principles already known, or taken as known; and it shows that the matter to be explained follows as consequence from these. But it is clear that

[1] *Darwin and after Darwin*, vol. iii. c. i.

the reasoning which deduces their consequence from them is unaffected by the nature of our grounds for taking them as true. If they were nothing more than hypotheses, we might still argue from them to their consequence as if they were indubitably certain. Just as we may syllogize in the same way from true premisses and from false [1], so it is with any other kind of reasoning. Moreover, it was pointed out that many at least of the most general and fundamental of our scientific principles are accepted only because they explain the facts of our experience better than any we can conceive in their stead; they are therefore, or were at the outset, hypotheses, used in explanation of facts, and accepted because of their relative success in explaining them. We do not see why they are true but only why we must believe them to be true. They are established inductively, by the facts which they explain, and the failure of any rival hypothesis; they are not explained from the facts, but the facts from them.

It follows that whatever deductive reasoning enters into an explanation enters also into the inductive proof of an hypothesis which is shown to explain, and to be the only one that will explain [2], the facts. And many explanations are put forward, which do not appeal only to principles already known, but have it as their avowed object to *prove* one or more of the principles which they employ. Explanation then figures as an instrument of induction; and J. S. Mill spoke accordingly of a 'Deductive Method of Induction', and rightly attributed great scientific importance to the process which he called by that name.

No better instance of this operation can be given than the familiar instance of the Newtonian theory of gravitation. Sir Isaac Newton showed that the movements of the heavens could be explained from two principles or laws—the First Law of Motion, and the Law of Universal Gravitation. The former is, that every body preserves its state of rest or uniform rectilinear motion until it is interfered with by some other body; according to the latter, every particle of matter attracts every other particle with a force that varies directly as the mass and inversely as the square of the

[1] On this, cf. *supra*, pp. 331–334.

[2] I add these words, because it is important to realize that an hypothesis is not really proved by merely explaining the facts: cf. *infra*, p. 523. But many hypotheses are provisionally accepted, which are not proved, on the ground that they explain the facts, and without the performance of what would often be the impracticable task of showing that *no* other hypothesis could equally well do so.

distance. The former had already been established by Galileo, and Newton took it for granted; but the latter he proved for the first time by his use of it in explanation.

The theory which bears the name of Ptolemy though much older than he, represented the sun, moon, and stars as moving round the earth; and originally it was supposed that they moved in circles with the earth as centre. While the laws of motion were still undiscovered, no difficulty was found in their circular motion; indeed Aristotle supposed it to be naturally incident to the substance of which the heavenly bodies were composed, that their motion should be circular; for the circle is the perfect figure; movement in a circle is therefore perfect motion; perfect motion belongs naturally to a perfect body; and the substance of which the heavens are composed—the *quinta essentia*, distinct from the four primary substances, earth, air, fire, and water, that are found composing this globe—is perfect.[1] The only difficulty arose when it was found that the orbits of the heavenly bodies, other than the fixed stars, were not perfectly circular; and that was met by the hypothesis of epicycles referred to in an earlier chapter.[2] The substitution of the Copernican for the Ptolemaic hypothesis, though involving a reconstruction of the geometric plan of the heavens, did not necessarily involve any new dynamics[3]; Kepler's discovery that the planetary orbits were elliptical was however a severe blow to the traditional theory of epicycles, which had already by that time become highly complicated, in order to make it square with the observed facts. But when the first law of motion had been grasped, it was evident that a planet, if left to itself, would not continue moving in a circle, and returning on its own track, as Aristotle had thought to be natural to it, and as with more or less approximation it actually does: but would continue moving for ever forward with uniform velocity in a straight line. Circular motion, however uniform, was now seen to involve an uniform change of direction for which a dynamical reason was required. And as the planets

[1] According to Aristotle, every body left to itself had a natural motion, dependent on its own nature: that of the heavens was round a centre, that of earth and water to a centre, that of air and fire from a centre. The centre was the centre of this globe, and so (on his view) of the physical universe. Bodies need not be left to their own motion; a stone, for example, may be thrown towards the sky; but in such case their motion was not natural, but violent.

[2] *Supra*, c. xxi, p. 470.

[3] The heliocentric hypothesis was put forward in the history of Greek astronomy by Aristarchus of Samos.

were constantly changing direction towards the sun, a force exerted from or in the direction of the sun seemed necessary.

Now the greatness of Newton's achievement [1] did not lie in the conception that the orbital motion of the planets was the resultant of two factors, the inertia of their proper motions which, left to itself, would carry them forward with constant velocity in a straight line, and a 'centripetal force' which, left to itself, would carry them to the sun. The resolution of curvilinear into rectilinear motions had been accomplished before him, and the hypothesis of an attractive force had already been hazarded. It had even been suggested by others as well that such a force might vary inversely as the square of the distance; for the area of the spherical surface over which it might be conceived as spreading at any distance from the centre of the sun varies directly as the square of the distance, and its intensity might be supposed to decrease as the area increased. Neither was it Newton who ascertained the facts about the movements of the planets—no small or easy contribution to the solution of the problem. But he did three things. He conceived that the force which deflected the planets into their orbits was the same as that which made bodies fall to the earth: or, to put it differently, he identified celestial attraction with terrestrial gravity, and conceived the earth as continually *falling* out of a straight path towards the sun, and the moon towards the earth; he conceived that this attractive force was exerted between every two particles in the universe; and he invented a mathematical calculus by which he could work out what were the theoretical consequences of the principles which he assumed.

All these steps were of the highest importance. The first provided data to calculate from; the second made it possible to give a precise form to the doctrine of attraction; the third made the calculation possible. The amount of acceleration produced per second in near bodies falling to the earth was already known [2];

[1] It is instructive to note that his law now seems not unconditionally true.

[2] Strictly speaking, that acceleration should not be the same at 1,000 feet from the earth and at 100 feet: and in virtue of atmospheric resistance a cricket-ball should not fall as far in a given time as a cannon-ball; but the theoretical differences would be so small as to escape observation, and therefore the fact that acceleration is empirically found to be 32 feet per second for all bodies in the neighbourhood of the earth creates no difficulty. On the other hand, in the oscillations of a pendulum, which vary in the plains and in the neighbourhood of mountains, we do find evidence agreeable to the theory, of the same kind as those minute differences would afford if we could measure them. The logical bearing of these considerations will be

Newton proved that the resultant of the attractions of all the particles of a sphere was as if its mass were concentrated at the centre; and that enabled him to show that the same law of attraction would give the known acceleration of a falling body near the earth and also that of the moon towards the earth or a planet towards the sun in their fall from the path of the tangent.

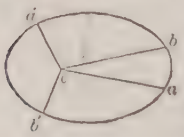
With his proof of this Logic is not concerned. Processes of reasoning are too numerous for Logic to study them all, and those of mathematics are for the mathematician to appraise; it is enough if the logician can satisfy himself in general regarding the grounds of mathematical certainty. But assuming the task of deducing from his principles their theoretical consequences to have been performed, we may look at the logical character of the reasoning in which Newton made use of that deduction.

The principal astronomical facts to be accounted for concerned the movements of the earth and other planets round the sun, and the movements of the moon round the earth.[1] The former body of facts had been already generalized by Kepler, in his three laws, (i) that the planets move in ellipses round the sun, with the sun in one of the foci; (ii) that they describe equal areas in equal times; (iii) that the cubes of their mean distances vary as the squares of their periodic times.[2] There was also a large body of recorded observations upon the movements and perturbations of the moon. But when Newton first worked out his theory he conceived that a sphere attracted bodies near it as if its mass were concen-

seen if it is remembered that a theory, though not proved by its conformity with facts, is disproved by any clearly established unconformity.

[1] Where the planets are mentioned they may be taken to include the moon, unless the context expressly forbids.

[2] Perhaps it should be explained that as a circle is a curve, every point on which is equidistant from a point within it called the centre, so an ellipse is a curve, the sum of the distances of every point on which from two points within it called the foci is constant; that the area described by a planet in moving from a point a to a point b on its orbit is the area comprised between the arc, and the lines joining those points to the centre of the sun: so that if the planet is nearer the sun, it will move faster, since if ac, bc are shorter, ab must be longer, to make the area abc the same; that the mean distance of a planet is its average distance from the sun during its revolution, and its periodic time the period of its revolution, so that if the cubes of the mean distance vary as the squares of the periodic time, it follows that a planet whose mean distance from the sun was twice that of the earth would have a 'year' or period of revolution, whose square was to the square of one (earth's year) as the cube of two to the cube of one—i.e. that its period of revolution would $= \sqrt{8} \times$ the earth's year.



trated near the surface. On this assumption, the force which gave an acceleration of 32 feet per second to falling bodies near the earth would not, if it varied inversely as the square of the distance, account for the period of the revolution of the moon. It was only after several years, when his attention had been recalled to the whole question by Halley, that he demonstrated that a sphere attracted as if its mass were concentrated at the centre[1], and found with this correction in his premisses that the theoretical results of the law of Universal Gravitation agreed with the observed facts. But it was further involved in his demonstration that any other rate of variation in an attractive force operating between all particles of matter would give results conflicting with those facts ; and therefore it had been shown not only that his theory might be true, but that if the planetary motions were to be accounted for by help of a theory of universal gravitation at all, the law of that attraction must be as he formulated it.[2]

The further confirmations which Newton's Law of Universal Gravitation has received, from its success in accounting for other physical phenomena, need not detain us ; we have to look to the steps involved in its establishment, and they can be sufficiently seen in what has been detailed already. First, there was the suggestion that the movements of the planets were to be accounted for by reference to two factors—the inertia of their proper motions, and a force of attraction ; this was not due to Newton. Next, it was necessary to determine or conjecture the way in which these two factors severally operated ; so far as the inertia was concerned, that had also been in part already done, and it was expressed in the first law of motion ; the actual velocity of each planet was ascertained by calculation from astronomical observations, and the velocity proper to each planet considered alone was determined by reference to the actual velocity and the velocity acquired by gravitation. But the velocity acquired by gravitation, or through the influence of the attractive force, had to be conjectured ; and though the law of its variation had been suggested before, unless the amount of its effect between some given masses at some given distance were known, the law of its variation left the matter quite indeterminate.

[1] Cf. Glaisher's Address in commemoration of the bicentenary of the publication of Newton's *Principia*, April 19, 1888, published in the *Cambridge Chronicle and University Journal* of April 20, 1888. I owe the reference to this to the kindness of Professor H. H. Turner.

[2] i.e. if it was to embody a simple ratio : cf. pp. 470–471, 507, *supra*.

The identification of the attractive force with terrestrial gravity and its formulation as a force operating between every two particles of matter, thus completed the necessary data; and principles and facts were now before Newton, sufficient, if a method of calculation were devised, to enable him to determine what should be the consequences of his hypothesis. The next step was the process of calculation. But he had to show, not barely what the consequences of his hypothesis would be, but that they would be the same as the observed facts: and moreover, that his was the only hypothesis whose consequences would be the same as the observed facts.[1] The comparison therefore of the facts with the theoretical results of his and of any other hypothesis was the step that succeeded the calculation; and having found that they agreed with his, and with no other, he reasoned thus—Assuming that the continual deflexion of the planets from a rectilinear path is due to such an attractive force, their actual motions, if my statement of the law of attraction is true, would be thus and thus; if it is false, they would be otherwise: but they are thus and thus, and therefore my statement is true.

Now of the steps in this whole logical process, some are not processes of *reasoning* at all—the suggested reference of the resultant motions to those two factors, the suggested identification of one of the factors with terrestrial gravity, the suggestion that it operates between all particles of matter, and the comparison of the theoretical results with the observed facts. Reasoning may have been employed in establishing the first law of motion; but that reasoning lies outside the present appeal to it. The reasoning involved in determining the theoretical results of the action of the factors assumed is deductive. But the final argument, in which the agreement of the facts with the results of this hypothesis and of no other is shown to require the acceptance of this hypothesis is inductive. Had the Law of Gravitation been already proved, we might have said that Newton was merely explaining certain empirical generalizations about the movements of the planets; had it been already proved and had the attraction of a sphere acted as he at first supposed, the apparent disagreement of its consequences with the records of the moon's and planets' movements would have led him not to lay aside

[1] Cf. previous page, n. 2. It was possible to show that no other rate of attraction would give results conformable to the facts, because the problem was a mathematical one; and in mathematics it is easier than elsewhere to prove not only that if *a* is true, *b* is true, but also the converse.

the theory, but to doubt the observations, or to assume (as Adams and Leverrier afterwards did for the perturbations of Uranus) the existence of some other factor to account for the discrepancy; but inasmuch as it was only now proved by its exclusive success in explaining the facts, he was arguing inductively to the proof of it.

If we look for a moment at the simpler inductive arguments which establish the cause of a phenomenon by appeal to 'grounds of elimination', we shall find in them too something of this double character, at once inductive and deductive. The facts appealed to as showing that a is the cause of x are themselves accounted for by that hypothesis. If, for example, facts do not allow us to doubt that malarial fever is conveyed by the bite of the Anopheles mosquito, then too the power of the Anopheles mosquito to convey malarial fever accounts for its appearing in persons bitten by that insect. It is impossible but that, if certain facts are the *ratio cognoscendi* of a causal principle, that principle should be the *ratio essendi* of the facts. But in these simple arguments there is nothing corresponding to the deductive reasoning which works out the joint consequence, in particular circumstances, of the action of two or more causes, from a knowledge (or conjecture) of the effect which each of these causes would produce singly. It is on account of this operation that J. S. Mill gave to reasoning that involves it, even when its primary object is the inductive establishment of a general principle, the name of the 'deductive method of induction'.

Such reasoning can only be used where the joint effect of several causes is calculable from the laws of their separate effects. Where the joint or complex effect cannot be determined by thinking, from a knowledge of what the separate effects would be, we rely entirely on the inductive method of elimination in order to show that such complex effect is to be attributed to the action of one particular conjunction of causes rather than another. But into the investigation of any complex effect of the other kind, in which the action of the several causes can be traced as combining to produce it, some measure of this deductive reasoning will always enter. Most obviously is this the case in regard to those complex effects which exemplify what has been called a *homogeneous* intermixture [1]—i.e.

[1] J. S. Mill gave the name of 'homogeneous intermixture of effects' to those cases where the joint effect of several causes acting together is of the same kind with their separate effects, and differs only in some mathematical respect from the effects which the same causes would produce singly; this happens, e.g., in the mechanical composition of forces—for which reason

where the complex phenomenon is quantitative, and there are many factors determining its quantity, some by way of increase and some of decrease. The simpler inductive methods are there quite inadequate: for there need be no two instances of the phenomenon in which its quantity is the same, nor, if there were, need the combination of factors be the same; neither can we infer from the non-occurrence of the phenomenon, or its presence only in an imperceptible degree, where the supposed cause is present, that what we had been inclined to ascribe it to does not produce it; since that cause might be present, but counteracted by another of contrary effect. Even the rule that cause and effect must vary concomitantly, and the rule that no such portion of the effect must be attributed to one among the factors making up the cause of the whole, as is already accounted for by other factors, are not sufficient to ensure success in such enquiries. It is necessary to be able to measure more or less precisely the complex effect, and to know with corresponding precision the amount of effect that the several supposed causes would produce alone, in order to prove that any particular one among them cannot be dispensed with, or rejected from being a part cause. And into this proof a deductive calculation will obviously enter. In the fiscal controversy, for example, initiated in Great Britain in 1903, it was alleged that the excess in the value of our imports over that of our exports was due to the crippling of our production by free trade; but this could only be proved by showing that the difference of value between exports and imports was unaccounted for, unless we were living on our capital; and that could not be shown unless the excess in value of imports were ascertained, which was attributable to other causes known to assist in producing their total excess-value—such as the fact that the valuation of our imports was swollen by the inclusion of the cost of carriage to our own ports (while our exports, being valued before transport, did not receive this

he spoke also of Composition of Causes in such a case. Where the joint effect differs in quality from the separate effects (and so cannot be calculated from a knowledge of them) he called it heterogeneous or heteropathic. He illustrated this from chemical combination, in which the chemical properties of the compound (unlike its weight) are not homogeneous with those of its constituents; though he quite overlooked the fact that elements were not the 'cause' of a compound in his usual sense of that term. But though homogeneous intermixture of effects allows of deductive reasoning, such reasoning may also occur where the complex effect is not the sum or difference or mathematical resultant of the separate effects. And it is the deducibility of it from a knowledge of the several principles involved that differentiates this kind of 'intermixture of effects'. Cf. *System of Logic*, III. vi.

addition): and by the value of the goods that paid for ships sold abroad or for the service which the country performs as ocean-carrier, although nothing appears in the total for exports on those heads: and by the value of the goods that represent payment for the use of British capital invested abroad, or pensions charged on the Government of India. The difficulty of determining the amount by which these causes should make our imports exceed our exports in value rendered it exceedingly hard to prove, at least on this line of argument, that we could not be paying out of the year's production for all that we imported in the year.

To sum up—Explanation considered in itself is deductive: it consists in showing that particular known facts, or laws, or general causal connexions, follow from principles already established, in the circumstances of the case; it discovers therefore nothing new, except as it makes us understand the reason for that which we had hitherto only known as a fact. But explanation also enters into induction, so far as the principles, from which the facts, or laws, or general causal connexions, are shown to follow, were not previously established, but are so only now by showing that the actual facts, laws, or causal connexions would follow from them and not from any alternative principles. In such induction there are four main steps distinguishable: (i) conceiving the several agents, or causes, at work; (ii) determining or conjecturing how or according to what law each of them severally would act; (iii) reasoning from these premisses to the result which they should produce in common, as well as to the result which would follow on any rival hypothesis as to the agents at work, and the several laws of their operation; (iv) showing that the facts are what should follow from these, and not from any rival premisses.[1]

Many observations might still be made upon this type of argument —one of the commonest and most important in the sciences. Its applications are very various. It may be directed to establish that a known agent is concerned in the production of a familiar effect with which it has not hitherto been suspected to be concerned: as Darwin showed that earthworms play a part in the subsidence of buildings below the surface level. Or again, it may be used in supporting a theory as to the law or principle displayed in a set of variable facts: the Mendelian theory, that there are definite alternative factors, dominant and recessive, determining, according as only

[1] This is not always a separate step: *v.* p. 523, n. 1.

one kind or both are present in the fertilized ovum, various peculiarities in individual animals and plants, involves elaborate deduction of the proportions in which such peculiarities should be found over a large number of specimens, and of the possibility of establishing varieties that breed true in this or that respect, and it is recommended by its success in accounting for observed facts of this kind. Or it may be used to show the existence of an agent, whose mode of action, if it exists, is known, as Adams and Leverrier argued that there must be a planet hitherto unobserved to account completely for the perturbations of the known planet Uranus.[1] The more we can introduce number and quantity into our statement of the principles that are to account for facts, and can determine numerically and quantitatively the facts themselves, the more this type of argument is available. But we are using it, whenever an explanation of facts is offered, among the premisses of which is one whose truth is in question, and is inferred from the success with which by its help the facts are shown to be explicable. The question may be, what causes *can* produce such an effect, or which of the causes that can produce it *are* contributing to produce it now? We may wish to establish a general principle, or only some special fact as to the circumstances that are modifying the results of that principle in the case before us. It is possible too that the laws of the action of the several agents may some of them have been previously ascertained and established, while others are only conjecturally formulated; or, if the question be as to the agents contributing to the result in a particular case or class of cases, the laws of the several actions of them all may have been established previously. But without dwelling on these points, we may conclude the chapter with four considerations.

First, the inductive arguments of science display in every different degree that combination with deductive reasoning which has been now analysed. Thus, though we may represent in symbols the induction whose logical form is a mere disjunctive argument, and contrast it with this into which the deduction of a complex result from several premisses so prominently enters, yet in actual practice the contrast is not so sharp; in few inductive investigations is the reasoning merely disjunctive; but the amount of deductive reasoning that has to be performed before one is in a position to

[1] This celebrated argument is often also used to illustrate Mill's 'Method of Residues', as it very well does. For that Method is, as Mill himself recognized, partly deductive in character. Cf. *infra*, p. 560.

apply a disjunction, and to say that this hypothesis is true because the rest can be proved false, varies very greatly in different investigations.

Secondly, to show that the facts agree with the consequences of our hypothesis is not to prove it true. To show that is often called *verification*[1]; and to mistake verification for proof is to commit the fallacy of the consequent[2], the fallacy of thinking that, because, if the hypothesis were true, certain facts would follow, therefore, since those facts are found, the hypothesis is true. It is the same mistake as that of incomplete elimination, in the establishment of a simple causal relation: the same as results from overlooking what is called the Plurality of Causes. A theory whose consequences conflict with the facts cannot be true; but so long as there may be more theories than one giving the same consequences, the agreement of the facts with one of them furnishes no ground for choosing between it and the others.[3] Nevertheless in practice we often have to be content with verification; or to take our inability to find any other equally satisfactory theory as equivalent to there being none other. In such matters we must consider what is called the weight of the evidence for a theory that is not rigorously proved. But no one has shown how weight of evidence can be mechanically

[1] Mill supposes verification, i.e. showing that the facts agree with the consequences deduced from an hypothesis, to be always a separate stage in the whole process. But, as Professor Cook Wilson has pointed out, this is not so, if the only facts appealed to are those which the hypothesis was framed to explain; for then its consequences are not deduced first, and the facts ascertained and compared therewith after. Mill was thinking of what is very common in this sort of inquiry, viz. that we endeavour to verify our theory by considering what should happen, if it were true, in circumstances which have not been examined, nor perhaps hitherto existed, and then observing what happens in these circumstances, instituting them if necessary. Such procedure often involves very delicate and elaborate experiment, as well as very intricate calculation, especially in physics. If the result which our theory led us to anticipate actually occurs, the theory is said to have shown a power of successful prediction; and men are often more influenced in favour of a theory by its power of successful prediction, than by its explaining facts already known. This however is unreasonable. Just as an erroneous theory may successfully explain known facts, i.e. the facts may be such as would exist if the theory were true, and yet it may be in some respect false, so it may successfully predict unknown facts. But what really gives to a theory a greater title to our belief is greater comprehensiveness, i.e. power to explain a wider and more varied range of facts; for the bigger the system, the harder it is to find several principles that equally satisfy the facts of it. And when men seek to verify a theory by way of predicting what will occur in fresh circumstances, they commonly take circumstances unlike those of what it was first formed to explain.

[2] Cf. p. 596, *infra*.

[3] Cf. p. 423, *supra*.

estimated; the wisest men, and best acquainted with the matter in hand, are oftenest right.

Thirdly, there is no logical difference between the reasoning contained in explanation, and the inductive reasoning that involves explanation, except in one point: that the latter infers the truth of some premiss assumed in the explanation from its success in explaining the actual facts and the impossibility of explaining them without assuming it. Where this impossibility is not shown, and we content ourselves with verification—that is, with showing that the facts consist with the assumption—there the logical difference is still slighter; it amounts to this, that in explanation the premisses are taken as previously known, and in the other case something in the premisses, not taken as previously known, is accepted on the strength of its use in the explanation.[1]

Fourthly, we may answer here the second of the two questions raised at the end of c. xvii. Demonstration is explanation from principles that are self-evident, or necessarily true. If it be said that in that case very little of what we believe is demonstrated, we must admit it. We can demonstrate little outside mathematics. But we have an ideal of demonstration, and it seems to be that; and it is not syllogistic, as Aristotle thought it to be.[2]

[Dr. Bosanquet, in a paper already referred to, 'On a Defect in the customary logical Formulation of Inductive Reasoning' (*Proceedings of the London Aristotelian Society*, N. S. vol. xi, 1910–11, p. 29), has expressed the opinion that 'the restriction of Inductive proof to

[1] J. S. Mill, to whose work the above chapter is not a little indebted (*v. Logic*, III. x–xiii), fails to mark sufficiently the difference between showing that the facts agree with a theory, and showing that the theory is true. And he does not bring out clearly enough the relation between what he calls the Deductive Method of Induction (c. xi) and what he calls the Explanation of Laws of Nature (c. xii). He neither notices how they differ, nor how closely they agree, though he gives the same investigation (the Newtonian theory of gravitation) as an example of both of them (xi. 2, xiii. 1). Moreover, in resolving into three steps his 'Deductive Method of Induction', he leaves out the first of the four mentioned on p. 521, but cf. xi. 1 and vii.

[2] For syllogism, as has been argued above, pp. 308–311, implies the application, to a particular case, of a general principle known independently; with complete insight, the necessity which connects the different elements in a complex fact should be manifest *in the case before us*, and the general principle or major premiss not brought in *ab extra*, but rather visible in and extricable from that case (cf. p. 311, *supra*). This much however Aristotle would probably have admitted, and would have called it syllogism, to show what character it was, in a subject of a given kind *S*, that involved its having the predicate *P*; but most demonstration cannot even so be put into the form of syllogism, connecting one term with another through a third by the relation of subject and attribute.

[the disqualification of competing hypotheses is a fundamental error of principle', and adds that he here finds himself in opposition to the doctrine of this book. With a great deal of his paper I cordially agree. It is an attack on the sufficiency of the principle 'Same cause, same effect' in inductive enquiry. Though provoked especially by the writings of M. Bergson, from whose *L'Évolution créatrice* he quotes a 'typical passage'[1], it seems to me to hold good equally against Mill's presentation of induction in his exposition of his 'Inductive Methods'. Induction is not commonly nor for long so simple a business as the pairing off of a determinate cause a with a determinate effect x, each repeated unchangingly amid varying circumstances from among which it has to pick them out. Our 'causes' are commonly variables, with whose variations is connected a corresponding variation in the effect, and we seek a principle from which we may determine what variation in the one is connected with what variation in the other. And they are commonly co-operant, so that we need also to trace their consequences through divers combinations into very divers complex effects. Hence, as Dr. Bosanquet says, the intelligence will 'bind different to different in binding same to same'; and the universality and generality at which it aims 'is not measured by millions of repeated instances, but by depth and complexity of insight into a subsystem of the world' (*loc. cit.* p. 34). 'The value of an Inductive conclusion, as of any piece of knowledge, lies in the amount of reality which it enables us to grasp, and this is very slightly tested by the number of cases in which the nexus is repeated in fact' (ib. p. 39). This seems to me very true, and I think the foregoing chapters are in accordance with it.[2] I also agree that what Dr. Bosanquet calls 'this work of the universal' is 'the true spirit and mainspring of the inductive advance of knowledge' (ib. p. 34). I have indeed said something of the kind above, pp. 469–470. That the work of the mind in framing theories cannot be reduced to rule I have pointed out in the same chapter, when discussing the formation of hypotheses. That indeed is its most originative work. It may be compared with the activity of artistic creation. There the mind, ruminating as it were upon a hint of the beauty which it seeks to articulate, somehow advances to a fuller apprehension of it; and in scientific activity, ruminating upon certain facts, it advances to the thought of a system in which they might be connected. This is the genuine work of intelligence[3], but our minds are not fully intelligent. Much suggests itself to the artist which is not suited to his theme; sometimes he

[1] p. 218: 'L'intelligence a pour fonction essentielle de lier le même au même, et il n'y a entièrement adaptable aux cadres de l'intelligence que les faits qui se répètent.'

[2] Cf. e. g. pp. 402, 408–409, 458, n. 1, 483. Cf. also Mill, *op. cit.*, III. xi. 3.

[3] Cf. the paper on 'Mechanism, Intelligence and Life', *Hibbert Journal*, xii. 3, April 1914, pp. 626–629.

[rejects it, sometimes he retains it, and others may recognize it as a defect in the work of his art. There is no general criterion here, any more than there is a general criterion of scientific truth, as Dr. Bosanquet says (ib. p. 38). A theory, as I have urged, may make coherent a wide range of fact, and yet not be true. But I think that Dr. Bosanquet underrates at this point the part played by the 'eliminative test'. It is the function of the intelligence to trace the connexion between one feature and another in the real, and this it does even if the hypothesis that these features are exemplified in this or that existent situation be false.[1] But where, as in the inductive sciences, our hypotheses include so much that, even if true, seems to us mere brute fact, a connexion of features in the real that is not self-evident, nor what we have any prospect of coming to find so, there we must, as it seems to me, rely for our acceptance of such alleged connexions upon the elimination or the lack of alternative theories. Dr. Bosanquet says that 'the only criterion of truth is the fuller truth—the science at a more developed stage' (ib. p. 38). If by a more developed stage of a science he means one at which we are aware of fresh facts to be connected in a system with those previously known, then with our earlier theory of the principles displayed in them these fresh facts will either be consistent or not. Supposing that they are not (as the varying distances of the planets from the earth were not consistent with the theory that they moved round it in concentric spheres), they lead us to abandon the theory; and we are applying here the eliminative test. Supposing that they are (as fresh facts revealed about the apparent movements of the planets by more accurate observation were consistent with the theory of epicycles), they do not prove the theory true. By the more developed stage of a science may be meant a stage at which we have not only ascertained fresh facts, but thought out principles of connexion that will account for them, whereas those accepted at a less developed stage will not. But again it seems to me that we now reject those by the eliminative test, because if they were true, the newly ascertained facts would not be as we find them; and we accept the principles that have displaced those, unless indeed they be self-evident, because they alone account for all the facts now known to us. Should we discover further facts inconsistent with them, we should give them up also; and therefore they remain subject to the eliminative test. The mere elaboration of a theory to keep pace with the accumulation of fresh facts, which might be called a development of the theory, is no criterion of its truth. It need not be true because it admits of such elaboration, and the modifications introduced may only make it more erroneous. If I understand him rightly, Dr. Bosanquet holds that as our knowledge of the facts belonging to some 'sub-system of the world' increases, and our theories of their systematic

[1] Cf. *supra*, pp. 333–334.

[connexion are modified and elaborated accordingly, we learn that we are getting nearer to the truth as to their connexion merely by comparison of our theories in respect of systematic comprehensiveness. I agree that the more systematically comprehensive is preferred to the less, because it accounts for facts which lead us to reject the other as not accounting for them. But nothing in its principles or starting-points which is not self-evident seems to me in the last resort to have any other warrant than that it alone enables us to find systematic connexion in the facts. This seems to me to be the only *inductive proof*, and if a competing hypothesis enabled us equally well to find systematic connexion in the same set of facts, I do not see how we should decide between them, until we discovered a 'crucial instance'[1], a fact which overthrew one of them, because that could not find a place for it. And even so, the other could not be *proved*, unless we could show that all possible competing hypotheses had been overthrown. This view of the nature of inductive proof does not of course involve that such eliminative argument is the most important part of inductive enquiry, or even the most important argument in it. There is much work, both of thought and otherwise, besides argument in inductive enquiry. I have enlarged on this in Chapter XXI. The activity of the intelligence which results in the formation of fruitful hypotheses, on whose importance Dr. Bosanquet so well insists, is not a process of argument. Nor can rules be given for it; Bacon promised to show how his method of 'Exclusions' could be applied to the formation of *bonae ac verae notiones*, as well as to the rejection of *notiones* that were not *bonae ac verae*, but he never showed it, because it cannot be done. And of argument the deductive processes spoken of in the present chapter are more difficult, and often bulk much more largely in a scientific investigation, than the mere eliminative argument, which is characteristically inductive because involved in every attempt to establish a principle of connexion, neither self-evident nor explicable from other principles, by appeal to facts that are to show its truth. Such a principle need not connect 'the same with the same', with no provision for variation and diversity. To trace its exemplification may be, as Dr. Bosanquet says, 'like the continuation of a varying curve from the datum of a given fragment of it' (ib. p. 35).[2] But suppose we were given a series of points lying on a curve, and required to find others through which it would pass at certain distances when continued; a number of curves might satisfy the *data*, but give different positions for the *quaesita*; how except by fresh *data* should we be able to decide between our alternatives, and then how else than by rejecting those with which these fresh *data* were inconsistent? That, in the last resort, seems to me to typify our case in regard to those scientific generalizations which rest merely on inductive proof.]

[1] Cf. *infra*, p. 565, n. 1. [2] Cf. H. Poincaré, quoted *supra*, p. 411, n. 2.

CHAPTER XXIV

OF INDUCTION BY SIMPLE ENUMERATION AND THE ARGUMENT FROM ANALOGY

THERE are many reasonings which do not prove their conclusion. It is not merely that we have to use dubitable premisses; for this, though it destroys the strictly demonstrative character of our knowledge, does not invalidate the reasoning, so long as the conclusions are what *must* be drawn, if the premisses are true. It is that we often draw, and act upon, conclusions, about which we cannot say even this much, that they must be true if the premisses are. And in so doing, we often find ourselves right; nor, if we refused to do it, could the affairs of life be carried on. Descartes, when he set himself to examine all which he had hitherto believed, and to doubt everything which could be doubted, determined with himself that he would not let this demand for demonstration in things of the intellect prevent his following the most probable opinion in practical matters.[1] But it is not only in these that we have to hazard an assent to conclusions which our premisses do not strictly justify. Many branches of science would not progress at all, unless we did the same there. In the first place, by committing ourselves to a conclusion, and working upon the assumption that it is true, we may be led to results that will help either to confirm or to overthrow it; whereas if we had merely withheld our assent from any conclusion, because the evidence was inconclusive, we might have remained indefinitely long possessed only of that inconclusive evidence. 'Truth', said Bacon, 'is more readily elicited from error than from confusion'[2]; and perhaps we might add, than from indecision. Only we must in such cases let our assent be provisional, and hold our opinion not as demonstrated, but as in default of a better. The advice of the politician, that a man should make war with another as with one to whom he may be reconciled, and peace as with one with whom he may become at variance, may without

[1] *Discours de la Méthode*, Troisième Partie.
[2] *Nov. Org.* II. 20.

suspicion of cynicism be adapted to the assent or dissent with which we receive conclusions that are based on insufficient evidence. But secondly, the sciences differ very much in the amount of evidence which they can hope to obtain for their conclusions. A fairly rigorous science may be content to use provisionally principles which are known to be insufficiently proved (and that means really, not *proved* at all); but some sciences hardly ever obtain rigorous proof of their positions, as for example Anthropology; and yet much at any rate of their teaching is generally accepted as authoritative. Aristotle said that it was 'the business of education to teach a man to demand rigorous proof of anything according to the nature of the subject; for it is as foolish to ask demonstration of the orator, as to accept plausibilities from the mathematician'[1]; and he would have allowed that for this purpose education must include both a training in 'Analytics'[2] and an acquaintance with the kinds of subject-matter to which these different attitudes are appropriate. It is often said that a man whose studies are too exclusively mathematical is at sea when he comes to deal with matters that do not admit of demonstration; and that contrariwise, if he is trained only in sciences where rigorous proof is impossible, he becomes incompetent to see what is required in matters of a stricter sort.

There are no logical criteria by which to judge the value of such reasonings, unless what is called the Theory of Probability may claim to be such a criterion. But the Theory of Probability is primarily a branch of mathematics; many of the assumptions which underlie its applications are open to suspicion on logical grounds; and its use is at any rate confined to subjects that admit of numerical treatment. The object of the present chapter however is to consider briefly two kinds of argument, which while being of this inconclusive character are very common, and have attracted considerable attention from logical writers accordingly.

Induction by Simple Enumeration consists in arguing that what is true of several instances of a kind is true universally in that kind. Simple enumeration means *mere* enumeration; and

[1] *Eth. Nic.* a. i. 1094[b] 23 *πεπαιδευμένου γάρ ἐστιν ἐπὶ τοσοῦτον τἀκριβὲς ἐπιζητεῖν καθ' ἕκαστον γένος, ἐφ' ὅσον ἡ τοῦ πράγματος φύσις ἐπιδέχεται· παρα-πλήσιον γὰρ φαίνεται μαθηματικοῦ τε πιθανολογοῦντος ἀποδέχεσθαι καὶ ῥητορικὸν ἀποδείξεις ἀπαιτεῖν.*

[2] Aristotle called by this name his treatises on syllogism and demonstration, presumably because in them he sought to analyse the argumentation of ordinary debate and of scientific proof, and so to show what conditions must be fulfilled in order to justify or compel assent to conclusions in either field.

such an argument differs from scientific induction in the absence of any attempt to show that the conclusion drawn is the only conclusion which the facts in the premisses allow, while it differs from induction by *complete* enumeration in that the conclusion is general, and refers to more than the instances in the premisses. It should however be noted here, that induction by complete enumeration, if the conclusion be understood as a genuinely universal judgement, and not as an enumerative judgement about all of a limited number of things, has the character of induction by simple enumeration. The name of *empirical generalization* is also given to such arguments by simple enumeration.

Bacon's strictures upon this form of reasoning have been already referred to.[1] Regard it as a form of proof, and they are not undeserved. Yet it is still in frequent use, in default of anything better. It has been inferred that all specific characters in plants and animals are useful, or adaptive, because so many have been found to be so. So many 'good species' have become 'bad species' (i. e. species incapable of any strict delimitation) in the light of an increased knowledge of intermediate forms, that it has been inferred that all species, if we knew their whole history, would be found 'bad'.[2] The familiar generalization that we are all mortal, though not based solely on enumeration, draws some of its force thence. Most men's views of Germans, or Frenchmen, or foreigners generally, rest upon their observation of a few individuals. The 'four general rules of geography', that all rivers are in Thessaly, all mountains in Thrace, all cities in Asia Minor, and all islands in the Aegaean Sea, are a caricature of this procedure, drawn from the experience of the schoolboy beginning Greek History. The history of the theory of prime numbers furnishes one or two good examples. More than one formula has been found always to give prime numbers up to high values, and was assumed to do so universally: $x^2 + x + 41$ worked for every value of x till $40 : 2^{2^7} + 1$ worked for long, but it broke down ultimately.[3] It is needless to multiply illustrations.

What is the assumption which underlies arguments of this kind? It is the old assumption that there are universal connexions in nature; and the conjunction of attributes which our instances present is taken as evidence of a connexion. The arguments are

[1] *Nov. Org.* I. 105. Cf. *supra*, pp. 380, 391–392.
[2] Romanes, *Darwin and after Darwin*, ii. 282.
[3] *v.* Jevons, *Elementary Lessons in Logic*, pp. 221–222.

weak, because the evidence for the connexion is insufficient. If *a b c d*, instances of *x*, present the property *y*, it does not follow that *y* is connected with those features on account of which they are classed together as *x*. Yet a large number of instances furnishes some presumption. For some reason must exist, why all these instances exhibit the same property. If it is not in virtue of their common character *x*, it must be in virtue of some other common feature. When the variety of circumstances is great, under which the instances are found, and the differences many which they present along with their identity as *x*, it is harder to find any other common features than what are included in classing them as *x*. Therefore our confidence in the generalization increases, although it may still be misplaced. All men are mortal; for if men need not die except through the accident of circumstances that are not involved in being man, is it not strange that no man has avoided falling in with these circumstances? There is force in the question. The number and variety of our observations on the point are such, that almost everything can be eliminated: almost everything that has befallen a man, except what is involved in being man, has also not befallen other men: who therefore ought not to have died, if it were because of it that men die. Something involved in being man must therefore surely be the cause of dying.

Induction by Simple Enumeration rests then on an implied elimination; but the elimination is half-unconscious, and mostly incomplete; and therefore the conclusion is of very problematic value. But where the instances do serve to eliminate a great deal, the openings for error are correspondingly reduced in number, and the conclusion is received with greater confidence accordingly. General considerations of this kind, however, will not stand against definite opposing facts; therefore such an empirical generalization is at once overthrown by a contradictory instance.[1] Neither will they overbear more special considerations drawn from acquaintance with the subject-matter to which the induction belongs. Pigmentation is known to be a highly variable property in many species; therefore the overwhelming range of instances to show that all crows are black was felt to be insufficient to give the conclusion any high degree of value. Again, a difficulty in conceiving how two properties could be causally connected will incline us to attach less weight to the fact of their conjunction.

[1] Ἔνστασις, *instantia*, meant originally a *contradictory* instance.

And contrariwise, where the connexion to which the conjunction points is one which seems conformable with other parts of our knowledge, we are much more ready to generalize from the conjunction. Many general statements are made about the correlation of attributes in plants and animals, which rest on simple enumeration; but the theory of descent suggests an explanation of the constancy of such a conjunction; for what was correlated in a common ancestor might well be correlated universally in the descendants. We are therefore readier to suppose that attributes found several times accompanying one another in a species (such as deafness with white fur and blue eyes in tom-cats, or black colour with immunity to the evil effects of eating the paint-root in pigs [1]) are correlated universally, even though we can see no direct connexion between them, than we should be if no way of explaining the constancy of the conjunction presented itself to us.

The **Argument from Analogy** (at least in the usual sense of the term) is of the same inconclusive character as Induction by Simple Enumeration; and like it, rests on the general belief in universal connexions, and takes a conjunction of attributes as evidence of their connexion.

Analogy meant originally identity of relation. Four terms, when the first stands to the second as the third stands to the fourth, were said to be analogous, or to exhibit an analogy. If the relation is really the same in either case, then what follows from the relation in one case follows from it in the other; provided that it really follows from the relation and from nothing else. Where the terms are quantities, or are considered purely on their quantitative side, and the relations between them are also quantitative, there the reasoning is of course mathematical in character: analogy in mathematics being more commonly called proportion. And such reasoning is necessary, like any other mathematical reasoning. If in respect of weight $a : b :: c : d$, and if a weighs twice as much as b, then c must weigh twice as much as d. So soon however as we connect with the relation $c : d$, on the ground of its identity with the relation $a : b$, a consequence which is not known to depend entirely on that relation, our reasoning ceases to be demonstrative. Suppose that by rail the distance from London to Bristol bears the same relation to the distance from London to Plymouth as the distance from London to Darlington bears to the distance from London to

[1] *v.* Darwin, *Origin of Species*, c. i, 6th ed. p. 9.

Aberdeen: and that it costs half as much again to send a ton of timber to Plymouth from London as to Bristol; we cannot infer that the rate from London to Aberdeen will be half as much again as it is to Darlington; for the rate need not depend entirely on the relative distance, which is all that is alleged to be the same in the two cases.

There are many relations however between terms which are not relations of quantity. Here too, four terms may stand in an analogy: and what follows from the relation of the first to the second may be inferred to follow from the relation of the third to the fourth. It might be said that the relation of his patients to a doctor is the same as that of his customers to a tradesman, and that therefore as a customer is at liberty to deal at once with rival tradesmen, so a man may put himself at once in the hands of several doctors. And if the relations were the same, the argument would be valid, and indeed in principle syllogistic; for the common relation would be a middle term connecting a certain attribute with a man's position towards his doctor. 'Those who employ the services of others for pay are at liberty to employ as many in one service as they pay for': such might be the general principle elicited from our practice in shopping, and proposed for application to our practice in the care of our health. The case of patient and doctor is 'subsumed' under the principle supposed to be exhibited in the case of customer and tradesman. Even however if it were not possible to disentangle a general principle, and reason syllogistically from it, we might use the analogy; thinking that there was an identity of relations, and that what was involved in the relation in the one case must be involved in it in the other.

Unfortunately however the identity of the relations may be doubted. Relations are not independent of their terms. Quantitative relations are no doubt independent of everything except the quantitative aspect of their terms, and are on that account usually stated as between quantities in the abstract. But with other relations it may be very difficult to abstract, from the concrete nature of the terms between which they hold, the precise features which involve the relation. Hence we may say that two relations are similar, and yet doubt whether they are similar in the way that would justify the inference. They may be partially the same, but the difference may just invalidate the consequence[1]; and reasoning by analogy cannot then possess the character of necessity.

[1] Cf. *infra*, pp. 589–590.

David Hume held that virtue and vice are not attributes of any act or agent, but only feelings which an act may arouse in a spectator; so that if nobody approved or disapproved my actions, they could not be called either virtuous or vicious. And one of the arguments by which he endeavoured to sustain this opinion was as follows. A parricide, he said, is in the same relation to his father as is to the parent tree a young oak, which, springing from an acorn dropped by the parent, grows up and overturns it; we may search as we like, but we shall find no vice in this event; therefore there can be none in the other, where the relations involved are just the same; so that it is not until we look beyond the event to the feelings with which other persons regard it, that we can find the ground for calling it vicious.[1] Doubtless there is an analogy here; but the relations are not altogether the same; for the relation of a parent to a child is spiritual as well as physical, and in the parricide there is an attitude of the will and the affections which cannot be ascribed to the oak.

Many arguments from Analogy, in the sense of this loose identity of relations, have become famous; and they are a favourite portion of the orator's resources. How often have not the duties of a colony to the mother-country been deduced from those which a child owes to a parent; the very name of mother-country embodies the analogy. Yet it is by no means easy to find the terms which stand in the same relation. The soil of Britain did not bear the soil of Australia; and the present population of Australia are not the descendants of the present population of Britain, but of their ancestors. To whom then does the Commonwealth owe this filial regard, and why? Doubtless the sentiment has value, and therefore some justification; but this argument from analogy will not quite give account of it. Alexis de Tocqueville again said of colonies, that they were like fruit which drops off from the tree when it is ripe. Here is another analogy, and two of the terms are the same as in the last. The relation of a colony to the mother-country suggests different comparisons to different minds, and very different consequences: which cannot all of them follow from it. We may take another instance, where the relations are really closer, and the argument therefore of more value. To grant that Natural Selection *may* be able to do all that is claimed for it, and yet object to it on

[1] *Treatise of Human Nature: Of Morals*, Part I. § 1, Green and Grose's ed. vol. ii. p. 243.

the ground that the facts which are accounted for by it may *equally well* be ascribed to intelligent design, is, it has been urged, as if a man were to admit that the Newtonian theory of the solar system works, and yet were to continue to suppose with Kepler that each planet is guided on its way by a presiding angel; if the latter therefore be irrational, so must the former be.[1] Or consider the following passage[2]:—'It has been objected to hedonistic systems that pleasure is a mere abstraction, that no one could experience pleasure as such, but only this or that species of pleasure, and that therefore pleasure is an impossible criterion' [viz. of good: i. e. it is impossible to judge what is good by the amount of pleasure which it affords]. 'It is true that we experience only particular pleasurable states which are partially heterogeneous with one another. But this is no reason why we should be unable to classify them by the amount of a particular abstract element which is in all of them. No ship contains abstract wealth as a cargo. Some have tea, some have butter, some have machinery. But we are quite justified in arranging those ships, should we find it convenient, in an order determined by the extent to which their concrete cargoes possess the abstract attribute of being exchangeable for a number of sovereigns.' The force of this argument will depend on whether the particular concrete pleasurable states do stand to their common character as pleasures in the same relation as the concrete cargoes of ships stand to the abstract attribute of wealth or exchangeability for sovereigns. Doubtless the relations are partly the same, for each, pleasure and wealth, characterizes its concrete subjects. But the concrete subjects, tea, butter, machinery, are *measurable* in terms of wealth, by the fact of being exchangeable for a definite number of sovereigns; and the question is whether there is anything that renders the others similarly measurable in terms of pleasure. On the value of this argument doctors will probably disagree: and this again shows how arguments from analogy are inconclusive.

There is however another sense in which the terms *analogy* and *argument from analogy* are used. The analogy may be any resemblance between two things, and not merely a resemblance of the relations in which they respectively stand to two other things; and the argument from analogy an argument from some degree of resemblance to a further resemblance, not an argument from the

[1] Romanes, *Darwin and after Darwin*, i. 279.
[2] McTaggart, *Studies in Hegelian Cosmology*, § 113. Cf. *supra*, p. 193, n. 3.

consequences of a relation in one case to its consequences in another. Expressed symbolically the argument hitherto was of the following type: *a* is related to *b* as *c* is to *d*; from the relation of *a* to *b* such and such a consequence follows, therefore it follows also from the relation of *c* to *d*. The present argument will run thus: *a* resembles *b* in certain respects *x*; *a* exhibits the character *y*, therefore *b* will exhibit the character *y* also. Argument of this type is exceedingly common.[1] 'Just as the flint and bone weapons of rude races resemble each other much more than they resemble the metal weapons and the artillery of advanced peoples, so,' says Andrew Lang, 'the mental products, the fairy tales, and myths of rude races have everywhere a strong family resemblance.'[2] The fact that mental products, which resemble certain material products in being the work of rude races, resemble them in the further point of exhibiting the strong family likeness that is known to characterize the latter, is here perhaps suggested to be something more than a coincidence. Or take this argument from Sir Henry Maine. He is discussing the various devices by which in different systems of law the lack of a son to perform for a man the funeral rites can be supplied. We are familiar with adoption. But adoption in England does not carry the legal consequences of legitimate sonship. The Hindu codes recognize adoption and various expedients besides; and the son so obtained has the full status of a real son, can perform satisfactorily the important ceremonies of the funeral rites, and succeed to property as the real son would succeed. One of their expedients is known as the Niyoga, a custom of which the Levirate marriage of the Jews is a particular case. The widow, or even the wife, of a childless man might bear a son to him by some other man of the family, and the son became his son, and not the natural father's. How did Hindu thought rest content in so fictitious a relation? 'All ancient opinion,' says Maine[3], 'religious or legal, is strongly influenced by analogies, and the child born through the Niyoga is very like a real son. Like a real son, he is born of the wife or the widow; and though he has not in him the blood of the husband, he has in him the blood of the husband's race. The blood of the individual cannot be continued, but the blood of the household flows on. It seems to me very natural for an ancient authority on customary law to hold

[1] It was called by Aristotle *παράδειγμα*: cf. *Anal. Pri.* *β*. xxiv, *Rhet. a.* ii. 1357[b] 25–36, and pp. 540–541, *infra*.

[2] *Custom and Myth*, p. 125, ed. 1901 ('The Silver Library').

[3] *Early Law and Custom*, p. 107.

that under such circumstances the family was properly continued, and for a priest or sacerdotal lawyer to suppose that the funeral rites would be performed by the son of the widow or of the wife with a reasonable prospect of ensuring their object.' We may find in the exacter sciences this sort of argument from analogy employed. Before it was known that light travelled in waves, it was known that sound did so. Light and sound were both capable of being reflected, and the direction of their reflection obeyed the same law, that the angle of reflection is equal to the angle of incidence. From these facts it was inferred by analogy that light, like sound, travelled in waves': as it was afterwards shown to do. Among the properties of gold was long enumerated fixity, i.e. that it was incapable of volatilization. As one element after another was successfully volatilized, it might have been inferred by analogy that gold could be volatilized too.

We may now compare this with the former type of argument from analogy; and afterwards consider their logical value, and their relation to induction by simple enumeration.

Since analogy properly involves four terms, the latter and looser but commoner sense of the expression *argument from analogy* seems at first sight difficult to account for. Why should a resemblance which is not a resemblance of relations be called an analogy at all? Perhaps the answer is that where the relation is no longer a quantitative one, it is apt to be regarded as a property of the subject that stands in the relation. The quantitative relation of one thing to another does not affect the intrinsic character of the thing; but other relations do. We should not regard it as constituting a resemblance between a child and a young elephant that one weighed half a hundredweight, and the other half a ton; but that they both had mothers (though that is also a resemblance of relations) would seem to constitute a resemblance. Such a relation rests on and involves important characters in the thing related of a less purely relational character than quantitative predicates are. And in this way the term *analogy* may well have come to be extended to resemblances generally, even where the resemblance is not a resemblance of relations.[1]

[1] I give in a note another possible explanation of the change that has taken place in the logical use of the term analogy, but one that seems to me less likely than the foregoing. The 'rule of three' is in a sense an argument from analogy. Starting with the conception of an analogy, in the strict sense, it supplies from three given terms the fourth term which will complete

Even in the stricter sense then, the argument from analogy does not commonly mean the mathematical argument from an identity of ratio: the relations are only similar, and must be conceived to involve intrinsic attributes of the things related.[1] In considering the value of the argument therefore we may for the future ignore the distinction pointed out between the two types of inference to which the name is given, and may take the second (to which the first tends to approximate) as fundamental. The argument from analogy is an argument from a certain ascertained resemblance between one thing and another (or others) to a further resemblance; because a and b are x, and a is y, $\therefore$ b is y. What is the logical value of this argument?

It is plainly not proof. As Lotze has pointed out[2], there is no proof by analogy. Many conclusions drawn in this way are afterwards verified; many are found to be false. Arguments from analogy can often be found pointing to opposite conclusions.

The *Parmenides* of Plato, a dialogue of his later period, discusses various difficulties with regard to the relation between the universal and the particular, which many scholars consider to be criticisms upon his own 'doctrine of ideas' as presented in his earlier writings. One of these is identical with an objection afterwards frequently urged by Aristotle against the Platonic doctrine as he understood it.[3] It has been suggested that the dialogue incorporates criticisms which Aristotle may have originated as a young man of about 17, when a pupil in the Academy. Are the points Plato's own, or are they borrowed from his pupil? On the one hand it may be said that when he wrote the *Parmenides* Plato was too old to revise his

the analogy. It is therefore an argument from the general conception or form of analogy to the actual analogy (or complete terms of the analogy) in a particular case. Now when I argue that because a and b both exhibit the property x, and a exhibits besides the property y, therefore b will also exhibit the property y, I may be said to be completing an analogy. The presence of x in a is to the presence of y in a, as is the presence of x in b to that of y in b. In this case, the argument would be from the existence of an analogy to the fourth term of it. But if the looser usage of the term be interpreted thus, it bears less resemblance to the earlier usage than upon the interpretation in the text.

[1] Metaphysical criticism could easily raise difficulties against the view that relations as such are extrinsic and attributes intrinsic to their subject. But we are concerned here rather with a common way of regarding the matter than with its ultimate tenability; and I think we do commonly so regard it.

[2] *Logic*, § 214.

[3] *Parm.* 132 D–133 A. It is possible that the argument was originally neither Plato's nor Aristotle's.

system, as this interpretation of the dialogue conceives that he was doing; on the other, that at 17 Aristotle was too young to develop criticisms so original and profound.

But Kant's chief works, embodying the system which has made him famous, were written after he was 50; and Berkeley at the age of 20 was entering in his Commonplace-book important and original criticisms of Locke.[1] One analogy supports the attribution to Plato, the other that to Aristotle.

If it is not proof, has argument from analogy any value? Can we give any rules by which to judge its value in a given case? Here we must remember that the argument rests altogether on a belief that the conjunction we observe discovers to us a connexion; the presence of both x and y in the subject a points to such a connexion between them as will justify our inferring from x to y in the subject b. If we definitely thought that x and y were irrelevant to one another, it would be foolish to expect b to exhibit one because it exhibited the other. But though the argument thus presumes a connexion between x and y, it makes no pretence of showing that y depends on x rather than on some other property z in a, not shared with a by b. There is no elimination. If however there were any implicit, though not formal, elimination: or again, if there were anything known to us which seemed to support the hypothesis of a connexion between x and y: we should attach more weight to the argument. Hence if the ascertained resemblance between a and b is very great, we may think the argument from analogy stronger. For there must be something in a to account for the presence of y; and if y is not connected with x, we must look for that something in the remaining nature of a; but the more we include in x (the ascertained resemblance), the less there is that falls outside it, and the fewer therefore the alternatives open to us, to account for the presence of y in a. Still it must be admitted that so long as we rely merely on this sort of consideration, it remains to the end possible that y is unconnected with x, and therefore that y will not be found in b. Of much more weight is the consideration, that the connexion between x and y implied in the argument is one for which our previous knowledge prepares us. The fact that the angle of reflection is equal to the angle of incidence might well be supposed due (as indeed it is) to the propagation of sound in waves; and if

[1] Cf. D. G. Ritchie, *Plato*, pp. 108, 120. I have not reproduced the exact use which he makes of the analogies.

so, we should expect the same fact in the case of light to be produced by the same cause.

It will be seen that the considerations which must influence us in determining what weight we are to attach to an argument from analogy are the same as those by which we must estimate the value of an induction by simple enumeration. Both point to a general principle, which if it were true would account for the facts from which we infer it; neither proves its truth; and to try to prove it must be our next business. Mill rightly says that, however strong an analogy may be, any competent enquirer will consider it 'as a mere guide-post, pointing out the direction in which more rigorous investigations should be prosecuted'. And the same might be said of an empirical generalization. The next sentences from the same passage of Mill's *Logic* may well be quoted: 'It is in this last respect that considerations of analogy have the highest scientific value. The cases in which analogical evidence affords in itself any very high degree of probability are, as we have observed, only those in which the resemblance is very close and extensive; but there is no analogy, however faint, which may not be of the utmost value in suggesting experiments or observations that may lead to more positive conclusions.'[1]

How then does argument from analogy differ from induction by simple enumeration? In the latter, because a number of instances of a kind x exhibit the attribute y, we infer that all x are y; in the former, because two particulars a and b agree in certain respects x, we infer that y, which is exhibited by a, will be exhibited by b also. In the latter, from the limited extension of an attribute over a class, we infer to its extension over the whole class; in the former, from a partial agreement between two individuals in intension, we infer to a further agreement in intension. But the one passes gradually into the other; for the former may be called the application to a particular case of a general principle inferred in the latter from a larger number of instances than in the former. This is very plain in an illustration which Aristotle gives of the 'Example' (his name for the argument from analogy). A man might have inferred that Dionysius of Syracuse designed to make himself tyrant, when he asked the people for a bodyguard; for Peisistratus at Athens asked for a bodyguard, and made himself tyrant when he got it; and likewise Theagenes at Megara. Both

[1] *System of Logic*, III. xx. 3 *med.*

these fall under the same general principle, that a man who aims at a tyranny asks for a bodyguard.[1] One of the instances of argument from analogy given above concerned the volatilization of gold; and it might perfectly well be said that it would be contrary to all analogy for gold to be incapable of a gaseous form. But we might equally well say that our experience of other elements warranted the empirical generalization that they could all be volatilized, and therefore gold must be capable of it. This affinity between the two processes of inference is however often concealed by the fact that the points of resemblance in two (or more) subjects, which form the basis of an inference to a further resemblance, have not given rise to any special denomination; there is no general name by which the subjects can be called on the strength of the resemblance, and the resemblance may even be one that we recognize but cannot precisely describe. In gold, we might pick out the fact of its being an element, as justifying the expectation that it can be volatilized. In Dionysius, his asking for a bodyguard is the circumstance that classes him with Peisistratus and Theagenes, and excites our fear that he aims at a tyranny. But a weatherwise man might be unable to describe what it is in the appearance of the sky that makes him fear a great storm, though he can say that it was on just such a night as this that some other storm broke out. The general proposition (the induction as some would call it), which mediates his inference from that past occasion to the present, cannot be formulated; and so he may appear to work without it, and the affinity between such a process and induction by simple enumeration may be unobserved. Yet it exists, and, as has been said, the one process passes imperceptibly into the other, as the number of instances increases from which the conclusion is inferred; though where we cannot formulate a general principle, we should certainly speak of the argument rather as one from analogy.

It is of some importance to realize that a general principle is always involved in such an argument, because it has been contended that all inference goes really from particulars to particulars.[2] There may be psychological processes in which a man's mind passes

[1] *Rhet. a.* ii. 1357[b] 25–36. To make the inference to Dionysius necessary (it is of course Dionysius I who is meant), the principle would have to be, that a man who asks for a bodyguard aims at a tyranny; and that is really what the suspicious citizen of Syracuse would have had in his mind.

[2] Mill, *System of Logic*, II. iii. 3, and *supra*, c. xiv, pp. 300–310: cf. also Bradley's criticism, *Principles of Logic*, Bk. II. Pt. ii. c. ii.

direct from *a* to *b*, and he predicates of the latter what he was predicating of the former, without grounding it on anything recognized to belong to them in common; just as a man who passes a letter-box in the wall may look round at it to see the time. Psychologists explain such actions as due to the 'Association of Ideas'. But this has nothing logical about it, and is not inference. Any one must admit when questioned, that unless he supposed *b* to share with *a* the conditions on which the presence of *y* depends, he could not rationally infer it in *b* because he found it in *a*; and a process which cannot rationally be performed can hardly be called a process of reasoning. But that supposition is the supposition of a general connexion; and therefore *inference* from particular to particular works through an implicit universal principle.

CHAPTER XXV

OF MATHEMATICAL REASONING

Mathematics is frequently and rightly called a deductive science. Yet it has been said to rest on generalizations from experience, and for this reason to be fundamentally inductive. There are also certain particular processes of reasoning in mathematics to which the name 'induction' is habitually given.

One of these is just induction by complete enumeration, which does occur sometimes in mathematics. A proposition may be proved independently of a right-angled, an obtuse-angled, and an acute-angled triangle, and therefore enunciated of the triangle universally: or of the hyperbola, the parabola, and the ellipse[1], and therefore enunciated of all conic sections. The formula for the expansion of a binomial series is proved separately to hold good when the exponent is a positive integer, negative, and fractional; and only therefore asserted to hold good universally. The peculiar nature of our subject-matter in mathematics enables us to see in each case that no other alternatives are possible within the genus than those which we have considered; and therefore we can be sure that our induction is 'perfect'. The nature of our subject-matter further assures us, that it can be by no accident that every species of the genus exhibits the same property; and therefore our conclusion is a genuinely universal judgement about the genus, and not a mere enumerative judgement about its species. We are sure that a general ground exists, although we have not found the proof by it. This kind of mathematical induction needs no further consideration.

The case is different where some proposition is inferred to hold good universally because it is proved to hold good in one or two instances. This sort of inference occurs in geometry, when we prove something about a particular square, or circle, or triangle, and conclude that it is true of any square, circle, or triangle; and again in algebra, when a formula for the summation or expansion

[1] The circle being treated as the limiting case of the ellipse.

of a series, and such-like, being shown to hold good for certain values of x, is inferred to hold good for any value. The former kind of procedure is too familiar to need illustration; of the latter, the simplest illustration is the proof of the formula for the sum of the first n odd numbers—i. e. of the odd numbers, beginning with 1, and taken continuously up to any term that may be chosen. The sum is always n^2; and this is shown as follows. It is found by addition that the sum of the first three, four, or five odd numbers is 3^2, 4^2, or 5^2; and then proved that *if* the sum of the first $n-1$ odd numbers $= \overline{n-1}^2$, *then* the sum of the first n odd numbers must $= n^2$. For the $n-1^{\text{th}}$ odd number is $\overline{2n-3}$. Let

$$1+3+5+7+\ldots+\overline{2n-3} = \overline{n-1}^2 = n^2-2n+1.$$

Add to each side $\overline{2n-1}$ (which is the next or n^{th} odd number)

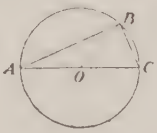
$$\therefore\ 1+3+5+7+\ldots+\overline{2n-3}+\overline{2n-1} = n^2-2n+1+2n-1 = n^2.$$

If the formula holds for $n-1$ places therefore, it holds for n places: that is, it may always be inferred to hold for one place more than it has been already shown to hold for. But it was found by addition to hold (say) for 5 places; therefore it holds for 6; therefore again for 7, and so on *ad infinitum*; and therefore universally.

It is instructive to compare this reasoning with the induction of the inductive sciences. In one respect it presents the same problem, viz. What is our warrant for generalization? Yet it cannot be said that the reasoning is of the same kind.

We saw that in the inductive sciences all generalization rests on the existence of universal connexions—whether we express that as the Law of Causation, or the Uniformity of Nature, or in some other manner. But the particular problem of any inductive enquiry is to determine what are the conditions with which a determinate phenomenon x is connected universally; and that is only to be done by an exhaustive process of showing with what, upon the evidence of the facts, it is not connected universally, until there is only one alternative left unrejected, which we are therefore bound to accept. Now it is by no such process of elimination as this, that we demonstrate the properties of a figure, or the sum, for any number of terms, of a series. We do not conclude that the angles of a particular (rectilinear) triangle are equal to two right angles, because we have tried and found that there is nothing else to which they can be equal; but we so far understand the nature of space

as to see, by means of drawing a line through the apex parallel to the base,[1] that the mere three-sidedness of the figure necessarily involves that equality. The geometrician sometimes appeals to the conclusion of a previous demonstration, without realizing to himself the reasons for the necessity of that conclusion; thus, for example, in proving that the angle in a semicircle is a right angle, he appeals to the fact that the three angles of the triangle in which it is contained are equal to two right angles, and to the fact that the angles at the base of an isosceles triangle are equal to one another, and shows now only that the angle in the semicircle must therefore necessarily be equal to the other two angles in the triangle in which it is contained. So far as he thus appeals to the conclusion of a previous demonstration, and applies it to the figure before him, he syllogizes; but when he realizes the necessity of that conclusion, he does not syllogize, but sees immediately that it is involved in the truth of other space-relations; and this he finds out by help of *drawing the figure*. It is felt that a *reductio ad absurdum* is a defective proof in geometry just because we should be able to show that such and such a proposition is true by direct reference to conditions which necessitate it, and not indirectly by the refutation of the contradictory. Thus the reasoning proceeds directly from the apprehension of certain necessary relations among characters in the subject of our study to the apprehension of other relations seen to be bound up with those[2]; not, as in induction, from the observation of facts to belief in the only connexions with which they cannot be shown to be incompatible. In our constructions, we have insight into the necessary implication of one fact with another within a system of spatially related points, lines, surfaces, and figures. Our reasoning therefore is deductive; and its premisses are proper premisses, ἴδιαι ἀρχαί—geometrical truths which explain other



[1] Or, from the intersection of one side with the base, a line parallel to the other side.

[2] We might say that our reasoning proceeds from conditions to their consequences; but it must be remembered that in mathematics different facts in the system of spatial or quantitative relations mutually condition one another; and therefore the order of demonstration is often indifferent, and condition and consequence may change places. Still the reasoning is deductive, since our premisses display to us the rational necessity of the conclusion, and do not leave it merely as one which the facts force us to accept, if there is any principle of connexion in them at all, but which we do not see from the nature of the terms to be necessary: cf. p. 437, n. 1, *supra*.

geometrical truths. It is the same with any process of calculation in arithmetic or algebra. There too we argue deductively; and there too our premisses are proper premisses, truths about relations of quantity which render necessary other relations of quantity. Nor is there any special difficulty about the 'mathematical induction' employed in proving the formula for the summation or expansion of a series, &c. When we prove that a formula which holds for $n-1$ terms holds for n terms, n represents any number in just the same way as the circle on a blackboard represents any circle. Geometrical proofs rest on the intuition of spatial relations, and algebraic on the intuition of quantitative relations, and so far the two sciences differ. But that is not more surprising than the fact that moral philosophy, in which our proofs rest on insight into relations neither of quantity nor space, differs both from geometry and from algebra.

Yet we may return to the question, What warrant have we for generalizing? We must grant that the reasoning by which I prove that the angle in this semicircle ABC is a right angle, or that a formula which holds for the sum of the first $n-1$ odd numbers holds for the sum of the first n odd numbers, is different from that by which I prove connexions of cause and effect in the inductive sciences. Yet why do I conclude that the angle in *any* semicircle is a right angle, or that the formula for the sum of the odd numbers, which holds up to the term next to the $n-1^{\text{th}}$, holds up to *any* next term, when I have only proved it about *this* semicircle, and the series up to the next to the $n-1^{\text{th}}$ odd number?

Probably most people's natural impulse would be rather to express surprise at the question than any sense of difficulty in the matter. What difference can it make, they would ask, what circle is taken? What difference can it make that in proving that what holds for so many places of odd numbers holds for one place more, the place you take is represented by $n-1$? Such counter-questions would be a very proper rejoinder. But it may be useful to see whereon our confidence rests, and so where the real difficulty about generalization lies in the inductive sciences.

Our confidence rests, as has been indicated already, on our power, in regard to the relations of points, lines, surfaces, and figures in space, or to the relations of quantities or numbers, to apprehend what must be: a power lacking in regard to the subject-matter of the inductive sciences. In geometry and mathematics we have

a direct insight into the special nature of our facts. This insight is expressed in a variety of 'necessary judgements' as to the connexion of one character with another in any particular quantitative or spatial subject. Some of these judgements have attracted attention as axioms or postulates. Others, which are implicit in our reasoning, have often passed unnoticed. Geometrical definitions, for example, are not made without insight into the possibility of constructing what is defined. If it is an axiom that two straight lines cannot enclose a space, the definition of a plane triangle implies the axiom that three can. Again, we constantly take it as evident, in a geometrical demonstration, that certain lines must intersect, with no warrant but our insight into the nature of our subject.[1] The very first proposition of Euclid assumes that the circumferences of two circles described with the same straight line for radius, and its opposite extremities for centres, will intersect; others assume that the diagonals of a quadrilateral will do the same, either within the figure, if it has no re-entrant angle, or without it, if it has one. Every simple numerical equation, such as $2+2=4$, asserts a relation seen to be necessary. Every application of algebraical induction involves that we see that one term has necessarily a certain relation to the next term or the term so many places from it in a series, no matter which term it be. Such insight we lack in regard to the connexions between one quality and another in concrete things, or between change in one thing and change in another or one change and another in the same thing; though we believe that there are necessary connexions here also, and that because what is necessary is universal, we can use our experience of particular things and events to determine what the connexions are.

But there is another point to consider. In discussing the principle of Universal Causation, and its relation to the so-called Uniformity of Nature, we saw that the necessity involved in the causal relation between a change of one kind and a change of another was irrespective of the repetition of such changes. An unique cause would produce its unique effect necessarily. The repetition of like changes, the multiplicity of like things, is important not because otherwise there would be no causal connexions, but because otherwise we could not discover them. And we could not otherwise discover them just for the reason that we have not that direct

[1] My attention was called to this by Professor Cook Wilson, from whom I borrow the illustrations in the next sentence.

insight into the connexions of terms here which we have in respect of geometrical and mathematical terms; we rely on the repetition of the like for eliminating the irrelevant, since that is not relevant which is not repeated as connexion would require.[1] But in the mathematical sciences we have this insight, and hence the repetition of like instances is superfluous to the process of proof. That the angles at the base of an isosceles triangle are equal would not only be as necessary, but would as easily be seen to be necessary, though there could be only one isosceles triangle; that $1+2+3+\&c.+n = \frac{n(n+1)}{2}$ would as easily be seen to be necessary, though (*per impossibile*) only one value and application could be given to n. Only in this case we should not generalize. That we do generalize depends therefore not merely on our direct insight into necessary relations, but on our apprehending that the terms between which they lie are not unique. Indefinite repetition with no qualitative variety belongs to the nature of space, and also of the numerical series. Any space is divisible into spaces which are smaller, but not otherwise different, and is a portion of a space which is larger, and not otherwise different. Therefore whatever space-relations are exemplified in one part of space may be equally exemplified in any other. This homogeneity or, as we might say, indifference of space is of course taken for granted in all the physical sciences; for we never regard mere difference of position as affecting the state of a body, but only difference of relation to other bodies involved in difference of position.[2] So with the number-series; at any point in it there is the same difference between one number and the next; a ratio found in one part of the series can be found in another[3], and so on; otherwise our x and y and n could not be general symbols.

But this insight into the homogeneity of space, or the uniform construction of the numerical series is, after all, only of a piece with the insight into the nature of our subject-matter which we display when we see the necessary truth of particular mathematical pro-

[1] Though the repetition need not be without variation, if we can find a formula to connect the differences of our variables: cf. *supra*, p. 525.

[2] Some theories of non-Euclidean space, just because they reject this indifference of it, have to represent the consequences by saying that bodies would be distorted through translation: i.e. distorted in terms of Euclidean space. That they have to represent the consequences in terms of Euclidean space seems to show that we cannot really conceive the possibility of the other.

[3] Not of course in any other: e.g. the ratio 5 : 7 can be found beginning at 10, but not beginning at 8.

positions. To see what is irrelevant is but the other side of seeing what is sufficient to a given consequence.[1] If we understand that the equality of the angles at the base of a triangle is necessarily involved just with the equality of the sides, we also understand that the length of the sides or the place of the triangle can make no difference. If we understand that the connexion between the truth of a formula for n terms of a series and its truth for $n+1$ terms depends just on their being next terms, we also understand that it has nothing to do with the magnitude of n. We believe indeed that, as in respect of number and quantity and of space, so in the attributes and changes of bodies there is a fixed system of necessary relations; but we cannot tell what it is by thinking. If we could, so as to see for example that gold must be heavier than lead, as we can see that the angle at the centre of a circle must be double that at the circumference, the physical sciences would be deductive, where they are now empirical.

But it has been said that the *principles* of geometry and of mathematics are themselves generalizations from experience, and these sciences therefore at bottom empirical and inductive, like the physical sciences[2]. This question was referred to at the outset of the chapter. Now, were it so, it is hard to see why the same should not as well be said of the *inferences* in mathematics.[3] Their demonstrative force arises from the fact that the nature of space or quantity allows us to see immediately the consequences involved in certain conditions. But any one who requires repeated experience to convince him of the truth of a geometrical *principle* (such as that two straight lines cannot enclose a space) may just as well require repeated experience to convince him of the truth of a geometrical *deduction*; we have to do with the mutual implication of spatial conditions in both cases. And so it is also in the science of pure quantity. The multiplication table up to 12×12 might be said to contain principles, and the multiplication of 266×566 to apply them; but whatever reason there is to doubt that $6 \times 6 = 36$, there will be the same reason to doubt whether it follows that $60 \times 60 = 3600$.

[1] We may indeed prove something of a subject which is not the commensurate subject, as if we proved that the external angles of a square were equal to four right angles, when it is true for any rectilinear figure; but even here we see that nothing relevant is *omitted* from the conditions of the property. Cf. Arist. *Anal. Post.* a. v.

[2] Mill, *System of Logic*, II. v–vii. Cf. *Autobiography*, p. 226.

[3] Or of any form of inference: cf. *supra*, pp. 312–313, *infra*, p. 567.

However, it will be sufficient if we confine ourselves to the consideration of the alleged inductive character of the process by which we ascertain mathematical principles, without attempting to determine how much would have to be regarded as principles, and how much as valid consequence.

What is really meant by the allegation is, that whereas every mathematical principle, such as the axiom of parallels, or $2+2=4$, is universal, our reason for accepting it as universally true lies in the fact that we have always found it to hold good in experience. Two apples and two apples make four apples; it is the same with cows or sovereigns, window-panes or waterpots. And whenever we have seen a straight line falling on two other straight lines and making the alternate opposite angles measurably equal, we have found—if we have tried—that however far we produced the two other straight lines, so long as they continued apparently straight, they remained at the same measurable distance from one another. All experience confirms these principles, and none is contrary to them; so we accept them as empirical generalizations, possessing, on account of the extent and variety of the circumstances under which they have been found to hold good, the same degree of certainty as if they had been proved by a rigorous elimination of all other hypotheses.

It is really sufficient answer to this view, to recur to what was said upon a similar attempt to treat the Law of Causation as empirically established. If the Law of Causation is true, the facts of our experience help us to determine what are the particular causal connexions in nature; if we start by doubting it, the facts will never bring us any nearer the proof of it. Similarly, if we start by doubting whether the relations between the same spatial or numerical terms are constant, the facts will never begin to prove it. Grant that the sum of $2+2$ is always the same, and it is worth while to see what it is; and whatever countable things we take to reckon with will make no difference. But question whether it is always the same, and proof that it is so becomes impossible. For you have no ground for supposing that if $2+2$ could sometimes make 5, cases of the occurrence would have occurred in your experience. Everything becomes problematical; the frequency of any particular sum of $2+2$ is quite indeterminate, if the sum is indeterminate; and your experience may assure you that you have never found them making anything else than 4, but cannot assure you that you are never

likely to do so. And so it is with geometrical principles also. If geometrical relations are not necessary and universal, we have nothing but a conjunction of facts empirically ascertained. In each place and time the conjunction may be different; there is no reason to suppose that what occurs here and now conveys any instruction about the occurrences at other times and places. If each place and time is loose and independent, the next may always contradict even the uniform results of previous experience.

Other lines of refutation are also possible. It might be pointed out that in point of fact we do not look for confirmation of our principles to repeated experience; but we interpret experience in the light of our principles. Two drops of quicksilver and two drops of quicksilver will make one drop of quicksilver; but we insist that the four drops are there, in a new figure. The angles between the end-lines and the side-lines of a tennis-court may seem each to be a right angle, and the sides to be drawn straight; but if we find that one end-line is shorter than the other, we say that we know that the angles cannot be true. It may be said that by this time our principles are well established, and facts in apparent conflict with them are therefore reinterpreted so as to be consistent with them. But facts in apparent conflict must have been frequent from the beginning. Again, it is hard to see what meaning can really be attached to the statement that $2+2$ might conceivably make 5, or that lines making equal angles with a third straight line might conceivably remain straight and yet converge; for such a thing cannot be represented to thought as possible.

It is of course true that in the application of mathematical reasoning to what is concrete, our conclusions will only be true if our premisses were so. If a wheel which I assume to be circular is not circular, conclusions based on the assumption will prove false. If I am wrong in my linear measurement of a floor, I shall be wrong as to the number of square feet of floor-cloth required to cover it. But this does not shake the certainty and universality of mathematics; indeed nothing else would be consistent therewith.

It is also true that without experience of counting numerable things, and of constructing figures in space, I should be unable to apprehend or understand the truth of mathematical principles. But this does not make their truth empirical, or my mode of ascertaining it inductive. For these principles are seen to be intrinsically necessary as soon as they are understood; whereas inductive

conclusions are never seen to be intrinsically necessary, but only to be unavoidable. Nor does further experience add anything to our assurance, when we have once made the construction or the calculation in which their truth becomes manifest to us; whereas further experience of the same conjunction amidst variation of circumstance is precisely what does add to our assurance of the truth of an empirical generalization.[1]

We must conclude that in mathematics there is (or at least should be[2]) no generalization from experience. To suppose mathematical principles to be such generalizations is like supposing the Law of Causation to be so. Their universality is the counterpart to the reign of law in physical nature. But the deductive character of mathematical science is due to the nature of the subject-matter, and our peculiar insight into the rational connexion of its parts. What is implied in our possession of this insight is a metaphysical question lying beyond our purview.

[The nature of mathematical certainty is a question of far-reaching metaphysical importance; and J. S. Mill, in his *Autobiography* (*loc. cit.*), frankly acknowledges that the chief strength of the opposition to the truth of the Empirical Philosophy had always seemed to lie here. It was on this account that he sought to show that mathematical principles in their turn were generalizations from experience. He held the same with regard to logical principles. It is logically important to see that there can be no knowledge unless there are truths not empirical—i. e. not open questions, for a decision on which we must go to the tribunal of sense-perception or events. And no one will understand the structure of knowledge, who does not see that mathematical principles are truths of this kind. But it may be asked what their relation is to logical principles. There are some who have represented logic as at bottom a branch of mathematics; and others seem inclined to suppose that mathematics can be reduced to formal logic. A non-mathematician is not well fitted to discuss these matters in print: and the discussion belongs in any case to a more advanced stage of logical science than this book pretends to attain. But I ought perhaps to say that I do not understand how either theory can be true.[3]]

[1] Cf. p. 531, *supra*. [2] Cf. p. 530, *supra*.

[3] On the circularity of attempts to define fundamental mathematical notions by help of what is regarded as the general logical notion of a class and its members, cf. H. Poincaré, *Science and Method* (E. T., F. E. Maitland, pp. 155-157). A similar circle seems to be involved at a critical point in Dedekind's treatise *Was sind und was sollen die Zahlen?* He explains (professedly without presupposing the thought of number) what a well-ordered

system is, and what an image of that system is. The alphabet, e.g., is a well-ordered system, and a cipher in which other symbols correspond to the several letters is an image of it. Now the image of a system may lie within itself. This would be the case with a cipher that put *b* for *a*, *c* for *b*, &c., and finally *a* for *z*. Here however every term in the original system occurs also in the image. Suppose this not to happen (e.g. that *a* did not occur in the cipher): and suppose also no term in the image to correspond to more than one in the original system; then we should say that a system cannot contain a complete image of itself. Dedekind however assumes a system containing a complete image of itself, with a separate term corresponding to each term in the original system, yet not employing the first term of it; and he develops certain properties of such a system, which are those of the number-series. Now *unless we thought at the outset of the number-series*, the whole of that procedure would be just words standing for nothing conceivable. Because the number-series is endless, therefore however many terms we take in it, starting from 1, we can find in it as many terms starting from 2, and there will be a 'one-one correspondence' between the two systems. But Dedekind's professedly logical considerations do not elucidate the number-series; on the contrary, it is required to elucidate them. One who finds paradox in the number-series will not think that it elucidates them completely; but they certainly do nothing to resolve such paradox.

CHAPTER XXVI

OF THE METHODOLOGY OF THE SCIENCES

We have seen that inferences cannot all be reduced to a small number of fixed types. They are not all syllogistic, not even all that are deductive. Their form is not altogether independent of their matter. All inference, according to Mr. F. H. Bradley, is a construction and an intuition.[1] The putting together of the premisses is the construction, but it is the terms which determine how it can be effected. The perception of something new to us in the whole which we have constructed is the intuition; and if we do not see its necessity, there is no help for us. But within the unity of this definition, we may examine any particular type of inference which, for its frequency or importance, seems to demand our special attention. Syllogism is one of these types; the disjunctive argument as applied to establish causal connexion is another. The relation of subject and attribute is one of the commonest which our thought uses, and therefore inferences based on it are common. The causal relation is not less important, and the type of inference used in its establishment equally deserved our study.

We found that this type of inference rested on some insight into the causal relation.[2] We considered very generally what that relation involved, and how we could satisfy ourselves that we were right in bringing any particular relation of facts under it. We noticed some of the difficulties which the complexity of nature places in our way; and some of the cautions which we must constantly bear in mind in interpreting facts in accordance with our conception. We found that general truths present themselves to the mind at first in the form of conjecture or hypothesis, and that often there is no means of testing such hypothesis except by first deducing—it may be by very elaborate reasonings—the consequences that should follow in specified circumstances

[1] *Principles of Logic*, p. 235. 'The process is a construction and the result an intuition, while the union of both is logical demonstration.'

[2] Not that all disjunctive argument involves that relation; but only disjunctive argument applied to the discovery of causes.

if it were true and if it were not. But all these matters were discussed and illustrated in a very general way.

Now different enquiries have their own peculiar difficulties, arising out of the nature of their subject-matter, and of the problems which it sets. And any rules for dealing with these peculiar difficulties will constitute rules of method, instructing us how to set about the task of singling out the laws or causal connexions from amidst the particular tangle in which the facts are presented in such science. The consideration of such rules, as distinct from the use of them, is *Methodology*; and so far as herein we consider how certain general logical requirements are to be satisfied in a particular case, it is sometimes called *Applied Logic*.[1]

To this subject belongs Mill's discussion of the proper method of studying the moral or social sciences.[2] He points out how methods of enquiry appropriate to certain chemical investigations (to which he therefore gives the name of the Chemical Method) are inapplicable in dealing with the sciences of human nature. The chemist, unable in a great degree to predict from his knowledge of the properties of elements the properties which will belong to their compounds, has to proceed by experiment conducted with every precaution to secure a precise knowledge of the conditions; and thus discovers the effect of a new condition or ingredient upon a whole of a certain kind. But we cannot experiment with society out of a merely speculative curiosity; the practical interests involved are too great; and were that not so, yet the thing is impossible. Our material is not under control. It would be most instructive to prevent the use of alcohol in England for a generation, and watch the difference in the amount of pauperism and crime; but there is no means of performing the experiment, for to pass a law is not to enforce it. Nor can we ever know precisely into what conditions we introduce the factor whose effects we wish to study; nor can we maintain those conditions unchanged in all but what is due to the influence of that factor during the course of our experiment. For these and other reasons, it is hopeless to expect much light to be thrown upon the laws of social phenomena, merely by watching what follows in different cases upon the adoption of the same policy, or by comparing the results of different policies. There are so

[1] Cf. Kant, *Introduction to Logic*, ii. 4 (T. K. Abbott's tr., p. 8), who gives a different sense to the term, but notices this use of it.

[2] *System of Logic*, VI. vii–x.

many factors which modify one another ; each effect depends on so many conditions, and each condition by its presence or absence makes a difference to so many effects by us regarded as distinct, that it is useless to suppose that the effect of any particular social experiment will stand out sharp and recognizable amidst its surroundings, or that we could say—Here is something which could not have occurred but for the measure we took.

We must have recourse then to deduction. From what we know of the laws of human nature, we must attempt to determine the effect which a measure must produce, or the conditions out of which a given state of society must have arisen. But again the great complexity of the subject imposes certain restrictions upon us. We must not expect to be able to trace any pervading feature of society to a single motive, as political obedience to fear,[1] or good government to a system by which the ruler's private interest is engaged in governing well. And Mill lays stress on one feature in particular of the method by which the course of human history is to be explained. Instead of working out first the theoretical consequences of certain general principles, and then checking ourselves by comparing our result with the facts, he holds that we should endeavour first to ascertain empirically the subordinate principles that manifest themselves in history, and check our formulation of them by considering whether they are consistent with the more ultimate laws of human nature and conduct from which in the last resort they must be derivable. For the facts of every period are so diverse and manifold, that the former procedure would probably be a waste of time. We may know the laws of human nature, but until we know the circumstances of a given state of society, we cannot tell what result these laws will produce. We never know them sufficiently for it to be worth our while to attempt to develop human history *a priori*, as the astronomer might attempt to develop *a priori* the course of a comet or of the tides. We must be content if such generalizations as we can frame *a posteriori* are confirmed by showing that they present nothing surprising when they have happened, although we might have been unable to predict them.[2]

[1] Cf. Bryce, *Studies in History and Jurisprudence*, Essay ix.

[2] Mill gives to this order of procedure the name of the 'Inverse Deductive, or Historical Method': by which he means the method appropriate to the study of history. The Historical Method now however commonly means interpreting present facts in the light of their past history. The contrast

In the chapter on Non-reciprocating Causal Relations, questions of methodology were really to some extent discussed. For we were engaged in considering the difference between the evidence required to establish a pure causal relation, where nothing irrelevant enters into the statement either of the cause or of the effect, and a non-reciprocating relation such as is implied when we speak of a Plurality of Causes. Now some sciences find it much harder than others to eliminate the irrelevant; and to them it is specially important to remember the sort of tests by which the non-reciprocating character of a relation may be detected.

In that chapter, two of the 'Rules by which to judge of Causes and Effects' which had been previously enunciated were reconsidered at some length, and it was shown that, although nothing which failed to satisfy their conditions could be in the strict sense the cause of any phenomenon, yet if cause were understood in a looser sense, as non-reciprocating, it was not safe to make the same assertion. But of the precautions to be attended to in the application of the other two Rules little was said.

These rules were, that nothing which varies when a phenomenon is constant, or is constant when it varies, or varies independently of it, is its cause; and that nothing is so of whose effect account has already been taken in other phenomena. Both these rules are especially useful where we are dealing with *measurable* effects, the total amount of which is dependent on a large number of conditions; and the investigations which employ them have been called 'Methods of Quantitative Induction'.[1] It may be worth while to consider some of the difficulties which beset the use of them; and that will furnish an example of a methodological problem; for a science which deals with measurable phenomena, in spite of the great advantage which their measurability brings, generally meets also with some special difficulties, which it needs particular precautionary measures to surmount.

What is measurable must so far be homogeneous. Sometimes it is for all practical purposes entirely homogeneous. A gas company

intended by the word *inverse* is that noted on the preceding page; but it does not really amount to more than that precise deductions of the consequences implied in our general principles, and their experimental verification, are impossible in social and political investigations, for the reasons above given. Kepler formulated his empirical generalizations about the planetary orbits before Newton deduced them from the laws of gravitation and inertia.

[1] Jevons, *Elementary Lessons in Logic*, XXIX.

supplies gas by metre; the gas is measured, and one cubic foot is practically indistinguishable from any other. Sometimes the homogeneity is less complete, but there can be no measurement except so far as it is found. It may be important for a general to know what percentage of men he is likely to lose by casualties other than in the field; these casualties may be of various kinds, and to the individual soldier it may make a great deal of difference whether he breaks down through dysentery or fatigue; but they are all alike in incapacitating men for service; and the general wants a measure of the extent to which that occurs. A valuer assesses the value of the personal property of a man deceased; it consists of pictures, plate, furniture, horses, stocks and shares, books, and all kinds of miscellaneous articles; but so far as these are all exchangeable for money they have a common property which can be measured in terms of money.

Now contributions may be made from many sources to any homogeneous quantity, but when you are merely told what the quantity is, there is nothing to show of how many parcels, so to say, it is made up. The total quantity is a sort of unity. Had one parcel been greater, the total would have been greater; should one parcel fluctuate in amount, the total fluctuates; but there is nothing to show which parcel is fluctuating and which is constant, and the variation seems to belong to the whole.

It follows that where an effect is quantitative, and there are a number of contributory factors which, one way or the other, influence its amount, fluctuations in these do not necessarily stand out in the result. There is no doubt that overcrowding affects the death-rate; yet the death-rate in a town may rise while overcrowding has diminished, if other causes operate to increase it faster than the improvement in housing operates to diminish it.

Hence a hasty application of the rule that nothing is the cause of a varying phenomenon which does not vary proportionately with it may lead us into grave mistakes. We might suppose, for instance, in the last example, that overcrowding had no influence on the death-rate, because the death-rate seemed to rise and fall independently. Doubtless the independence is only seeming; and if the other contributory factors could be kept constant, we should find the rise and fall proportionate. But we cannot keep them constant.

And even if we could, we should be exposed to other errors of interpretation. The death-rate, many as are the causes which contribute to it, is yet measured as a whole, and treated as one phenomenon. If all the causes which contribute to it were constant except one, and that one fluctuated, the whole result might be attributed to the one circumstance which exhibited proportional fluctuations with it. In this particular matter, indeed, we know too much to fall into such an error ; we know that overcrowding is not the only cause of death. But where our previous knowledge is less, it is very easy to attribute the whole of a varying effect to the factor which varies in proportion, instead of attributing to it only the increase or decrease beyond a fixed amount. The influence of education upon character is great ; and that is shown by the effects of giving and withholding it. But we cannot thence infer that it is all-powerful, or that the whole difference between the criminal and the good citizen and father is due to comparative defects in the criminal's upbringing.[1]

It is clear, then, in the case of a fluctuating effect which is the complex result of several causes, that though there must no doubt be a proportionate fluctuation in the cause, yet it is unsafe to reject from being a cause either a factor which fluctuates when the effect is constant, or one which is constant when the effect fluctuates. For we see the effect as a whole ; and the whole need exhibit no fluctuations proportionate to those of any one part. The rule of elimination is not false ; for if the separate effects of each factor were not lost and undistinguished in the total, we should observe the facts conforming to it. But this not being so, the rule is unsafe.

The best remedy lies in determining the precise amount of effect which each factor can produce ; and as each factor may perhaps be liable to fluctuation, what we need is a principle or law connecting each degree of its activity with a corresponding quantity of the effect. This is done, for example, in the Law of Gravitation. And could we thus calculate the amount of effect which the other causes at work, at the strength at which they were severally present, were capable of producing, we might then safely attribute any difference beyond this to some circumstance that fluctuated proportionately with it.

[1] The 'Perfectibilitarians', like Godwin, at the beginning of the last century, held very nearly this. Cf. Godwin's *Political Justice*, I. iv.

But in such a procedure we should no longer be appealing merely to the principle that the cause of a varying phenomenon must be something that varies in proportion. We should be invoking also the fourth of our grounds of elimination, that it can be nothing whose effect is already accounted for. Only because we have determined the amount of effect which the other factors can produce are we entitled to say that the residue is in no part due to them. And unless we know with fair accuracy what amount of effect may be justly assigned to other factors present, we cannot upon the strength of this principle attribute any part to some particular further factor *a*. The application of this rule therefore is involved in the same difficulties as that of the former, through the fact that the effects of many different causes are compounded and lost in one total amount.

Moreover, so long as all these causes are freely varying, and masking their separate effects in one total, the determination of the law of any single cause, much as it would help us to discover the others, is the very thing that is so difficult. Hence the necessity for experimenting with each suspected cause singly. It may be impossible to exclude the influence of any others; we must then endeavour to keep it constant; or we may employ what is called a controlling experiment at the same time. We may see what happens both when a certain factor is introduced, and when it is not, under circumstances which, though we cannot keep them constant, we have good reason to believe to be varying alike in either case. A farmer, for example, wishes to know whether some new dressing is of any use to his grass. He cannot remove the other causes which promote or hinder the growth of grass, and see how large a crop of hay this dressing could produce alone: for alone it would produce none at all. Neither can he control those other causes, so as upon the same field to use it one year and not the next, and maintain all other factors the same. But he can select two plots, or series of plots, on which he has reason to believe that the other causes all operate equally, and use the dressing on one and not on the other.

But even so, we have not got a great way towards determining the *law* of a cause. To show through all that masks it that some part of an effect is due to a particular cause is not the same as showing how much is due to it: still less as finding a mathematical expression that connects definite fluctuations in the one with definite fluctuations in the other. There are many cases where this last

achievement is impossible, even though the phenomena we study be quantitative and to some degree measurable; indeed it is impossible except in dealing with the physical properties of bodies. Elsewhere we must be content with a vague much and little. In time of war, the risk of capture at sea is a great deterrent to neutral commerce; but we cannot say precisely how great. The history of times of plague shows that increased uncertainty of life relaxes the bonds of custom and morality; but it would be impossible to give any measure of the connexion between the two facts, though the measurability of the facts, in the sense that as the death-rate from plague rises the frequency of criminal or reckless acts increases, helps to betray the connexion. The one fact may be, in mathematical parlance, a function of the other; but it is not a function of the other alone; and we cannot so disentangle the many causes and their complex result as to give precision to the degree in which one affects the other. Moreover, where the phenomena are more purely quantitative, the law of variation that connects them is not always by any means easy to establish; for a formula which holds good over a considerable range of variation may break down beyond those limits. The coefficient of expansion of a metal, which indicates the rate at which its bulk increases with successive increments of heat, no longer applies when the metal vaporizes. There are what have been called critical points, at which the change in an effect no longer observes the same proportion as hitherto to the change in the cause. Great caution must therefore be observed in formulating any law upon the evidence of concomitant variation between two phenomena, even where we are satisfied that we have excluded any variation due to other causes, and can give a precise measure of the phenomena in question.

The causes whose effects are merged in a total may not only vary independently of one another; some may be intermittent in their operation. And whether they are continuous or intermittent, they may be periodic; and one may have a longer period than another. There may again be causes which are both intermittent and irregular in their action, recurring at no definite and periodic intervals. Yet it is possible to cope with many of the difficulties which these facts present by taking averages. No one would expect the rainfall of one year to agree closely with that of another in the same locality; the circumstances affecting it are too numerous and inconstant. But we have no reason to expect that the average annual rainfall over

a considerable period of years should not agree closely for different periods; for though in one year there may be more circumstances than usual that are favourable to rain, in the next there may be fewer. If, then, the average rainfall for one considerable period of years were greater than for another, we should look for some definite reason for the difference: which we might find perhaps in a difference in the amount of forest standing in the district at the different dates; for the intermittent and irregular causes of whose operation we are aware would have roughly balanced in the two periods, though not perhaps in any two single years. Another method is to plot curves. A base line for example is taken, and perpendiculars drawn to it at equal intervals for the successive years. On each of these a point is taken whose height above the base is greater or less in proportion to the number of inches of rainfall in that year; and a line is drawn through those points. The line will rise and fall irregularly; but it is possible that in spite of these intermediate fluctuations there may be long-period fluctuations which stand out clearly; what may be called the crests and troughs of the curve may be at fairly equal intervals, though its course is not uniform from trough to crest. This would indicate the action of some cause having a similar period; and if we discovered any factor with a corresponding period of fluctuation, there would be a strong presumption that it was the cause.

The profitable use of statistics depends very largely on methods like these; but the devices for bringing out their teaching are often much more elaborate than has been indicated. These belong, however, to the detail of particular sciences rather than to the general principles of logical method. Enough perhaps has been said to indicate the misinterpretations of causal relation to which we might be led, regarding quantitative phenomena that vary in their amount, by too hastily applying rules true in themselves to any unanalysed total effect: as well as the difficulties that beset us in disentangling the component parts and fluctuations.

A few further and miscellaneous examples of the way in which precepts for the better prosecution of a particular science may be drawn from general logical principles will serve to conclude this chapter. It must not be supposed that the subject is at all adequately treated here; it is only illustrated.

What is called the *historical* or *comparative* method has in the last few generations revolutionized many branches of enquiry. It

is but an application of the general principle of varying the circumstances in order the better to discover the cause of a phenomenon. But of old, enquirers into matters of historic growth, such as language, or myth, or religion, or legal ideas, were content to attempt an explanation of the facts of some particular age or country by observations carried on within that age or country alone, or if beyond it, only in adjacent ages or countries of the same type. The historic method looks farther afield. It compares the institutions of widely different ages, or of peoples who though contemporaneous stand at widely different levels of civilization and of thought. In the light of such a comparison, facts may take on quite a new appearance. Legal or other customs for which a later age has found a reason in some supposed meaning or utility which they now possess are seen to have had a very different origin, in conditions no longer existing, and beliefs no longer entertained. Folk-lore is full of such surprises. The custom of throwing rice after a married couple as they drive away is sometimes explained by saying that rice is a symbol of fertility; Sir J. G. Fraser, comparing a number of other facts, thinks that the rice was originally intended to lure back the spirit of the bride or bridegroom to its body; it was supposed that at critical times—and everything connected with marriage was critical—the spirit left the body, in the form of a bird; the rice would attract it, and if it hovered about the body it would be more likely to re-enter. Whether this be the true explanation of the custom or not, only the comparative method could have suggested it. It is the same with myth; the account of the origin of Greek and Roman mythology popularized by Max Müller represented it as, in the language of Dr. Andrew Lang, a disease of language, the pearl in the oyster.[1] Names originally designating the attributes of earth or sun or moon were confused with words of similar sound but different meaning, and out of these other meanings myths arose. Apollo Lykios had no connexion with the wolf; he was only the Shining One; but when that was forgotten, some wolf story would be invented to account for the name. Such theories are however discredited when it is found that a myth occurs in forms substantially alike among widely different peoples, whose languages do not all admit of supposing it to have originated through confusion between similarly sounding words of different meanings. There is no new

[1] *Custom and Myth*, p. 1.

principle in the use of such an argument against the 'Sun-myth' theory of mythology; we simply say that the theory fails, because the phenomena it is intended to account for occur where it cannot be applied. But Aryan mythology is a large subject by itself; an enquirer might naturally think that it could be explained without going to the mythology of African or American savages; it has been found that this is not the case; the long descent of man connects his present with a past very dissimilar, and connects thereby with one another contemporary forms of civilization wide apart. Therefore it is important to insist upon studying the present in the light of history and comparing as extensive a range of facts as can be gathered together.

We hear sometimes of 'methodological assumptions'. By the term is meant assumptions made for the sake of getting forward with the investigation of a subject, but not necessarily regarded as true. For example, there is obviously some connexion between states of mind and states of body. The psychologist, seeing quite clearly that to suppose the former to be produced by the latter soon lands him in the most hopeless contradiction, and ignorant as to the true way of stating the relation between them, may think the hypothesis of interaction the most convenient assumption to make, with a view of increasing and systematizing his knowledge of the laws which determine the development of the individual mind; or instead of the hypothesis of interaction (which conceives mind and body as producing changes in one another) he may prefer the hypothesis of parallelism, according to which every mental change has a corresponding bodily change, and vice versa, but the two series proceed each uninfluenced by the events of the other. Either hypothesis, if not regarded as true, but only as facilitating enquiry, would be a methodological assumption.[1] Similarly, if he believes in the freedom of the will, the psychologist may still, as a methodological assumption, accept the doctrine of determinism; because so far as actions have not any cause sufficiently accounting for them in some pre-existing state of the agent, but spring from the activity of a will not acting according to any laws, it is hopeless to try to explain their occurrence. In his attempts to do this therefore he will assume what is necessary to

[1] But it is strange that a psychologist should think that the truth or falsity of such an assumption makes no essential difference to psychology. Cf. Professor G. F. Stout, *Manual of Psychology*[3], Pref. p. v.

the possibility of doing it, even though he may believe that it cannot be altogether done.

Lastly, general logical considerations may indicate the weak places in a particular science at a given time, and thus show what line of enquiry is logically of most importance to the science in question. The theory of Natural Selection assumed the existence of variations, that is, divergences from the parent type in offspring; and it assumed these variations to be accidental or non-adaptive. It concentrated itself at first on the task of showing how great a degree of adaptation between an organism and its environment could be brought about, through the operation of the struggle for existence among individuals varying slightly from type in all directions; and how by the accumulation of such small variations as happened to be favourable in each generation a profound modification of specific type might ultimately be produced. It was quite worth while to work this out even upon a basis of assumption as to certain of the facts. But the pressure of criticism has directed attention to the question whether variations are all of them non-adaptive; and one of the logical requisites of the theory of Natural Selection is a suitable collection of facts throwing light upon this point. The facts are not very easy to obtain or estimate; but biologists are working at this problem with great assiduity. A study of the contemporary state of biology from a logical point of view would have to consider with some care the kind of facts required on such a point as this, and the sort of instance that would be *crucial*,[1] i.e. decisive against one or other theory.

[1] From *crux*, a sign-post: as directing our choice between two (or more) theories: *v.* Bacon, *Nov. Org.* II. 36. A crucial instance, though it can disprove, can never prove a theory, except upon the assumption that there is no other theory with which it agrees. And it is easier to imagine instances fatal to the view that all variation is non-adaptive than to the view that adaptive variation sometimes occurs.

CHAPTER XXVII

APPENDIX ON FALLACIES

A FALLACY is an argument which appears to be conclusive when it is not; and the chief use of studying fallacies must be that we may learn to avoid them. Regarding Logic as a science, we might therefore justly say that we are not called upon to discuss them. The only way in which their study can help us to understand how our thought works is by the force of contrast. Show a man an argument which he recognizes to be unsound, show him where the unsoundness lies, and he may very likely realize more clearly, so far as they can be formally prescribed, what are the conditions of valid reasoning. On this account as we went along we contrasted examples of invalid with examples of valid inference. What more then is wanted? for the case is not as it is, for instance, with psychology. To the psychologist few things are more instructive than the study of marked abnormalities of mental life: just as to the physiologist diseases reveal much which cannot be seen in health. For psychology is an empirical science, so far as it is a science at all: it aims at discovering the principles in accordance with which the various manifestations of consciousness develop in the life of the individual; what these are it is to a large extent unable to anticipate, although the metaphysician may have his views as to the conditions under which alone their action—whatever they may be—is possible. Now insanity is just as much a fact as any normal mental development; the conditions under which it occurs must be equally ascertainable; and doubtless the same principles, in accordance with which this development proceeds under certain conditions normally and to a sane result, are often exemplified in the mental disturbances which other conditions evoke. They are exemplified there too in a more prominent way; so that such cases furnish what Bacon called a *glaring instance*[1] to assist us towards their discovery. But it would be absurd to say that the principles of rational thought are equally exemplified in fallacy as in sound thinking; and it would be absurd to hope to discover, in the procedure of a fallacious mind,

[1] *Instantiae Ostensivae*, or *Elucescentiae*. *Nov. Org.* II. 24.

the nature of true thinking. We have said once and again that Logic analyses the operations of thought which the mind has already performed in thinking about things; but it must not be supposed that it is on that account, any more than mathematics, an empirical science. The mathematician can only recognize the necessary relations of number or space by the help of some quantities or figures in which he finds them; yet he recognizes their necessity to be absolute and universal, and the fact that his non-mathematical friends make mistakes in their mathematical thinking is not taken by him as evidence that there are really two ways of thinking about the matter; he merely says that on such subjects they cannot really think. So also with Logic. Only in some thought in which they are found can the necessary relations involved in thinking be recognized; but their necessity too is recognized to be absolute, and we say that those who hold otherwise are incapable of thinking about how they think. If any one is inclined to hold otherwise, and to suppose that the laws of our thinking are psychological laws, exemplified no less in fallacy than in its opposite, let him reflect that even in doing so he is bound to assume the contrary. For he who in that opinion sets out to ascertain what the principles of thought, as a matter of empirical fact, are, will be unable by rights to know that the thought is valid by which he conducts that investigation. How then could he have any confidence in its results? Yet the fact that he intends to trust them implies that he assumes the principles of thought, in accordance with which he conducts the investigation, to be valid, whatever principles the investigation may report in favour of; and herein he takes for granted that he can recognize immediately what rational thought is, without reference to empirical facts revealed by psychology.

Nevertheless the insertion of a chapter on Fallacies may be defended. It has tradition in its favour; and without it, the nomenclature of fallacies—a nomenclature by no means fallen out of common use—would remain unexplained. There are practical uses in it also; and it would be ridiculous to say that because Logic is a science we may not turn the study of it to advantage in practice. Familiarity with some of the commonest types of fallacy is no security that we shall never fall into them ourselves; still less are we bound to fall into them unless we have acquired that familiarity. But it may help us to avoid them, by helping us more readily to perceive them. The overtones which a man has never noticed till

they were pointed out to him he may afterwards detect easily for himself. A flavour in a dish, a line in a picture, whose presence had gone unobserved, a man may be unable to ignore, if it has been singled out and presented to him in isolation. So it may be with a fallacy. There are many whose perception of the unsoundness of an argument is not unaffected by their belief in the truth or falsity of its conclusion: they will detect it where they think that what it proves is false; but let that be true—still more, let the supposed truth be precious to them, or familiar—and the same form of argument in its support may pass unchallenged. Yet if we have accustomed ourselves to the look, or type, of the fallacy, we are less likely to be the victims of such an imposition. It is true that, in the words of Archbishop Whately[1], 'After all, indeed, in the practical detection of each individual Fallacy, much must depend on natural and acquired acuteness; nor can any rules be given, the mere learning of which will enable us to *apply* them with mechanical certainty and readiness: but still we shall find that to take correct general views of the subject, and to be familiarized with scientific discussions of it, will tend, above all things, *to engender such a habit of mind*, as will best fit us for practice.' And, as Aristotle intimates,[2] a man who, if you give him time, may be well able to detect a fallacy by the light of nature, may be placed at a practical disadvantage by not being able to do it quickly enough: here the systematic study of fallacies will help him. Nor is it only in arguing with others that he may reap some benefit from the study; it will accrue to him also in the conduct of solitary thinking.[3] It was however chiefly with reference to the conduct of debate that Aristotle discussed the subject. It was from this point of view that he observed, that a man might be suspected of incompetence, who only found fault with an opponent's argument, and could not show in what the fault consisted.[4] It may be added, that so far as fallacies are referable to recognized types, it is a great abridgement of criticism to be able to name the types, and refer a particular fallacy to one of them.

These are practical considerations; and it would probably be found that importance has been attached to the doctrine of fallacies chiefly by those who have viewed Logic as an instrument for reasoning. But an use may be found in the doctrine, of a more theoretical kind. It is intellectually unsatisfactory to see that an argument

[1] *Logic*, p. 153, 8th ed.
[2] *Soph. El.* xvi. 175ª 23.
[3] Ib. 175ª 9.
[4] Ib. 175ª 14.

is faulty, and not to see precisely why. We desire for ourselves, no less than we owe to our opponent, an analysis of the error. Otherwise, and if we can only see it, and not see through it, the mind, as Aristotle expresses it, is entangled, and unable to proceed.[1] It is probable that some of the fallacies of which he finds the solution in different ambiguities of language did once constitute a more serious entanglement than they do to-day. This is partly because, as others have pointed out, such fallacies may disappear by translation into a foreign tongue ; and peoples more familiar than the Greeks were with a diversity of tongues have an advantage in detecting such. It is partly also because an analysis new in his day is common property in ours ; and many of its results are so incorporated into the currency of common thought and speech, that a man whose attention is called to them feels as if he was taught only what he already knew.

If however we are satisfied that Logic should treat of fallacies, it is very difficult to be satisfied with any treatment of them. Truth may have its norms, but error is infinite in its aberrations, and they cannot be digested in any classification.[2] The same inconclusive argument may often be referred at will to this or that head of fallacies. 'Since, in any Argument', says Whately, 'one Premiss is usually suppressed, it frequently happens, in the case of a Fallacy, that the hearers are left to the alternative of supplying *either* a Premiss which is *not true*, or *else*, one which *does not prove* the Conclusion. E.g. if a man expatiates on the distress of the country, and thence argues that the government is tyrannical, we must suppose him to assume *either* that "every distressed country is under a tyranny", which is a manifest falsehood, *or*, merely that "every country under a tyranny is distressed", which, however true, proves nothing, the Middle-Term being undistributed'.[3] The assumption of a false premiss is not indeed perhaps to be called a fallacy, as we shall see presently ; it is at any rate different in its nature from inconclusive argumentation. But the choice may equally well lie between two modes of inconclusive argumentation, when we have to classify a fallacy ; a man who attempts to refute by an enumeration of striking instances the proposition that some specific characters

[1] *Eth. Nic.* η. iii. 1146[a] 24.

[2] Cf. de Morgan, *Formal Logic*, p. 237. 'There *is* no such thing as a classification of the ways in which men may arrive at an error : it is much to be doubted whether there ever *can be*.'

[3] *Logic*, p. 159, 8th ed.

in plants and animals are not adaptive might either be charged with illicit process of the minor term, in drawing an universal conclusion where his premisses only entitle him to a particular one, or with what is called *Ignoratio Elenchi*, in supposing that a particular affirmative refutes a particular negative.[1] And not only is it impossible to make such a classification of fallacies as will never leave it in doubt to which class a particular example is to be referred; if that were all, it might be said that the types were distinct, and the classification so far a good one, although individuals could not be assigned to their types unambiguously: but it may be doubted as well, if the types of error can be exhaustively detailed, and the classification completed.

The reason for this is twofold. In the first place, there may be arguments so foolish and inconsequent, that they cannot even be said to simulate cogency; these cannot be positively characterized, but must be lumped together by the mere negative mark of inconclusiveness. And secondly, there are many fallacies, the detection of which requires not general logical training, but acquaintance with a particular scientific subject-matter. The latter point is of some importance, as connecting with what has been already said about demonstration.

We have seen that the syllogism cannot sustain the claim once made in its behalf, to be the type of all valid inference; but that —to say nothing of hypothetical and disjunctive argument—there are deductive reasonings whose validity lies in no conformity to a scheme exhibitable in the abstract, or symbolically, but rests for its apprehension upon acquaintance with the nature of the special subject-matter with which they deal. The readiest illustration of this, but by no means the only one, is furnished by geometry. Now what is true of valid is equally true of invalid reasonings. There are many which are not of a sort that can occur in reasoning on every subject-matter, but are bound up with misconceptions of the special subject-matter in which they occur. This too may be readily illustrated from geometry. 'Lewis Carroll' devised a proof that 'a right angle is sometimes equal to an obtuse angle'. The demonstration was in all other respects unimpeachable, but vitiated by one—of course intentional—error in the construction of

[1] Cf. Ar., *Soph. El.* xxiv. 179^{b} 17 *οὐδὲν δὲ κωλύει τὸν αὐτὸν λόγον πλείους μοχθηρίας ἔχειν* ('There is nothing to prevent the same argument having several faults'), and xxxiii. 182^{b} 10.

the figure, in which a line was drawn to one side of a point which must in fact fall on the other.[1] Just as a knowledge of geometry can alone show where this line must fall, so a knowledge of geometry can alone expose the inconsequence of the false demonstration. And similar inconsequences occur in every particular science, which only an understanding of that science can show to be inconsequences. Thus if it were argued that because a and b were halves of the same thing, therefore they were halves of one another, and since $a = 4$, b must $= 2$, it is only a perception of the nature of quantity that reveals (doubtless in this case to the least mathematical of us) the invalidity of the first step in the argument. It is less obvious that among a people who acknowledge kinship only through the female, a man would inherit not from his father but from his brother or maternal uncle. Yet a little reflection shows this to be the case, and shows therefore the fallacy of arguing, where female kinship prevails, that because A is in possession of a property, his son will possess it after him. Here the detection of the fallacy rests upon our perception of the system of relationships uniting the members of a society which takes account only of union by descent through the female line.

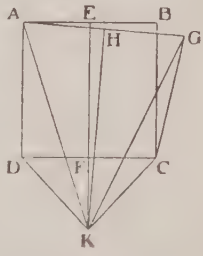
Aristotle, who noticed that every science afforded its own special opportunities for erroneous inference, gave to those that involved

[1] *v.* the *Lewis Carroll Picture Book*, edited by S. Dodgson Collingwood, (London, 1899), pp. 266–267. (*GK* must really fall to the right of *C*.)

'Let *ABCD* be a square. Bisect *AB* at *E*, and through *E* draw *EF* at right angles to *AB*, and cutting *DC* at *F*. Then *DF* = *FC*.

'From *C* draw *CG* = *CB*. Join *AG*, and bisect it at *H*, and from *H* draw *HK* at right angles to *AG*.

'Since *AB*, *AG* are not parallel, *EF*, *HK* are not parallel. Therefore they will meet if produced. Produce *EF*, and let them meet at *K*. Join *KD*, *KA*, *KG* and *KC*.



'The triangles *KAH*, *KGH* are equal, because *AH* = *HG*, *HK* is common, and the angles at *H* are right. Therefore *KA* = *KG*.

'The triangles *KDF*, *KCF* are equal, because *DF* = *FC*, *FK* is common, and the angles at *F* are right. Therefore *KD* = *KC*, and angle *KDC* = angle *KCD*.

'Also *DA* = *CB* = *CG*.

'Hence the triangles *KDA*, *KCG* have all their sides equal. Therefore the angles *KDA*, *KCG* are equal. From these equals take the equal angles *KDC*, *KCD*. Therefore the remainders are equal: i. e. the angle *GCD* = the angle *ADC*. But *GCD* is an obtuse angle, and *ADC* is a right angle.

'Therefore an obtuse angle is sometimes = a right angle.

'Q. E. D.'

mistakes in geometry the name of ψευδογράφημα, or false construction.[1] As an example he gives Hippocrates' method of squaring the circle by lunules. A lunule is a figure enclosed between arcs of two circles concave in the same direction. Hippocrates found a rectilinear area equal to a lunule whose upper arc was a semicircle, and its lower arc the *fourth* part of the circumference of another circle; he then found another rectilinear area equal to the sum of (*a*) three equal and similar lunules whose outer arcs were semicircles, and their inner arcs the *sixth* part of the circumference of another circle, and (*b*) a semicircle of the same diameter as the three lunules (i. e. of diameter equal to the chord of the arcs enclosing them); and he supposed that it would be possible, by subtracting from this rectilinear area an area equal to the three lunules, to obtain in the remainder a rectilinear area equal to the semicircle. He overlooked the fact that because you can find a rectilinear area equal to a lunule of the former sort, whose inner arc is a quadrant, it does not follow that you can find one equal to a lunule of the latter sort, whose inner arc is a sextant; and in fact a rectilinear area equal to these three lunules cannot be obtained.[2]

Now it will indeed be seen that, in this or any other case of erroneous reasoning dependent on misconceiving the consequences which follow from given conditions in a special subject-matter, the error can be expressed in a false *proposition*. It is false that because a rectilinear area can be found equal to one of these lunules, it can be found equal to the other: it is false that things which are halves of the same thing are halves of another: it is false that, if we take account only of kinship through the female line, a man will be in the same line of descent with his father. But we cannot see that any of these propositions is false, unless we understand something of the respective subject-matter. They are as it were false 'special principles', or ἴδιαι ἀρχαί.[3] It is not desirable to call every false proposition a fallacy, as e.g. that snakes eat dust, or that South

[1] *Soph. El.* ix, xi. There is not however any false construction made and used in this argument, as in that quoted in the last note; though it is falsely concluded that a required construction has been shown possible. Ψευδογράφημα practically means a fallacy involving special subject-matter: cf. the interesting example quoted from Proclus by Sir Thomas Heath, *Elements of Euclid*, i. 206, to prove that 'if two straight lines falling on another straight line make the interior angles on the same side of it together less than two right angles, those two straight lines being produced can never meet'.

[2] *v.* Poste's ed. of *Soph El.*, App. F, pp. 245–247.

[3] Cf. *supra*, p. 387.

America is an island ; nor can we extend the name to every valid argument that uses a false premiss. If the falsity of the premiss can only be ascertained empirically, there is error, but not fallacy. If however the falsity of the premiss is to be ascertained by thinking out the consequences of certain relations, or concepts, in the circumstances of a given case, then we are guilty of fallacy, or defect of reasoning, in overlooking it ; and that is what frequently occurs in the matter of any particular science.

There are indeed general heads, under which many such fallacies can be brought. In particular, they very often arise from overlooking some of the special circumstances of the case : from assuming that what is true under certain conditions will still be true when those conditions are in some way modified. Thus, if two things *a* and *b* are *equal* to the same thing, they are *equal* to one another ; from which we may conclude, that if they bear any *same* quantitative relation to a third thing, they bear *that* relation to each other ; and then it would follow that if they were halves of the same thing they would be halves of one another. But in fact, it is only when their same relation to a third is one of equality, not merely when their relation to it is the same, that they bear to one another the relation borne to it. We shall meet with this type of fallacy by and by under the name of *Secundum Quid*. That heading embraces a great range of examples. But though we can detect in them a common character, it is only by understanding something of the special matter of the argument, that we can see that the fallacy is being committed in a given case. The type, if one may say so, is fluid ; the instances are not so far of one form, that we can separate their common form from the variety of their matter, and exhibit it symbolically ; nor, though the type admits of all this diversity, can we subdivide it, and carry our classification down to *infimae species*. We recognize that its character differs in different cases ; but the differences cannot be formulated.

Our task then is one which does not admit of fully satisfactory performance. Still no doubt it can be better and worse done. What classification of fallacies are we to adopt ?

The earliest, and for long the accepted, classification is that of Aristotle, given in the last book of his *Topics*, called the *Sophistici Elenchi*. It is not free from defects ; and others, some of which will be referred to, have been propounded. But the subject is emphatically one upon which some consensus is desirable. If it is

useful to have a nomenclature of fallacies, it is useful to have a *standard* nomenclature. And it is remarkable how, even in rival classifications, many of the Aristotelian species of fallacy still hold their own. Later writers have given new meanings to certain of the Aristotelian names; or have invented new names for special forms of some of the Aristotelian fallacies; or have included in their list what are not forms of erroneous argument, but sources of error of a different kind[1]; yet it is surprising how little there is which cannot be brought within Aristotle's list. And if we consider not the enumeration of types of fallacy, but their classification, it will appear, I think, that there is no such merit in any alternative scheme as justifies us in sacrificing the advantage of keeping to the standard and traditional scheme of Aristotle.

Aristotle divided fallacies into two main groups—fallacies in **dictione**, *παρὰ τὴν λέξιν*, arising through ambiguity of language, and fallacies **extra dictionem**, *ἔξω τῆς λέξεως*, which do not have their source in such ambiguity. Although one of his species of fallacies *extra dictionem*—the fallacy of Many Questions—might perhaps be referred more naturally to the other group, yet the division, being dichotomous, is sound. It suffers, however, like all such divisions, from the defect of not positively characterizing one

[1] Thus the fallacy of Accident has practically been identified with *Secundum Quid* by many writers: that of Consequent has, e.g. by de Morgan and Jevons, been explained as 'the simple affirmation of a conclusion which does not follow from the premisses' (de Morgan, *Formal Logic*, p. 267): divers forms of *Ignoratio Elenchi* have received special names: Whately has explicitly included under fallacies, in defiance of his own definition, 'any false assumption employed as a Premiss' (*Logic*, 8th ed. p. 168: cf. def. on p. 153): Mill includes among fallacies such sources of error as Mal-observation—i.e. mingling inference with the report of what is perceived (*System of Logic*, V. iv. 5); and his first great group of fallacies, which he calls *A priori* Fallacies, or Fallacies of Simple Inspection, consists of a number of maxims which he considers erroneous (though it is not equally clear that they all are so), such as that what is inconceivable cannot be true, that effects must resemble their causes, that motion can only be produced by motion, that the same effect must always have the same cause (V. iii.); in iv. 1, Fallacies of Simple Inspection are called 'Prejudices, or presumptions antecedent to and superseding proof', and in ii. 2 they are called supposed connexions or repugnances between facts, 'admitted, as the phrase is', on their own evidence, or as self-evident. Whately (*op. cit.* p. 208) speaks of the fallacy of References, i.e. giving references in support of a statement to passages which do not really bear it out, in the trust that readers will not look up the references and discover this. Professor William James gives the name of the Psychologist's Fallacy to the mistake of supposing that a man who has a given psychical experience knows it, when he has it, to be all that I as a psychologist know or believe it to be (*Principles of Psychology*, vol. i. p. 196). Locke's *argumenta ad verecundiam, ad ignorantiam, ad hominem*, which he opposes to an *argumentum ad iudicium*, might be called heads of fallacies (*Essay*, IV. xvii. 19–22).

group.[1] Later writers, willing to remedy this defect, called the fallacies *extra dictionem* fallacies *in re*, or material fallacies. But this introduces a cross-division. For it cannot be said that fallacies *in dictione* are independent of the *res* or matter of the argument. On the contrary, inasmuch as they arise through giving different meanings to the same words either in the two premisses, or in premiss and conclusion, they disappear if we abstract from the matter of the argument and look only to the form in which it is cast. The proper antithesis to matter is form ; a fallacy not in the matter must be in the form : i.e. it must be independent of what the terms are, and must therefore persist, if symbols be substituted for the terms, and whatever term be substituted for the symbols. This cannot be said of the fallacies *in dictione*.

It is true that Whately gives a somewhat different interpretation to the expression *material fallacy*. He divides fallacies into **logical** and **material**. By the former title he means fallacies where the error lies in the fact that the premisses do not prove the conclusion ; by the latter, those in which the premisses prove the conclusion, but either the premisses are false, or such at least as we are not entitled to assume, or else the conclusion proved is not that which we profess or are required to establish.[2] He then subdivides *logical fallacies* into two groups, according as their defect of proof can be seen in the mere form of the argument (e.g. undistributed middle) or only if we attend to the ambiguity of the terms employed ; the former group he calls *purely logical*, and the latter *semi-logical*. Though the nomenclature here is unfortunate (for according to his own definition of a *logical* fallacy, those which lie in ambiguity of language are altogether and not only half logical), yet the division is sound. It includes however arguments which have no fault except that their premisses are false ; and it is true that in this he follows the words of Aristotle[3] ; but in the body of his treatise Aristotle

[1] Cf. *supra*, pp. 122–124.

[2] By the *matter* of an argument he means the propositions in it, not the terms of the propositions.

[3] *Top.* a. i. 100^{b} 23 ἐριστικὸς δ' ἐστὶ συλλογισμὸς ὁ ἐκ φαινομένων ἐνδόξων, μὴ ὄντων δέ, καὶ ὁ ἐξ ἐνδόξων ἢ φαινομένων ἐνδόξων φαινόμενος ('A contentious syllogism is one whose conclusion follows from premisses that appear to be endoxical but are not, or that appears to follow from premisses that are or appear to be endoxical') : cf. *Soph. El.* ii. 165^{b} 7 ἐριστικοὶ δὲ (λόγοι) οἱ ἐκ τῶν φαινομένων ἐνδόξων μὴ ὄντων δὲ συλλογιστικοὶ ἢ φαινόμενοι συλλογιστικοί ('Contentious arguments are those that conclude or appear to conclude from premisses that appear to be endoxical but are not'). The latter definition excludes unsound arguments from premisses really endoxical (i. e. probable or supported by opinion, and allowable in non-scientific discussion) : but this

proceeds as if he had not included them. And the practice of Aristotle appears preferable in this respect; for false premisses are certainly incapable of any classification, and the consideration of one does not help us to detect another. That, if the premisses are false, the conclusion, though valid, need not be true, every one should certainly realize; and it is good advice to a disputant to consider well the truth of the premisses he is asked to grant, or to a solitary thinker to consider well the truth of what he proposes to assume and build upon. Nevertheless there seems to be a real difference between a plausible but inconclusive *argument*, which we can see through by clearer and more attentive thinking, and a false *proposition* (whether or not plausible), which cannot be exploded by any more attentive consideration of itself, though it may by reasonings that are within our power. For this reason the extension of the term fallacy to cover 'any false assumption employed as a premiss' seems undesirable; the only sort of false proposition to which it ought to be applied is false *canons of proof*. If this correction is made, Whately is left with only two kinds of material fallacy (*Petitio Principii* and *Ignoratio Elenchi*), both of which are in Aristotle's list of fallacies *extra dictionem*; and there is no particular advantage in that regrouping of the species enumerated in both lists, which the adoption of Whately's principle of division carries with it. Whately certainly enumerates under the head of *purely logical* fallacies those breaches of syllogistic rule with which we long ago became familiar by the names of *undistributed middle*, *quaternio terminorum*, and *illicit process of the major* or *minor term*; and Aristotle makes no mention of these. But that is not because his classification provides no place for them; they are clearly fallacies *extra dictionem*. They were omitted because they did not, in Aristotle's view, simulate cogency; no one who could not detect these ought to undertake a disputation; and even a sophist, aiming only at appearing to confute his adversary and not at truth, would hardly dare to employ such methods as these. And so it was with

can hardly be supposed to be deliberate. The expression twice used in *Soph. El.* i. (164^{a} 23 *ὅτι μὲν οὖν οἱ μὲν εἰσὶ συλλογισμοί, οἱ δ' οὐκ ὄντες δοκοῦσι, φανερόν* —'It is plain then that there are real syllogisms and what appear to be syllogisms without being so': 165^{a} 17 *διὰ μὲν οὖν ταύτην τὴν αἰτίαν καὶ τὸς λεχθησομένας ἔστι καὶ συλλογισμὸς καὶ ἔλεγχος φαινόμενος μὲν οὐκ ὢν δέ*—'For this reason then and those that follow there are both apparent syllogisms and apparent confutations which are not really such') might perhaps by itself be more naturally understood to refer only to fallacious *arguments*, and not to include arguments that have no fault except in the falsity of their premisses.

the writers who for many centuries reproduced—often with increasing divergence—the Aristotelian doctrine. 'The pure syllogism and its rules were to them as familiar as the alphabet. The idea of an absolute and glaring offence against the structure of the syllogism being supported one moment after it was challenged, would no more suggest itself to a writer on logic than it would now occur to a writer on astronomy that an accidental error (which might happen to any one) of affixing four ciphers instead of five when multiplying by a hundred thousand would be maintained after exposure.'[1] A sophism, or sophistical confutation, as Aristotle called a fallacy (for he had in mind throughout the conduct of a disputation, and the methods by which one might attempt to confute a thesis maintained by an opponent: though these are of course equally methods of establishing a conclusion that confutes it), must be at least *φαινόμενος συλλογισμός*, apparently conclusive; these he wished in his treatise to enable the student to expose[2]; but a plain breach of syllogistic rule had not any appearance of conclusiveness, and enough had already been said in the *Prior Analytics* to enable any one to expose that.

We may therefore abide by the Aristotelian division into fallacies *in dictione* and *extra dictionem*. In each member of the division he enumerates a variety of types. The lists are as follows[3]:—

[1] de Morgan, *Formal Logic*, p. 240.

[2] Cf. *Soph. El.* i. 165[b] 26 *τῶν δὲ ψευδόμενον ἐμφανίζειν δύνασθαι.*

[3] Whately, as was observed above, regroups the fallacies here enumerated to suit his division. It is of course inadmissible to adopt the nomenclature of his division, and I retain Aristotle's grouping, as is done by Jevons in his *Elementary Lessons in Logic*, XX and XXI. He treats as *purely logical* fallacies the four breaches of syllogistic rule above mentioned; as *semi-logical*, Aristotle's six fallacies *in dictione*; and as *material*, Aristotle's seven fallacies *extra dictionem*. He does not therefore understand the distinction between logical and material as Whately does. 'The logical fallacies', he says, 'are those which occur in the mere form of the statement. . . . The material fallacies, on the contrary, arise outside of the mere verbal statement, or as it is said, *extra dictionem*' (p. 170). This is not of course what Whately meant. But clearly Jevons means by a *logical* fallacy one which can be detected in the form without consideration of the matter; it should therefore be capable of illustration in symbols, as his 'purely logical' fallacies are. A *material* fallacy, on the contrary, needs that we should understand the terms for its detection. From this point of view, it is nonsense to speak of 'semi-logical' fallacies; a fallacy either can be detected in symbols or not: it must either be 'logical' or not, and cannot be 'semi-logical'. The fallacies *in dictione*, which he ranks as 'semi-logical', he ought undoubtedly to have ranked as 'material'. On the other hand, some of those which he ranks as 'material'—the fallacy of the Consequent certainly (which however he misunderstands) and one type of *Petitio Principii*—can be exhibited in symbols, and ought to have been enumerated among the 'purely logical'. The

a. Fallacies *in dictione*, or παρὰ τὴν λέξιν.

1. Equivocation, or παρὰ τὴν ὁμωνυμίαν.
2. Amphiboly, or παρὰ τὴν ἀμφιβολίαν.
3. Composition, or παρὰ τὴν σύνθεσιν.
4. Division, or παρὰ τὴν διαίρεσιν.
5. Accent, or παρὰ τὴν προσῳδίαν.
6. Figure of speech, or παρὰ τὸ σχῆμα τῆς λέξεως.

b. Fallacies *extra dictionem*, or ἔξω τῆς λέξεως.

1. Accident, or παρὰ τὸ συμβεβηκός.
2. *Secundum Quid*, or παρὰ τὸ ἁπλῶς ἢ πῇ λέγεσθαι καὶ μὴ κυρίως.
3. *Ignoratio Elenchi*, or παρὰ τὴν τοῦ ἐλέγχου ἄγνοιαν.
4. *Petitio Principii*, Begging the Question, or παρὰ τὸ ἐν ἀρχῇ λαμβάνειν.
5. *Non Causa pro Causa*, False Cause, or παρὰ τὸ μὴ αἴτιον ὡς αἴτιον.
6. Consequent, or παρὰ τὸ ἑπόμενον.
7. Many Questions, or παρὰ τὸ τὰ δύο ἐρωτήματα ἓν ποιεῖν.

The fallacies *in dictione* are so many different forms of error that may arise through the double meanings of language. They differ according to the character of the ambiguity; and in a syllogism it may be any of the three terms which is ambiguous.[1] Obviously such arguments are invalid; and if the different meanings were expressed by different terms in each case, we should have a plain

fact is that, if the distinctions of *logical* and *material*, and *in dictione* and *extra dictionem*, are to be combined in one classification, they cannot be identified, as Jevons identifies them. We may either start with the distinction of fallacies into *logical* and *material*, according as they lie in the mere abstract form of the argument, and can be exhibited in symbols, or not: and then divide the latter into those *in dictione* and those *extra dictionem*, according as they arise through ambiguity of language, or not; but of course those fallacies *extra dictionem* which are *logical* in this sense must be removed from Aristotle's list of fallacies *extra dictionem*, if that title is used to indicate a subdivision of *material*. Or else we may begin by dividing them into fallacies *in dictione* and *extra dictionem*, and treat *logical* and *material* as subdivisions of *extra dictionem*. In the former case, what Jevons calls *semi-logical* (= Aristotle's fallacies *in dictione*) will enter by the latter name as a subdivision of *material*; in the latter, what he calls *purely logical* will enter as a subdivision of *extra dictionem*. Cf. the remarks in Mr. St. George Stock's *Deductive Logic*, c. xxx, who points all this out very clearly in discussing fallacies. It may be added that there may be in algebra fallacious arguments which use symbols, but are not on that account logical in the above sense, because the symbols are not logical symbols, standing for any term, but specifically symbols *of quantity*.

[1] Many arguments referable to Aristotle's heads of fallacy are not syllogistic.

quaternio terminorum, which would impose on nobody. As it is, the shifting of the meaning may sometimes pass unobserved; or the identity of the language seem to afford some proof of identity of meaning; and even where it is obvious that we are tricked by the argument, we may wish to be able to show how.

1. **Equivocation** is the simplest form of ambiguity, where a single word is used in divers senses. 'The sick man is well; for men who have recovered are well, and the sick man has recovered'[1]; here the equivocation is in the minor term, and arises from the fact that the expression 'the sick man' may mean either 'the man who is sick' or 'the man who was sick'. The following is an old example: 'Finis rei est illius perfectio: mors est finis vitae: ergo mors est perfectio vitae'; the equivocation in this case lies in the middle term. Trivial and punning examples of this fallacy, as of all those that depend on ambiguity of language, will occur to any one; but in many cases it is serious and elusive. 'It is the business of the State to enforce all rights: a judicious charity is right: therefore it is the business of the State to enforce a judicious charity.' 'A mistake in point of law', says Blackstone, 'which every person of discretion not only may, but is bound and presumed to know, is in criminal cases no sort of defence'[2]; the State must perhaps presume a knowledge of the law, and so far we are bound to know it, in the sense of being required under penalty; but a criminal action done in ignorance of the law that a man is *legally* bound to know is often considered *morally* discreditable, as if the knowledge of the law on the matter were a plain moral duty. How far that is so in a particular case may be a very doubtful question; the maxim quoted tends to confuse the moral with the legal obligation. In a long and closely reasoned argument, where important terms have been defined at the outset, it may still be very difficult to hold them throughout to the precise meaning set forth in the definition; and so far as this is not done, the fallacy of Equivocation arises. James Mill held that a so-called necessity of thought arises from the constant conjunction in experience of the 'ideas' between which a necessary connexion is asserted; his son, endeavouring to make this doctrine cover negative propositions which assert a necessity of thought, says[3] that here one idea is inseparably associated with

[1] Ar., *Soph. El.* iv. 165[b]39. [2] Quoted by Austin, *Jurisprudence*, i. 482.
[3] *v.* Jas. Mill's *Analysis of the Phenomena of the Human Mind*, i. 97, note 30, by J. S. Mill.

the idea of the absence of the other. But the idea of the absence of an idea means the *opinion* that an idea is absent.

2. **Amphiboly**[1] is ambiguity in a phrase, in which the words are used univocally throughout, but the meaning of the phrase as a whole changes through change of the construction in which the same words are taken. A traditional example in Latin is 'Quod tangitur a Socrate, illud sentit: lapis tangitur a Socrate: ergo lapis sentit'; in the major premiss, *illud* is the object of *sentit*; the conclusion is drawn as if it had been the subject. So we might say in English: 'Polyphemus what he best loves doth devour: the ram that leads the flock he loves the best: therefore the ram devours him.' Lawyers are well aware of the importance of avoiding ambiguity in the construction of a legal document (though under that head they would include the ambiguities which Aristotle assigned to Division and Composition, as well as Amphiboly and Equivocation too). Whately cites a good example from the rubric at the beginning of the Form of Service formerly ordered for use on Jan. 30, the anniversary of the execution of King Charles I: 'If this day shall happen to be Sunday, this Form of Prayer shall be used and the Fast kept the next Day following'; is the form of prayer to be used on Sunday and the Fast kept on Monday, or are both to be deferred? Another famous and deliberate example is in the oracle which Ennius said was delivered by Apollo to Pyrrhus —'Aio te, Aeacida, Romanos vincere posse.'[2] Ambiguous words and constructions are still not unfrequently used to deceive by those

> 'That palter with us in a double sense;
> That keep the word of promise to our ear,
> And break it to our hope'.

[1] The Greek word is *ἀμφιβολία*, which is said to be an *ἀπάτη παρὰ τὸν λόγον*, 'being misled by a form of words', as distinct from *ὁμωνυμία*, where the ambiguity is in an *ὄνομα* or word (*Soph. El.* vii. 169ᵃ 22). Hence arose the compound *ἀμφιβολολογία*, which became corrupted into Amphibology, as *εἰδωλολατρεία* became corrupted into Idolatry. There seems to be no reason for not saying Amphiboly in English; Amphibolia is frequent in Latin (e.g. Crackenthorpe, Aldrich). It will be seen that the fallacy of Composition may also turn on taking words together in different ways. I think that Aristotle (who notices their affinity *Soph. El.* xx. 177ᵃ 38) would have called a fallacy Amphiboly where the ambiguity arose through taking variously the government within the same group of words, and Composition where it arose through taking the same words now with these and now with those others in a sentence. Whately's example would in this case have been referred by him to Composition.

[2] Cf. Cic. *de Divinatione*, ii. 56. Cicero reasonably observes that Apollo did not speak in Latin. Cf. Augustine, *de Civ. Dei*, iii. 17 'Cui sane de rerum futuro eventu consulenti satis urbane Apollo sic ambiguum oraculum edidit,

3 and 4. **Composition** and **Division** are the converse one of the other. They consist in taking together in the conclusion (or one premiss) either words, or objects of thought, which in the premiss (or the other premiss) were not taken together, or vice versa. Plato in the *Republic*[1] argues, from the fact that a man can refuse the thing that he desires, that there must be a principle of reason as well as of appetite in the soul. For, he says, it is impossible to be contrarily affected at the same moment towards the same object in the same part of oneself (one cannot for example at once loathe and long for the same object); yet a man who is thirsty and refuses to drink is contrarily affected at the same moment towards the same object; he does not therefore refuse drink on account of the character of his appetites, but because of his reason; he reckons that to indulge his appetite would interfere with the pursuit of some other end which he prefers. Now a sophist might attack this conclusion as follows: 'Are you now drinking? No. Can you now drink? Yes. Therefore when you are not doing a thing, you still can do it? Yes. But if you can do a thing when you are not doing it, you can desire a thing when not desiring it? Yes. And so you can be contrarily affected in the same part of yourself (your appetitive nature) towards the same object at the same time.'[2] The fallacy is one of composition. The admission is that a man can when not desiring a thing desire it, i.e. that when not desiring it, he is capable of doing so; this is used as if it meant that he can *desire when not desiring it*, i.e. that he is capable of at once desiring and not desiring it; the words 'when not desiring it' are taken, or compounded, in one case with 'can' and in the other with 'desire'. If a man were to argue that three and two are five, and three and two are odd and even, therefore five is odd and even, and the same number may thus be both, he would be committing the same fallacy; when it is said that three and two are odd and even, it is true only if 'three and two' are not taken together, as the subject of which 'odd and even' are predicated, but 'three' is made the subject of 'odd' and 'two' of 'even'; but the conclusion is drawn as if they were

ut, e duobus quicquid accidisset, ipse divinus haberetur: ait enim, Dico te Pyrrhe vincere posse Romanos: atque ita sive Pyrrhus a Romanis sive Romani a Pyrrho vincerentur, securus fatidicus utrumlibet exspectaret eventum.' Cf. also *Henry VI*, Part 2, Act i. Sc. 4, ll. 60-65.

[1] *Rep.* iv. 436 A sq.

[2] Τὸ δύνασθαι μὴ γράφοντα γράφειν ('to be able to write without writing') is an example of fallacy παρὰ τὴν σύνθεσιν in *Soph. El.* iv. 166[a] 24. I do not know if the principle involved was ever brought against Plato's argument.

taken together. On the other hand, the same argument furnishes an example of the counter fallacy of taking separately in one premiss words which were taken together in the other; for three and two together are five, but it is separately that they are odd and even, and separately that in the conclusion each of them is declared to be both. And the reader will doubtless have observed that the previous example illustrates no less the division from one another in the conclusion of words that were combined in the premiss than the combination in the conclusion of words that in the premiss were divided.[1]

It was said above that in these fallacies either words or objects of thought are taken in one place in the argument together and in another separately. Of course the combination or separation of certain words carries with it that we think differently in either case of the things signified. But sometimes the illicit combination or division made in thought is not reflected by taking words together or apart. If any one were, upon the strength of the text in Gen. i. 27—'So God created man in his own image, in the image of God created he him; male and female created he them'—to argue that man was originally created bisexual,[2] and that the present division into male and female was the result of the Fall, and were to base on that a condemnation of marriage, he would be guilty of the fallacy of Composition; and quite as foolish arguments have been drawn from the words of Scripture upon such subjects. Now here the fallacy lies in referring the words 'male' and 'female' together to each person signified by 'them', instead of referring 'male' to one and 'female' to another. But the point is the same in the story of the showman who announced that children of both sexes were admitted free, and then charged admission to boys and

[1] It is difficult to keep Composition and Division apart. Aristotle gives the last example slightly differently as an example of Division—'Five is two and three, and therefore odd and even'; five is two and three together, and so inferred to be what they are separately. He gives also as an example of Division one which might equally well be called Composition, πεντήκοντ' ἀνδρῶν ἑκατὸν λίπε δῖος Ἀχιλλεύς, where the sophist charges you with saying that Achilles left 100 men out of 50 (*centum ex hominibus quinquaginta liquit divus Achilles*: the ambiguity cannot be reproduced in English)—*v. Soph. El.* iv. 166ᵃ 33–38. But one cannot wrongly combine certain words with these instead of those without also wrongly separating them from those instead of these. Note that in the numerical example in the text the ambiguity arises through understanding the same words of things separately or together, but is not reflected in an ambiguous grouping of words.

[2] Cf. the fancy in Plato's *Symposium*, 189 D E.

girls alike on the plea that neither of them were children of both sexes. Yet in the latter case there are no words that are wrongly taken together ; it is the sexes thought of, to which the showman pleaded that he had only promised to give free admission when combined. Words like *both* and *all*, which may have equally a distributive and a collective reference to the things signified by the substantives to which they belong, are specially adapted to facilitate this fallacy.[1] Another and a double example of the fallacy of Composition, in a business transaction, is afforded by the tale of a railway enterprise in one of the British Islands. A company is said to have been formed to build a railway, and to have announced in its prospectus that a guarantee of 3 per cent. on the share capital had been given by the Government, and a guarantee of 2 per cent. by the local authority ; and later in the same document to have stated that a guarantee of 5 per cent. had been given by the Government and by the local authority.

5. The fallacy of **Accent** meant to Aristotle one arising through the ambiguity of a word which has different meanings when differently accented. It was perhaps distinguished from Equivocation, because words differently accented are not strictly the same word. The Latin writers illustrate it in words which have different meanings when the quantity of a vowel is different ; e.g. ' omne malum est fugiendum, pomum est malum : ergo fugiendum '. The ambiguity is of course one which is more likely to occur in communication by writing than by speech.[2] In English, which does not distinguish words by tonic accent, the name is generally given to

[1] It illustrates again how much akin the different fallacies *in dictione* are, and how the same example may from different points of view be regarded as falling under different heads, that any one who likes can call the showman's trick, or others where words like *all* and *both* figure similarly, fallacies of Equivocation. Aristotle does not give any such instances under the head of *σύνθεσις* or *διαίρεσις* ; it has been however done by divers writers, and if we look to the nature of the thought involved, justly. And the fallacies in question might have been defined above as arising, when a conclusion is reached by taking those things together which we are only entitled to take separately, or vice versa ; for even where words are taken together or separately in one part of the argument, which were intended to be taken separately or together in the other, it is only as this leads to our so taking what they signify that fallacy results. But as this is reflected often in a definite combination and division of words, and as that fact probably led to the erection of these as particular species of fallacy based on ambiguous language, it seemed right to make express mention of such cases in describing them (cf. Crackenthorpe, *Logic*, ed. quart. p. 353, *cum quis ab iis coniunctis arguat, quae separatim vera sunt, non coniuncta*).

[2] Ar., *Soph. El.* iv. 166^{b} 1.

arguments that turn on a wrong *emphasis* of some particular word in a sentence; in which if the emphasis were placed differently, the meaning might be very different. The words of the Catechism in the 'Duty towards my Neighbour'—'to hurt no body by word nor deed'—have by laying stress on *body* been wrested to include the injunction to be kind to animals.[1]

6. The fallacy of **Figure of Speech** arises through the ambiguous force of some verbal inflexion, which is wrongly alleged to imply in one case what it really implies in others. If a man were to argue from the use of such an expression as 'I am resolved what to do', that, because the passive signifies not action but being acted on, as in 'I am beaten', 'I am praised', therefore a man's resolution is not his own free act, but the result of something done to him, he would be guilty of this fallacy. Arguments from linguistic usage of that sort are by no means uncommon or necessarily unsound: as that the object of sight is not a visual sensation, because you say that you feel a sensation, but no one would say that he felt a colour. In this case there is no ambiguous *inflexion*, which is what was held to constitute the differentia of the fallacy now under consideration. But let a man say that *important* is a negative notion, because *imperturbable* or *impenitent* is, and we have a case in point.[2] J. S. Mill in his *Utilitarianism*[3] affords an excellent example of this fallacy in a critical point of his argument. He is trying to prove that the chief good, or one thing desirable, is pleasure. 'The only proof', he says, 'capable of being given that an object is visible, is that people actually see it. The only proof that a sound is audible, is that people hear it: and so of the other sources of our experience. In like manner, I apprehend, the sole evidence it is possible to pro-

[1] This example was given me from personal recollection. Not unlike this fallacy, understood as consisting in basing on a wrong emphasis a conclusion not intended by the speaker or writer, is the error of inferring from the stress which a man lays on one element of a truth that he necessarily overlooks another. It might be said to be Hegel's conception of the progress of speculative thought, that it advances by emphasizing first one and then the other side of a contrast in such a way that the emphasis on one leads to overlooking the other: until a new conception is reached which unites the two. This indeed he considers inevitable in the development of philosophy. But many writers have been erroneously interpreted, because it was thought that when they insisted upon one aspect of a truth they intended to deny some other aspect. This error of interpretation however could hardly be classed with fallacies *in dictione*, since the misinterpretation does not arise through the doubtful stress-accentuation of particular words.

[2] A lady once observed: 'The question is, is he a postor or an impostor?'

[3] p. 52 (Routledge's ed., 'New Universal Library', p. 66).

duce that anything is desirable, is that people do actually desire it.' But *visible*, *audible* mean what *can* be seen or heard ; whereas Mill is trying to prove that happiness *ought to be* desired, or is the thing *worth* desiring. Yet the termination *-able* or *-ible* must be taken to have the same force in the word *desirable* as in *audible* or *visible*, if the argument is to have any force at all ; and the only thing shown is really that men *can* desire happiness : which was never in question.

To distinguish the different sources of the ambiguity in the different fallacies enumerated above is not a matter of first-rate importance ; but to be alive to the errors into which ambiguities of language may lead us is so. , ' Verba plane vim faciunt intellectui, et omnia turbant,' wrote Bacon.[1] Perhaps the disturbance which they caused was in some respects more serious of old than now. We do not suffer less from the subtle and unconscious shifting of the meaning of important terms in a sustained argument ; but some of the more trivial and (as we should say) obvious ambiguities may have been a more real puzzle in olden days. ' The genius of uncultivated nations ', says de Morgan,[2] ' leads them to place undue force in the verbal meaning of engagements and admissions, independently of the understanding with which they are made. Jacob kept the blessing which he obtained by a trick, though it was intended for Esau : Lycurgus seems to have fairly bound the Spartans to follow his laws till he returned, though he only intimated a short absence, and made it eternal : and the Hindoo god who begged for three steps of land in the shape of a dwarf, and took earth, sea and sky in that of a giant, seems to have been held as claiming no more than was granted. The great stress laid by Aristotle on so many

[1] *Nov. Org.* I. 43. The false ideas about nature generated through language Bacon called *idola fori*. These false ideas or *idola* were classified by him according as they had their sources in universal properties of human nature, in idiosyncrasies of the individual, in language, or in false theories of science and philosophy. The division was not logically perfect, and the enumeration in each group is doubtless not complete. This illustrates in a parallel field the difficulties above acknowledged to render a perfect classification of fallacies impracticable. Bacon himself calls attention to the parallel that exists between his undertaking and a classification of fallacies : ' *Doctrina enim de* idolis *similiter se habet ad* interpretationem naturae, *sicut doctrina de sophisticis elenchis ad dialecticam vulgarem* ' (I. 40). The ' interpretation of nature ' involved more than reasoning ; it required the use of the senses in observation, the recording of facts, the formation of conceptions, or hypothesis, the invention of a nomenclature, &c. There are obstacles in the way of the successful performance of these operations, no less than of reasoning. The fallacies of the common Logic waylay us in the work of reasoning. His *idola* arise from circumstances that waylay us in all these tasks.

[2] *Formal Logic*, p. 244.

forms of verbal deception may have arisen from a remaining tendency among disputants to be very serious about what we should now call play upon words.' Just as many people tend to think that in conduct the claims of veracity are satisfied or broken, according as the facts can or cannot, by some verbal quibble, be brought within the four corners of what they said or promised, so with argument men may think that there is something in it, though the conclusion turns upon an ambiguity of language. Not but what men are often also too ready to assume that a controversy is merely verbal when it is not.

In the enumeration of the fallacies which he recognizes, Aristotle obviously had before him the practices of disputants in his own day.[1] One man, the 'respondent', undertook to defend a thesis; the other, the 'questioner', attempted to extract admissions from the respondent which involved the contradiction of his thesis. But we find that a man might endeavour to discredit his opponent by confuting him on a side issue; and that it was a recognized device to get him to admit something easier to attack than his original thesis; though when Aristotle wrote, men had learned to reply to the entrapping question by asking what it had to do with the original thesis.[2] Similarly we are told that answers in the form of a plain yes or no were less insisted on when he wrote than formerly; whereby a bountiful source of unfair confutations was cut off.[3] The questioner is advised also not only to endeavour to involve the respondent in a contradiction of his own thesis, but to bring out its inconsistency with what is held by those whose authority he or others may respect, or by mankind at large, or by the majority of mankind, or by his own school.[4] Nowadays formal disputation has gone out of fashion. Men still harangue; and we understand by a debate a series of set speeches, in which a proposal is attacked and defended. Many of the devices which can be employed to produce the appearance of confuting an adversary are common to rhetoric and dialectic—to the harangue and to the interchange of question and answer. But if we were more familiar with the latter

[1] Minto, in the first chapter of his *Logic, Inductive and Deductive*, speaks as if Aristotle worked out his system of logic as a whole chiefly with the conduct of disputation in view. He seems to me to have very much overstated his case; but so far as the treatise on Sophistical Confutations is concerned, it is true.

[2] *Soph. El.* xii. 172^{b} 16–24.

[3] Ib. 175^{b} 8–10. Cf. on the fallacy of Many Questions, p. 597, *infra*.

[4] Ib. xv. 174^{b} 19–23.

mode of trying an issue, we should perhaps understand better the scope that exists for some of the sophistical confutations that Aristotle mentions. Such disputation is seen chiefly to-day in courts of law, when counsel cross-examines a witness; and an unscrupulous counsel can still confuse a timid witness, and discredit him before the jury, by involving him in contradictions more apparent than real. And there have been times when matters, which to-day are submitted to the judgement of the public by means of speeches to and fro, reported in the newspapers, were argued by chosen disputants according to fixed rules of debate before an audience whose verdict, as to which side got the best of the discussion, was of high practical importance. Not a few controversies of that sort were argued during the Reformation, at Leipsic or at Marburg or at Zurich or elsewhere.

The fallacies *in dictione* have to some extent become of less importance through the decay of the habit of disputation. The same cannot be said of those *extra dictionem*.[1] These are not united by any common character, as the others were by springing from ambiguity in language.

1. The first in the list is the fallacy of **Accident**. The following are some of the examples referred by Aristotle to this head: 'This dog is yours: this dog is a father: therefore he is your father.' 'Do you know Coriscus? Yes. Do you know the man approaching you with his face muffled? No. But he is Coriscus, and you said you knew him.' 'Six is few: and thirty-six is six times six: therefore thirty-six is few.' His solution of the error involved seems to be this. A subject has divers accidental predicates, i.e. predicates indicating attributes which are not commensurate with it nor essential to it; what is predicable of the subject may or may not be predicable of these accidents, and vice versa.[2] Thus the dog is a father, and is yours; but it does not follow that the father is yours—that he is yours as a father, as he is yours as a dog. Coriscus is approaching with his face muffled; to be a man approaching with his face muffled is an accident of Coriscus; and it does not follow that, because Coriscus is known, a man approaching with his face muffled is known to you. It is an accidental way of regarding thirty-six things, that they are six groups of six things; and though the groups are few, the thirty-six are not therefore few. The defect of the solution offered is, that

[1] Except perhaps 'Many Questions'; but cf. *infra*, p. 598.

[2] *Soph. El.* v. 166^{b} 30–32, xxiv. 179^{a} 27–31.

it does not enable us to distinguish between those cases in which what is predicated of the accidents of a subject may be predicated of the subject itself, or vice versa, and those in which it may not. 'This dog is yours, and this dog is property (or, a spaniel): therefore he is your property (or, your spaniel)': why is this argument valid and the former one not? If you say that the former is invalid because it equates subject and accident[1] when they are incommensurate, why do you allow the latter, which does so just as much? A term and its definition may be equated: they are interchangeable, and wherever one occurs in a proposition you may substitute the other without detriment to its truth. But you cannot extend that rule to terms which have any less close relation; for these you may be led into error by such substitution or you may not; the rule would not be infallible.

We learn from Aristotle himself that other solutions than what he formulated were offered for some of the fallacies referred by him to the head of Accident[2]; and as Poste says[3], 'the fallacy per accidens has been generally misunderstood.' It has been very commonly expounded in a way that does not really distinguish it from the fallacy next to be considered, *Secundum Quid*. Indeed what has happened is that the notion of the former has been dropped, being somewhat ill defined, and the name of the latter, being somewhat clumsy; so that what to-day is commonly called *Accident* is what the Aristotelian tradition called *Secundum Quid*. But because the tradition recognized them as two, a distinction between the *direct* and the *converse* form of the latter fallacy was drawn, which is really quite unsubstantial.

[1] The phrase is from Poste's ed. of *Soph. El.* (v. p. 73): cf. esp. his remarks on p. 158, from which the above interpretation and criticism are borrowed. Cf. also H. Maier, *Die Syllogistik des Aristoteles*, 2. Teil, 2. Hälfte, pp. 280, 288–291. It should be observed that the subject and predicate, in whose equation the error lies, are not terms of thought, but terms verbal (cf. *supra*, p. 21). No one, e.g., would suppose that for a substance could be substituted one of its attributes: that being a father could take the place of a dog, and wag a tail or bite. But a man might suppose that in a proposition one term could take the place of another, when each is predicable of the same subject-term: and so might proceed to enunciate a new proposition which seemed as if it ought to follow from the others, though what is meant by it was not implicated in what they mean. Because it is true to say of a certain dog that he is your dog, and also that he is a spaniel, or property, or a father, therefore it might seem equally true to say of him that he is your spaniel, or your property, and that he is your father. Thus this fallacy, though not traceable to the ambiguity of definite words, is not independent of the part which language plays in the conduct and expression of thought.

[2] *Soph. El.* xxiv. [3] *Op. cit.* p. 158.

2. The fallacy of **Secundum Quid**, or—to give the formula in full—*A dicto simpliciter ad dictum secundum quid* (from which the argument *a dicto secundum quid ad dictum simpliciter* is sometimes distinguished as its converse [1]), is one of the subtlest and commonest sources of error. It consists in using a principle or proposition without regard to the circumstances which modify its applicability in the case or kind of case before us.[2] Water boils at a temperature of 212° Fahrenheit; therefore boiling water will be hot enough to cook an egg hard in five minutes: but if we argue thus at an altitude of 5,000 feet, we shall be disappointed; for the height, through the difference in the pressure of the air, qualifies the truth of our general principle. A proposition may be stated *simpliciter*, or without qualification, either because the conditions which restrict its truth are unknown, or because, though known, they are thought seldom to arise, and so are neglected; and we may proceed to apply it where, had it been qualified as the truth required, it would be seen to be inapplicable. Perhaps it holds good normally, or in any circumstances contemplated by the speaker; the unfair confutation lies in taking advantage of his statement to bring under it a case which, had he thought of it, would have led him to qualify the statement at the outset. But it is not only in disputation that the fallacy occurs. We are all of us at times guilty of it; we argue from principles that hold good normally, without even settling what conditions constitute the normal, or satisfying ourselves that they are present in the case about which we are arguing. Freedom is good, and therefore it is supposed that every community should have free institutions, though perhaps there are some races only fit for a very moderate degree of

[1] No real distinction can be made here. It is sometimes said that in the direct fallacy we argue from a general rule to a special case, in the converse from a special case to a general rule. But the former is not fallacious unless because the rule is applied in a sense in which it is not strictly true; and the latter, if it misleads, is erroneous generalization, which is by no means the converse of *Secundum Quid*. We may distinguish between applying a rule to a case to which it is inapplicable because of the presence of conditions whereby the rule was not qualified, and applying it to one where it is inapplicable because of the absence of conditions whereby it was qualified; but the latter error is hardly likely to deceive, and if it did, it would do so rather by suggesting a false statement of particular fact. To argue that because wine is pernicious its use should be forbidden may be plausible (cf. de Morgan, *Formal Logic*, p. 251); but to argue that because wine is pernicious in excess, *X* ought not to drink it, is hardly plausible, unless it is taken to be meant that *X* cannot drink wine without drinking to excess. If that is true, there is no fallacy; if it is false, the fallacy is not antithetical to the other.

[2] Cf. Dicey, *Law and Opinion in England*, p. 487, on the extension of principles to fresh cases in 'judge-made law': also Ar. *Eth. Nic.* e. x. 4, 1137^{b} 14–19.

'freedom'. A man should be allowed to do what he will with his own; and that is often urged as a conclusive argument against any interference either with his disposition of his property, or his education of his children. Paris did nothing wrong in carrying off Helen, for her father left her free to choose her husband; but the freedom allowed her extended only to her first choice, like the authority of her father.[1] There are trivial examples of this as of any other fallacy, as that if it be maintained that an Ethiopian is black, it is contradictory to say he has white teeth.[2] But there is no fallacy more insidious than that of treating a statement which in many connexions is not misleading as if it were true always and without qualification.

3. **Ignoratio Elenchi** means proving another conclusion than what is wanted. The name does not literally mean that, but 'ignorance of confutation'. But the business of any one undertaking to confute a statement is to prove the contradictory; and if I prove anything else, I show that I do not know what confutation requires. Of course every fallacious confutation shows that I am ignorant of, or ignore, what is required.[3] But other fallacies have other defects; in this, the argumentation may be perfectly sound, and the sole defect lie in the fact that the conclusion proved does not confute the thesis maintained. Or—since it makes no difference whether we regard a man as undertaking to confute one thesis or to sustain another contradictory to it—we may say that the fallacy lies in proving what is not the precise conclusion which we are called upon to prove. Against a minister who proposes to put a small duty on corn to-day it is no sufficient answer to prove that the people are much more prosperous under free trade than in the days when corn stood at 60 or 80 shillings a quarter; against a free-trader it is no sufficient answer to prove that foreign nations injure us by their tariffs. Subterfuges of that kind are however so frequent a resource of the orator, that it is hardly necessary to illustrate them. Every reader of Plato's *Apology* will remember how Socrates refused to appeal to his judges with tears and entreaties, or to bring his wife and children into court to excite their commiseration; for his part was to persuade them, if he could do it, of his innocence and not of his sufferings.[4]

Such appeals as Socrates declined to make are sometimes called *argumenta ad misericordiam*, arguments addressed to show that a

[1] Ar., *Rhet.* β. xxiv. 1401[b] 34, quoted Poste, *op. cit.* p. 117.
[2] *Soph. El.* v. 167[a] 11.
[3] Cf. *Soph. El.* vi. 168[a] 17 sq.
[4] *Apol.* 34 C, 35 B C.

man is unfortunate and deserves pity, when it ought to be shown that he is innocent, or has the law on his side. Other favourite forms of irrelevant conclusion have also received special names. The best known is the *argumentum ad hominem*, in which, being called upon to confute an allegation, I prove something instead about the person who maintains it. The politician who attacks an opponent's measures by showing that they are inconsistent with his former opinions commits this fallacy; it is the same if I condemn Home Rule for Ireland on the ground that Parnell was an adulterer. But the *argumentum ad hominem* need not be altogether irrelevant. A barrister who meets the testimony of a hostile witness by proving that the witness is a notorious thief, though he does less well than if he could disprove his evidence directly, may reasonably be considered to have shaken it; for a man's character bears on his credibility. And sometimes we may be content to prove against those who attack us, not that our conduct is right, but that it accords with the principles which they profess or act upon. Christ replied to those who censured him for healing on the Sabbath, by asking which of them, if his ox or his ass had fallen into a ditch, would not pull it out on the Sabbath day.[1] Their practice was sufficient to justify him to them, whatever were the true theory of our duties on the Sabbath. And Aristotle answers the Platonists, who held all vice to be involuntary, by showing that they could not discriminate in that respect between vice and virtue; there was no more reason for calling one involuntary than the other; virtue, however, they called voluntary; and whatever be the true state of the case, *their* position at least was not sustainable.[2]

4. The nature of **Petitio Principii** is better expressed in the English name, **Begging the Question**.[3] It consists in assuming

[1] Luke xiv. 1–6.

[2] *Eth. Nic.* γ. vii. 1114ª 31–ᵇ25. In the game of disputation, we may be held to score a victory if we force an opponent to an admission inconsistent with the thesis he propounded. But in the search for truth, to convict any one of inconsistency is irrelevant; we have to determine what is true.

[3] Gk. *τὸ ἐν ἀρχῇ λαμβάνειν, τὸ ἐξ ἀρχῆς αἰτεῖσθαι*, to assume or claim the admission of the very thing propounded for debate at the outset—the *πρόβλημα*. The word *petitio* belongs to the terminology of disputation, where the questioner *sought* his premisses in the admissions of the respondent. He had no right to ask the respondent to admit the direct contradictory of his thesis; let the thesis, for instance, be that the Pope cannot remit the temporal punishment of sin in Purgatory: the opponent may not ask the respondent to admit that he can. If by some verbal disguise he gets the respondent to admit it, it is only a sophistical confutation; the respondent did not see what he was granting, and would have refused to grant it if he had seen—not because it

what is to be proved, in order to prove it. To do this within the compass of a single syllogism—assuming in the premisses the very thing to be proved, and not merely some thing which depends on that for its proof—is only possible by the use of synonyms. If I argue that *C* is *A* because *B* is *A* and *C* is *B*, and if the middle term *B* is identical either with the major or the minor, then I use the proposition to prove itself; for let *B* be the same as *A*: then, by substituting *A* for *B* in the minor premiss, I get '*C* is *A*' as a premiss; or let *B* be the same as *C*: then by substituting *C* for *B* in the major premiss, I again get '*C* is *A*' as a premiss; and in either case therefore the conclusion is among the premisses. Thus let the syllogism be that to give to beggars is right, because it is a duty to be charitable; so far as charity is taken to include giving to beggars, we have no business to assume that it is a duty; for the question whether it is a duty and the question whether it is right are the same question: to call it a duty is to call it right. Here the major premiss, that duty is right, is a tautology, and the minor contains the *petitio*. On the other hand, if I defend legacy duties by saying that property passing by will ought to be taxed, I beg the question in the major; for a legacy duty is a tax on property passing by will, and to say that such property should be taxed is only to assert in other words the justice of a legacy duty.[1]

But the fallacy is generally committed less abruptly. The premiss unduly assumed is generally not the conclusion itself differently expressed, but something which can only be proved by means of the conclusion; and arguing thus is often called *arguing in a circle*. If I argued that early Teutonic societies were originally held together by kinship, because all societies were so held together originally,[2]

led to the contradictory of his thesis, for a man is often fairly refuted by showing that he cannot reasonably deny something which does that: but because it *was* the contradictory of it. It is quite fair to try to get a man to admit a general principle, and then to show that his thesis is inconsistent with it, *provided* that the general principle does not really require the disproof of his thesis in order to its own establishment. Hence the term *principium* is a mistranslation. The fallacy lies in *begging* for the admission not of a principle to be applied to the determination of the matter, but of the very matter, in *question*. As occurring in a book or speech, where a man puts forward his own premisses, and has not to get them by the admission of a respondent, it consists in *assuming* among the premisses either the conclusion itself which a show is made of proving, or something more or less directly depending thereon. Cf. Mansel's ed. of Aldrich's *Artis Logicae Rudimenta*, App. E.

[1] It is also possible to beg the question when the conclusion is negative, but then only in the major premiss; and to beg it in other figures than the first (for details see Poste, *Soph. El.*, App. A). Cf. also *supra*, p. 578, n. 1.

[2] For the general statement see Sir Henry Maine, *Early Institutions*, p. 64.

I might be accused of arguing in a circle ; for the major premiss, it might be said, is only arrived at by enumeration ; early Teutonic societies have to be examined in order to show that it is true. Of course to show that the generalization was not enumerative would be to rebut the accusation ; but, as we saw in discussing the view that all syllogism is *petitio principii*, every syllogism whose major premiss is an enumerative judgement is so.[1] The circle is fairly manifest in such cases ; but in others it may often escape the notice of its author. 'There are certain people', says Dr. McTaggart,[2] 'who look on all punishment as essentially degrading. They do not, in their saner moods, deny that there may be cases in which it is necessary. But they think, if any one requires punishment, he proves himself to be uninfluenced by moral motives, and only to be governed by fear. . . . They look on all punishment as implying deep degradation in some one,—if it is justified, the offender must be little better than a brute ; if it is not justified, the brutality is in the person who inflicts it. This reasoning appears to travel in a circle. Punishment, they say, is degrading, therefore it can work no moral improvement. But this begs the question. For if punishment could work a moral improvement, it would not degrade but elevate. The humanitarian argument alternately proves that punishment can only intimidate because it is brutalizing, and that it is brutalizing because it can only intimidate.' Romanes finds an example of *petitio* in an argument of Huxley's, adduced to show that all specific characters are adaptive.[3] 'Every variety which is selected into a species is favoured and preserved in consequence of being, in some one or more respects, better adapted to its surroundings than its rivals. In other words, every species which exists, exists in virtue of adaptation, and whatever accounts for that adaptation accounts for the existence of the species.' Here the fallacy lies in substituting, for 'every variety which is selected', 'every species which exists' ; the statement in the first clause is true for every variety which is selected, since selection means the survival of those best adapted to the conditions of life. But the question is whether every species which exists has originated by 'selection'. One more instance may be cited, from a work on the squaring of the

[1] pp. 304, 305, *supra*.

[2] *Studies in Hegelian Cosmology*, § 142. By punishment here is meant 'the infliction of pain on a person because he has done wrong' (§ 137). And it is of corporal punishment that we most often hear this view expressed.

[3] *Darwin and after Darwin*, ii. 307.

circle, called *The Nut to Crack*, by James Smith.[1] Smith held the ratio of circumference to diameter to be $3\frac{1}{8}$, and proved it thus: 'I think you will not dare to dispute my right to this hypothesis, when I can prove by means of it that every other value of π will lead to the grossest absurdities; unless indeed you are prepared to dispute the right of Euclid to adopt a false line hypothetically, for the purpose of a *reductio ad absurdum* demonstration, in pure geometry.' That is, he argued first that if $3\frac{1}{8}$ be the right ratio, all other ratios are wrong; and then, that because all other ratios are wrong, $3\frac{1}{8}$ is the right ratio. And he conceived that he had established his conclusion by a *reductio ad absurdum*—by showing that the denial of his thesis led to absurdity. But the absurdity, in such an argument, ought to be ascertained independently, whereas here it rests upon the assumption of the truth of what it is used to prove.

5. The fallacy of **False Cause** is incident to the *reductio ad absurdum*. That argument disproves a thesis by showing that the assumption of its truth leads to absurd or impossible consequences, or proves one by showing the same for the assumption of its falsity.[2] In *False Cause*, the thesis alleged to be discredited is not really responsible for the absurd or impossible consequences, which would follow equally from the other premisses, whether that were affirmed or denied. 'It is ridiculous to suppose that the world can be flat; for a flat world would be infinite, and an infinite world could not be circumnavigated, as this has been.' Here the supposition inconsistent with the fact of the circumnavigation of the world is not that the world is flat, but that it is infinite; it might be flat and still circumnavigable, if it were finite; the thesis of its flatness is therefore unfairly discredited.

From a passage in the *Prior Analytics* it would seem that Aristotle regarded this fallacy as of frequent occurrence.[3] But the fact that later writers have largely given a different meaning to the name suggests that it is not really a prominent type. It is often identified with the fallacy *Post hoc, ergo propter hoc*: i.e., supposing that one event is due to another, merely because it occurred after

[1] Cf. de Morgan, *Budget of Paradoxes*, p. 327.

[2] James Smith argued, not that 'if A is false, B will be true: but B is false, $\therefore$ A is true'; but 'if A is true, B will be false—(as to which nothing was known)—$\therefore$ A is true'.

[3] *Anal. Pri.* β. xvii. 65^a 38 τὸ δὲ μὴ παρὰ τοῦτο συμβαίνειν τὸ ψεῦδος, ὃ πολλάκις ἐν τοῖς λόγοις εἰώθαμεν λέγειν, κτλ., 'that the false statement does not arise from the premiss alleged, as we are accustomed often to say in argument, &c.' Cf. Poste's *Soph. El.*, App. B, on this passage.

it ; as the countryman is said to have declared that the building of Tenterden Steeple was the cause of Goodwin Sands, because the sands only appeared after the steeple was built. Such, as Bacon truly says, is the origin of almost every superstition—of men's astrological fancies, and their fancies about omens or dreams. The story which he quotes may well be repeated in his own words. 'Itaque recte respondit ille, qui, cum suspensa tabula in templo ei monstraretur eorum qui vota solverant, quod naufragii periculo elapsi sint, atque interrogando premeretur, anne tum quidem deorum numen agnosceret, quaesivit denuo, *At ubi sunt illi depicti qui post vota nuncupata perierint ?*'[1]

Inferences of this kind are undoubtedly both frequent and fallacious ; and *Post hoc, propter hoc* is a definite type or *locus* of fallacy. That is, it is a general or dialectical principle—a principle applicable in divers sciences, and not exclusively appropriate in one : and it is a false principle, the application of which is as likely to lead to error as to truth. Nor is it peculiar to this fallacy, that it can be expressed as a false principle. *Equivocation* proceeds on the false principle that a word is always used with the same meaning : *Accident*, on the principle that a term and its predicate are interchangeable : *Secundum Quid*, on the principle that what is true with certain qualifications is also true without them. And the fact that these different types of fallacious inference severally depend on a false, or misleading, principle is what was meant by calling them *loci* of fallacy.[2] But the locus *Post hoc, propter hoc* is not quite the same as that of *Non causa pro causa* : in other words, the type is a little different. In *False Cause* we are dealing with the logical sequence of premisses and conclusion ; the fallacy lies in connecting

[1] *Nov. Org.* I. 46. Bacon cites the story in illustration of one of the 'Idola Tribus', the tendency to overlook or despise facts which do not agree with an opinion which we have once adopted. J. S. Mill would call this the fallacy of Non-observation (*System of Logic*, V. iv). The meaning *Post hoc, propter hoc* does occur in Aristotle, *Rhet.* β. xxiv. 1401[b] 29-34 ἄλλος παρὰ τὸ ἀναίτιον ὡς αἴτιον, οἷον τῷ ἅμα ἢ μετὰ τοῦτο γεγονέναι· τὸ γὰρ μετὰ τοῦτο ὡς διὰ τοῦτο λαμβάνουσι, καὶ μάλιστα οἱ ἐν ταῖς πολιτείαις, οἷον ὡς ὁ Δημάδης τὴν Δημοσθένους πολιτείαν πάντων τῶν κακῶν αἰτίαν· μετ' ἐκείνην γὰρ συνέβη ὁ πόλεμος ('Another *locus* of fallacy is through taking what is not the cause as cause, because one thing has happened together with or after the other ; for what arises after something is taken as arising through it, especially in political argument, as Demades for example said that the policy of Demosthenes was the cause of all their ills ; for after it came the war').

[2] The *Sophistici Elenchi* is the concluding book of Aristotle's *Topics*. The false principle is *exemplified* in the fallacious argument; it is not one of the premisses of the argument.

the conclusion with a particular premiss which might, so far as getting the conclusion is concerned, have been equally well included or omitted; and because the conclusion is false, we erroneously infer this premiss to be false also. In *Post hoc, ergo propter hoc* we are dealing with the temporal relation of cause and effect; the fallacy lies in connecting the effect with a particular event which might equally well have happened or not happened, so far as the effect in question is concerned; and we erroneously suppose that the effect, which did occur, occurred because of that event. But if any one likes to use the name *False Cause* as equivalent to *Post hoc, propter hoc*, there is not much harm done; for the fallacy which in the *Sophistici Elenchi* Aristotle describes under the name is not one that we have much occasion to speak of.

6. It is otherwise with the fallacy of the **Consequent**, which some modern writers have also misunderstood.[1] For this is one of the very commonest, and we have already had occasion to notice it in discussing inductive reasoning.[2] It consists in supposing that a condition and its consequent are convertible: that you may argue from the consequent to the condition, no less than vice versa. If a religion can elevate the soul, it can survive persecution: hence it is argued that because it has survived persecution, such and such a religion must elevate the soul; or perhaps (for we may follow Aristotle[3] in including under the name both the forms of fallacy

[1] e.g. de Morgan, *Formal Logic*, p. 267; Jevons, *Elementary Lessons in Logic*, p. 181.

[2] p. 523, *supra*.

[3] Cf. *Soph. El.* xxviii. 181[a] 27 παρ' ὃ καὶ ὁ τοῦ Μελίσσου λόγος· εἰ γὰρ τὸ γεγονὸς ἔχει ἀρχήν, τὸ ἀγένητον ἀξιοῖ μὴ ἔχειν, ὥστ' εἰ ἀγένητος ὁ οὐρανός, καὶ ἄπειρος. τὸ δ' οὐκ ἔστιν· ἀνάπαλιν γὰρ ἡ ἀκολούθησις ('with this accords the argument of Melissus; for he thinks that if what is generated has a beginning, what is ungenerated has not; so that if the heaven is ungenerated, it is also infinite. But this is not so; for the sequence is the other way'); i. e. from '*A* is *B*' you cannot infer 'not-*A* is not-*B*', but only contrariwise, 'not-*B* is not-*A*'. It appears by the same chapter that Aristotle would bring the illicit, viz. simple, conversion of an universal affirmative judgement under the same heading. This illustrates the close parallelism between the *modi ponens* and *tollens* in hypothetical, and *Barbara* and *Camestres* in syllogistic reasoning (cf. pp. 339–342, *supra*). But that Aristotle did not identify them might perhaps be inferred from the fact that he does not include Undistributed Middle and Illicit Process of the Major in his list of sophistical confutations, while he does include, under the name of the fallacy of the Consequent, the corresponding though not identical errors which may be committed in hypothetical reasoning. It may be noted that such inferences would only not be fallacious where condition and consequent reciprocated—a relation which corresponds to that of commensurate terms in an universal affirmative judgement. Hence Aristotle says that the fallacy of the Consequent is

to which hypothetical reasoning is liable) that because it is incapable of elevating the soul, it will succumb to persecution. Such fallacies are committed whenever a theory is assumed to be true for no better reason than that the facts exist, which should exist if it were true—i. e. whenever verification is mistaken for proof [1]; and whenever the refutation of an argument advanced in support of a theory is supposed by itself to be fatal to the theory. If it can be shown that no other theory accounts for the facts, or that no other argument can be advanced in support of the theory, then the matter is different; but without some reason to believe this, such inferences are worth nothing. Nevertheless, they are inferences which we are all very apt to make.[2]

7. There remains lastly the fallacy of **Many Questions.** This consists in putting questions in such a form that any single answer involves more than one admission. If one admission be true and another false, and the respondent is pressed for a single answer, he is exposed to the risk of confutation, whatever answer he makes. 'The execution of Mary Queen of Scots was brutal and sacrilegious —was it, or was it not?' If it was brutal but not sacrilegious, what is a man to answer? He will be accused by saying no of denying the brutality, by saying yes of affirming the sacrilege. Sometimes, instead of submitting two problems for decision together, the question appears to submit only one; but that is one which would not arise except on the assumption of a certain answer to another: and so the respondent again cannot answer it without committing himself to more than he intended, or on a matter which has not been definitely submitted to him. Of this sort is the famous enquiry, 'Have you left off beating your mother?', as well as any question that asks for the reason of what has not been admitted to be true. It is often recounted how Charles II asked the members of the Royal Society why a live fish placed in a bowl already full of water did not cause it to overflow, whereas a dead fish did so; and how they gave various ingenious reasons for a difference which did not exist. If

a case of that of Accident (*Soph. El.* vi. 168^b 27). Under it in turn might be brought *Post hoc, propter hoc.* If Goodwin Sands were caused by building Tenterden Steeple, they would have appeared, as they did, so soon as the steeple was built; but they might equally have done so, if the building of the steeple had nothing to do with their appearance.

[1] Cf. p. 523, *supra.*

[2] This fallacy is 'logical', or formal; it can be expressed in symbols. So can an argument in a circle sometimes be; e. g. if it is of the form '*A* is *B*, *B* is *C* ∴ *A* is *C*: and *B* is *C* because *A* is *C* and *B* is *A*'.

one were to enquire why a protective system encourages the industry of the country which adopts it, or how dowsers are made aware by their feelings of the presence of subterranean waters, the fallacy would be the same. It may be said that a respondent is always able to give an answer which will save him from any misconstruction; to the question 'Have you left off beating your mother?' the answer 'no' might seem to be an admission of the practice; but why should not a man reply 'I never began it'? To this it may be rejoined, first, that in the old disputations, and in some situations, such as the witness-box, to-day, a man might be more or less precluded from 'explaining himself', and required to give a 'plain answer' to a question which does not admit of it. With the use of the fallacy under this sort of duress may be compared the custom of 'tacking' in the American legislature. The President of the United States can veto bills, and does veto them freely; but he can only veto a bill as a whole. It is therefore not uncommon for the legislature to tack on to a bill which the President feels bound to let pass a clause containing a measure to which it is known that he objects; so that if he assents, he allows what he disapproves of, and if he dissents, he disallows what he approves.[1] But secondly, even where no unfair duress is employed, the practice of presupposing a certain answer to one question in the form of putting another throws the respondent off his guard, and makes him apt to admit without considering it what, if it had been explicitly submitted to his consideration, he might have doubted or denied.

The fallacy therefore is not a trivial one; such questions are a real source of error, when we put them to ourselves: of unfair confutation, when we put them to others. But it is doubtful whether it is a fallacy *extra dictionem*. For when a question is so put as that it must be answered by *yes* or *no*, and misconception is unavoidable on either answer, the error arises from the way in which the question is worded; and the same may be said of the acquiescence in false assumptions, into which in other cases we are entrapped.

The foregoing remarks have been directed to explain what are the types of fallacy which have been traditionally distinguished, and are still many of them very commonly referred to by their traditional names. The types are not all equally distinct, frequent, or important; but the original meaning of each name has been given as far as pos-

[1] Bryce's *American Commonwealth*, Part I, c. xx: vol. i, p. 214.

sible, because nothing but misunderstanding can result when different writers employ such terminology each in his own meaning, and there did not for the most part seem sufficient reason to prefer any later interpretation for a standard. In a few cases later interpretations which have much to be said for them have been given as well. No doubt Fallacy is a subject on which successive generations to some extent need new treatises: not because the principles change, but because the fields change in which they are most prolific. Many suggestive illustrations of the dominion which fallacy holds in important subjects of modern thought may be found in the pages of Whately, Mill, or de Morgan, to which reference has already several times been made.

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